

ERRATA and IMPROVEMENTS

The columns titled **Aut-hor** contain abbreviations of the names of the people who have graciously pointed out the error following. They are:

- RB** ▷ **Robert B. Burckel**, burckel@math.ksu.edu
MR ▷ **Mike Revesz**, revesz@math.utexas.edu
SB ▷ **Sterling Berberian**, berb@math.utexas.edu
PC ▷ **Piotr T. Chrusciel**, Piotr@gargan.Math.Univ-Tours.fr
MM ▷ **Marti Llauro Magrinya**, mllm1@wanadoo.es

Please send errors you found or suggestions to:

- KB** ▷ **Klaus Bichteler**, kbi@math.utexas.edu

Many of the errata pointed out came with suggested changes, for which I thank their authors. If a change below contains yet more mistakes, they are mine; I may not have followed the suggestion or may have made up a change of my own.

To ease reading the tables, plain falsehoods and errata that obscure the mathematics start in boldface and are highlighted like this.

Typos and slight improvements appear in unadorned fashion.

Chapter I

Pa-ge	Li-ne	Au-thor	Old Text	▷ New Text	Com-ment
6	-11	RB	▷ of functions	▷ of real valued functions	
8	-12	PC	▷ dense in \mathcal{E}	▷ dense in \mathcal{E} in the topology of uniform convergence	
9	3-4	RB	▷ \mathbb{R}^n	▷ \mathbb{R}^m (avoiding two meanings of n)	
9	19	RB	▷ to ϕ in the sense	▷ to $\bar{\phi}$ in the sense	
9	-9	RB	▷ is then	▷ is, then	
10	-6	RB	▷ and $t^2 - \phi_k \leq \epsilon^2 \dots$	▷ and $t^2 - \phi_k(t) \leq \epsilon^2 \dots$	
10	-4	RB	▷ $((k-1)\epsilon, k\epsilon)$	▷ $((k-1)\epsilon, (k+1)\epsilon)$	
13	1	RB	▷ vanish then	▷ vanish, then	
13	3	RB	▷ point $z \in S$ then	▷ point $z \in S$, then	
13	-15	RB	▷ $\{\phi_{s_i,t} : 1 \leq i \leq n\}$	▷ $\{\phi_{s_i,t} : 1 \leq i \leq n\}$	extraneous [
13	-3	RB	▷ interval S ,	▷ interval $S \stackrel{\text{def}}{=} [-M, M]$	
15	-4	MR	▷ pseudometric	▷ <i>pseudometric</i>	
166	13	RB	▷ For $\phi \in \mathcal{E}_+$	▷ For $\phi \in \mathcal{E}$	
166	17	RB	▷ vanishes on $\phi \geq 1$	▷ vanishes on $[\phi < 1]$	
166	17	RB	▷ $[(2\phi - 1)_+ \geq 1]$	▷ $[(2\phi - 1)_+ < 1]$	
166	23	RB	▷ $\alpha s^{\alpha-1} \wedge \delta^{(\alpha-1)/\alpha}$	▷ $(\alpha s^{\alpha-1}) \wedge \delta^{(\alpha-1)/\alpha}$	1
166	-3	RB	▷ $r_i \cdot e^{\alpha_i t}$	▷ $r_i \cdot e^{-\alpha_i t}$	
167	2	RB	▷ $e^{\alpha t}$	▷ $e^{-\alpha t}$	
167	11	RB	▷ after I.3.28 insert: We treat the case that \mathcal{E} is a ring.		2
20	15	RB	▷ A collections	▷ A collection	
20	-16	RB	▷ left-continuous,	▷ left-continuous with right limits,	3
20	-6	RB	▷ are	▷ is	
21	-16	RB	▷ $-\infty < \int^{\natural} f$	▷ $-\infty < \int^{\natural} f$	
166	-3	MR	▷ everyone	▷ every one	
26	-1	RB	▷ continous	▷ continuous	also on p.27 line 8
27	-7	RB	▷ $\sum_n b_n - a_n < \epsilon$	▷ $\sum_n (b_n - a_n) < \epsilon$	

Chapter I Continued

Pa-ge	Li-ne	Au-thor	Old Text	New Text	Com ment
27	-4	RB	< satisfies	> possesses	
27	-0	RB	< insert exercise 6.17		4
169	3	RB	< for	> For	

169 -20 KB < replace the text from (ii) Let ... to line -13 by:

(ii): The functions $\phi_n \stackrel{\text{def}}{=} \sum_{i=1}^n 2^{-i} (i-1, i] \in \mathcal{E}[\mathbb{R}]$ form a $\|\cdot\|^{\natural}$ -Cauchy sequence. Assume f is a $\|\cdot\|^{\natural}$ -norm limit. Then f cannot vanish outside $(-M, M]$, whatever $M \in \mathbb{N}$: if it did then $\psi_n \stackrel{\text{def}}{=} \phi_n \cdot (-M, M] \xrightarrow{n \rightarrow \infty} f \cdot (-M, M] = f$ in $\|\cdot\|^{\natural}$ -norm and $\|\phi_n - \psi_n\|^{\natural} \rightarrow 0$; yet $\int \phi_n - \psi_n \geq 2^{-M}$ for $n > M$: $f \notin \mathcal{F}^{\natural}$, so it is not Riemann integrable, a contradiction. Thus (ϕ_n) has no $\|\cdot\|^{\natural}$ -limit.

1 Actually, it does not matter for the argument whether the old text is read as $(\alpha s^{\alpha-1}) \wedge \delta^{(\alpha-1)/\alpha}$ (intended) or as $\alpha(s^{\alpha-1} \wedge \delta^{(\alpha-1)/\alpha})$. The same change should be made on page 23, lines -13, -10, -9.

2 The point of this change is to make sure that the “ \in ” of “Then $\phi_n \stackrel{\text{def}}{=} \phi'_n \cdot \sigma \cdot \rho_n \in \mathcal{E}$ ” in line 17 is justified. The exercise is rather simple if \mathcal{E} is a self-confining vector lattice closed under chopping: let $\mathcal{E} \ni \phi_n \rightarrow \bar{\phi} \in \bar{\mathcal{E}}_{00}$ uniformly. There exist $\psi \in \mathcal{E} = \mathcal{E}_{00}$ confining $\bar{\phi}$, without loss of generality $\psi > \sup_n \|\phi_n\|_{\infty}$ on $[\bar{\phi} \neq 0]$. Then $(-\psi) \vee \phi_n \wedge \psi$ belongs to \mathcal{E} , is confined by any function confining ψ , and converges uniformly to $\bar{\phi}$.

3 **Robert B. Burckel** gives the counterexample $\phi = \sum_{n \in \mathbb{N}} 1_{(\frac{1}{2n+1}, \frac{1}{2n}]}$ to the “if” part of the claim as stated.

4 Insert:

exercise 6.17 Show that if a function $f \in \mathcal{F}^{\natural}$ has $\|f\|^{\natural} = 0$, then the set $[f \neq 0]$ is Lebesgue-negligible. (Suggested by **Robert B. Burckel**)

Chapter II

Pa-ge	Li-ne	Au-thor	Old Text	New Text	Com ment
32	12	RB	< stays	> persists	
32	13	RB	< (a)	> (i)	
32	14	RB	< The theorem	> The putative theorem	
32	16	RB	< (b)	> (ii)	
34	3	RB	< decreasing	> positive	
34	-7	RB	< Equation (1.1)	> Inclusion (1.1)	
34	-2	RB	< $\epsilon \cdot (M + \ \psi_1\ _u)$	> $\epsilon \cdot (2M + \ \psi_1\ _u)$	
37	19	RB	< $K \subset S$ compact	> K a compact subset of S	
37	F ²	RB	< point set topology	> point-set topology	
171	5	RB	< $\mathcal{C}_n^1 = \mathcal{C}_n^2 = \mathcal{C}$	> $\mathcal{C}_n^1 \subseteq \mathcal{C}_n^2$	
39	17	RB	< sets in \mathcal{R}	> sets in \mathcal{R} , each contained in one of the A_i ,	
41	33	RB	< $\leq \ m_1\ \cdot \ m_1\ $	> $\leq \ m_1\ \cdot \ m_2\ $	
42	-8	RB	< take the value ∞	> take the value $+\infty$	
43	-12	RB	< $\{\phi_{1,k} + \phi_{2,k} : k \in \mathbb{N}\} \subset \mathcal{E}$	> $\{\phi_{1,k} + \phi_{2,l} : k, l \in \mathbb{N}\} \subset \mathcal{E}$	
44	1	RB	< By Lemma 1.1	> Due to the σ -additivity of \int (Lemma 1.1)	
44	2	RB	< $\lim_{n \rightarrow \infty} \int \hat{\phi}_n > r$.	> $\lim_{n \rightarrow \infty} \int \hat{\phi}_n = \int \phi > r$.	
44	8	RB	< $\int^* h + h' \dots \int^* \phi_n + \phi'_n$	> $\int^*(h + h') \dots \int^*(\phi_n + \phi'_n)$	

Chapter II Continued

Pa- ge	Li- ne	Au- thor	Old Text	▷ New Text	Com ment
45	10	KB	▷ p.19	▷ page 19	
45	15	RB	▷ of functions	▷ of real-valued functions	
45	19	RB	▷ realized that	▷ realized in Lemma 3.1 that the σ -additivity established in Lemma 1.1	
45	28	RB	▷ seminorm	▷ functional	
45	31	RB	▷ (f_n)	▷ (f_n) of real-valued functions $\boxed{\infty}$	
47	12	RB	▷ ring	▷ ring	
47	12	MR	▷ function h	▷ function $h \geq 0$	
47	16	RB	▷ add: [Hint: Reduce to the case that h has compact support and consider $h_n(x) \stackrel{\text{def}}{=} \inf\{h(y) + nd(x, y) : y \in S\}$]		
47	-12	RB	▷ extend the line:	▷ (Use equation I (4.3) to define f^{\natural} on all functions, keeping in mind the convention $\inf \emptyset = +\infty$ (page 4).)	
47	-5	RB	▷ all positive numerical	▷ all numerical	
49	1	KB	▷ A function	▷ A numerical function	
49	-2	RB	▷ Replace lines -2,-1 by		$\boxed{\infty}$
(iii): Let $A \stackrel{\text{def}}{=} [f' = f'']$. Then $\ f'\ ^* = \ Af' + A^c f'\ ^* \leq \ Af'\ ^* + \ A^c f'\ ^* = \ Af'\ ^* + 0 \leq \ f'\ ^*$, whence $\ f'\ ^* = \ Af'\ ^* = \ Af''\ ^* = \ f''\ ^*$.					
50	-11	MR	▷ defined.	▷ defined, and how to read $f \leq g$ a.e. etc. when f, g are merely a.e. defined.	
51	9	RB	▷ $\leq 2^{-k}$	▷ $\leq 2^{-k-1}$	
51	-10	RB	▷ $\ G \cdot \sum_{k=K}^{\infty} f'_{n_{k+1}} - f'_{n_k}\ ^*$	▷ $\ G \cdot \sum_{k=K}^{\infty} (f'_{n_{k+1}} - f'_{n_k})\ ^*$	
51	-2	RB	▷ $f_{n_k} \xrightarrow{n \rightarrow \infty} f$	▷ $f_{n_k} \xrightarrow{k \rightarrow \infty} f$	$n \rightarrow k$
53	15	RB	▷ $\ \phi_n\ ^\lambda = 2^{-n} \rightarrow 0$	▷ $\ \phi_n\ ^\lambda = 2^{-m} < 2/n \rightarrow 0$	
53	-9	RB	▷ . This	▷ . This is	
54	13	RB	▷ $\sup_K \ \sum_{k=1}^K f_{n_{k+1}} - f_{n_k}\ ^*$	▷ $\sup_K \ \sum_{k=1}^K (f_{n_{k+1}} - f_{n_k})\ ^*$	
54	15	RB	▷ (5.3) (iii)	▷ (5.3) (ii)	
55	6	RB	▷ Theorem 6.1	▷ Theorem I.6.3	
55	8	RB	▷ permance	▷ permanence	
55	16	RB	▷ $(f \wedge 1)_-$	▷ $-(f \wedge 1)_-$	
55	-5	MR	▷ and all	▷ and for all	
172	-19	RB	▷ $\phi_k \rightarrow h$	▷ $\psi_k \rightarrow h$	
172	-19	RB	▷ $\ h\ ^* \geq \sup \ \psi_k\ ^* = \infty$	▷ $\ h - \psi_1\ ^* \geq \sup_k \ \psi_k - \psi_1\ ^* = \infty$	
172	-10	RB	▷ $\lim_n \ g_N\ ^* = \lim \inf_n \ f_n\ ^*$	▷ $\lim_N \ g_N\ ^* \leq \lim \inf_n \ f_n\ ^* \quad n \rightarrow N$	
172	-2	RB	▷ mesurability	▷ measurability	
56	-9	RB	▷ Extend: \mathcal{L}^1 is a module over $\overline{\mathcal{E}}$ and over \mathcal{L}_b^1 .		
56	-6	RB	▷ Delete Exercise 5.12		$\boxed{1}$
58	-7	KB	▷ $ \int \phi_n - \phi'_n $	▷ $ \int (\phi_n - \phi'_n) $	
59	3	RB	▷ $\int f = \int^* f = \ f\ ^*$	▷ $\int f = \ f\ ^*$	delete: $\int^* f $
59	6	RB&MM	▷ the whole item should read as follows:		$\boxed{2}$
<i>(iv) Suppose the sequence (f_n) of integrable functions converges $\ \cdot\ ^*$-a.e. to f and $\ \sup f_n \ ^*$ is finite. Then $f_n \rightarrow f$ in mean, f is integrable, and $\int f_n \rightarrow \int f$.</i>					
59	-10	RB	▷ $\int f = \lim \int \phi_n$.	▷ $\int f = \lim \int \phi_n $.	
59	-6-5	RB	▷ (iii) ... \blacksquare	▷ (iii) and (iv) are evident from Theorem 5.6. \blacksquare	

Chapter II Continued

Page	Line	Author	Old Text	New Text	Comment
59	-2	KB	Replace this line by:		∞
<p>Assume \mathcal{E} is a vector lattice closed under chopping as well as a ring. (i) Define, in the manner of Definition 4.4, \int^* and \int_* for numerical functions that are merely $\ \ \ ^*$-a.e. defined, and show that the properties listed in Proposition 3.2 and Exercise 3.14 persist. (ii) Then show that an a.e. defined numerical function f is $(\mathcal{E}, \ \ \ ^*)$-integrable if and only if</p>					
60	1	KB	Delete: (see Exercise 3.14)		
60	2	RB	Insert: Exercise 6.2'	Assume that $\int = \ \ \ ^*$ on \mathcal{E}_+ . If (f_n) is an increasing or decreasing sequence of integrable functions and the sequence $(\int f_n)$ is bounded then the pointwise limit $f \stackrel{\text{def}}{=} \lim f_n$ is integrable and $f_n \rightarrow f$ in mean. [Hint: Apply 6.1 (ii) and the MCT to $(f_n - f_1)$ or $(f_1 - f_n)$.]	
173	6	RB	after II.6.2	insert the following lines:	∞
<p>(i): Set $\tilde{f} = \hat{f} = f$ on the set D where f is defined, $\tilde{f} = -\infty$ and $\hat{f} = +\infty$ on the negligible set D^c. If $\int^* \tilde{f} < s$ then there is an $h \in \mathcal{E}^\uparrow$ with $\tilde{f} \leq h$ and $r \stackrel{\text{def}}{=} \int^* h < s$. As $\hat{f} - \tilde{f} = \infty \cdot D^c$ is negligible, there is an $h_0 \in \mathcal{E}^\uparrow$ with $\hat{f} - \tilde{f} \leq h_0$ and $\int^* h_0 < s - r$. Then $\mathcal{E}^\uparrow \ni h + h_0 > \hat{f}$ and $\int^* h + h_0 < s$. Thus $\int^* \hat{f} \leq \int^* \tilde{f}$ and $\int^* \tilde{f} = \int^* \hat{f}$. It is thus unequivocal to define $\int^* f \stackrel{\text{def}}{=} \int^* f'$, where f' is any function defined everywhere and equal to f on D. $\int_* f$ is of course defined similarly. To check the properties claimed is now a matter of (a lot of) bookkeeping. [We see here that upper and lower integrals are rather user-unfriendly and the employ of means is much preferable.]</p> <p>(ii): In view of (i) neither hypothesis nor conclusion change if f is replaced with a real-valued function that is everywhere defined. Continue: Assume ...</p>					
60	4	RB	dominated	▷ dominated by an integrable function	
60	6	RB	evident,	▷ evident from Theorem 5.6,	
60	6	RB	This	▷ The italicized conclusion	
60	11	RB	Extend this line:	mean. (The mean is of course the Daniell mean for the measure in question, and integrability is understood with respect to that.)	
173	8	RB	$f \leq \phi + h_n$	▷ $f \leq \phi_n + h_n$	
173	20	RB	and II.6.7	▷ II.6.2 (i), and II.6.7 below	∞
173	22	RB	$2g_n$	▷ $g + g_n$	
173	22	RB	Replace Therefore with		
<p>By applying the preceding conclusion to $f - f_n$ in the role of f_n and $g + g_n$ in the role of g_n, it follows that</p>					
173	-14	RB	with	▷ with $h_n \geq f$ and	
173	-12	RB	(MCT)	▷ (Exercise 6.2') see p60 12	
173	-11	RB	$\lim_{N \rightarrow \infty} \bigwedge_{n=1}^N h_n$.	▷ $\lim_{N \rightarrow \infty} \bigwedge_{n=1}^N h_n$, which is integrable by Theorem 5.5 and the MCT (page 54).	
173	-7	RB	\int^* is subadditive on \mathcal{F}^* , but not in general. Replace answer II.6.7 by:		
<p>Assume f is the integrable function. If $\int^*(f + f') < r \in \mathbb{R}$ then there is a function $h \in \mathcal{E}^\uparrow$ exceeding $f + f'$ and having upper integral $\int^* h < r$. By 6.2', h is integrable. Then $f' \leq h - f \in \mathcal{L}^1$ and consequently $\int^* f' \leq \int^*(h - f) = \int(h - f) = \int h - \int f$, which implies $\int^* f' + \int^* f \leq r$. Thus $\int^*(f + f') \geq \int^* f + \int^* f'$. From this $\int^* f' = \int^*[(f + f') + (-f)] \geq \int^*(f + f') + \int^*(-f) = \int^*(f + f') + \int(-f) = \int^*(f + f') - \int^* f$, which is the reverse inequality.</p>					
61	3	RB	S	▷ \mathbb{R}	
61	4	RB	Delete exercise 6.9		3
173	-1	RB	and delete this answer of it		
61	-10	MR	A set	▷ a set	
61	-9	MR	are immediate	▷ are thus immediate	
62	20	MR	the el... σ -finite	▷ <i>the el...σ-finite</i>	
62	-18	RB	countable many	▷ countably many	
62	-13	MR	integrands whose	▷ integrands in \mathcal{E} whose	
62	-12	MR	mean is	▷ mean on \mathcal{E} is	

Chapter II Continued

Pa-ge	Li-ne	Au-thor	Old Text	▷ New Text	Com ment
174	8	RB	▷ sets covers	▷ sets (see Proposition 7.5) covers	
64	4	RB	▷ A_n	▷ $A_{n,k}$	also on lines 5,11
64	8	RB	▷ Riemann	▷ Riemann	
64	17	RB	▷ $\int s_n$	▷ $\ s_n\ ^*$	also on line 19
66	1	RB	▷ her	▷ his	silly, yes
174	24	RB	▷ f	▷ 1_A	
174	27	RB	▷ $U = \bigcup_{i,n=1}^{\infty} (a_n^i, b_n^i + \epsilon 2^{-n-i-1})$	▷ $U = \bigcup_{n=1}^{\infty} \bigcup_{i=1}^{I(n)} (a_n^i, b_n^i + \epsilon 2^{-n-i-1})$	
174	-9	RB	▷ $\dots B_k \cap (-1, 1)$	▷ $\dots B_k \cap (-1, 1)$, B_k as on page 67	
67	-2	MR	▷ sequence f_n	▷ dominated sequence (f_n)	
67	-1	MR	▷ zero	▷ zero that is dominated by an elementary integrand and	

☒ **Robert Burckel** found several instances of expressions like $f - g$ where both f and g could take the value $+\infty$. The sign ☒ points out that this is the reason for the change made.

1 **Robert Burckel** gives the counterexample $\ell^1(\mathbb{N})$ to (ii) as stated. If exercise II.5.12 is to be kept, \mathcal{L}^1 should be replaced by L^1 , the assumption $\|1\|^* < \infty$ must be added, and the whole thing should be moved as Exercise II.3.2 on page 108.

2 The condition that $\lim \int f_n$ be finite is not sufficient, as witness the mean $\|f\|^* = \int^* |f| + |f(0)|$: it majorizes the Lebesgue integral, yet $f_n \stackrel{\text{def}}{=} n\{0\}$ defeats the claim. One must require in addition that $\|\phi\|^* = \int \phi$ for $\mathcal{E} \ni \phi \geq 0$. The deleted claim is dealt with in Remark 6.3 on the next page anyway.

3 This is patently false. **Robert Burckel** gives the counterexample $\mathcal{E} = \{0\}$, $\mathcal{L}^* = \mathcal{L}^1(\mathbb{R})$.

Chapter III

Pa-ge	Li-ne	Au-thor	Old Text	▷ New Text	Com ment
70	7	RB	▷ elementary integrands	▷ following functions from $\overline{\mathcal{E}}$	
70	12	RB	▷ $k \cdot \phi_k$	▷ $k \cdot \phi_k $,	
70	17	KB	▷ \mathcal{E}^\uparrow	▷ $\overline{\mathcal{E}}_+^\uparrow = \mathcal{E}_+^\uparrow$	
70	-9	RB	▷ to f	▷ to f'	
70	-8	RB	▷ $U^c \cup [f \neq f']$	▷ $U^c \cap [f \neq f']$	
71	-6	RB	▷ f b a	▷ f be a	
72	-13	RB	▷ Extend: An integrable function is measurable		
72	-9	MR	▷ every function	▷ every real-valued function	
73	11	RB	▷ alternate	▷ alternative	
73	12	RB	▷ everywhere defined	▷ everywhere-defined	
73	-9	MR	▷ refer	▷ defer	
73	-8	MR	▷ permance	▷ permanence	
73	-3	RB	▷ a function	▷ some function $\overline{\phi}_n$	also on p.74 1.16
174	-5	RB	▷ integrable	▷ integrable	
73	-3	MR	▷ everyone	▷ every one	

Chapter III Continued

Page	Line	Author	Old Text	New Text	Comment
174	-3	MR	everyone	every one	same on line -1
175	2	RB	Proposition II.7.1	Propositions II.7.1 and II.7.5	
175	3	RB	A	A_1	
74	15	MR	everyone	every one	
74	-1	RB	tt Extend: If f_n is measurable and $\ f_n - f\ ^* \xrightarrow{n \rightarrow \infty} 0$ then f is measurable.		
75	9	MR	and check	to check	
76	11	MR	conclude	conclude	
77	8	MR	$\bar{\alpha}^1, \bar{\alpha}^2,$	$\bar{\alpha}^1, \bar{\alpha}^2 \in \bar{\mathcal{E}},$	
77	20	RB	$0 \leq \phi_n \leq 1$	$\phi_n \geq 0$	
77	21	RB	...where.	...where (see II.7.5).	
77	-15	MR	Means	Means.	
77	-8	MR	$A = [\phi > r],$	$A = [\phi > r], \phi \in \mathcal{E},$	
77	-6	RB	Delete: "of functions"		
77	-5	RB	$\psi_n = A \cdot \phi_n$	$0 \vee A \cdot \phi_n \wedge 1$	
77	-2	RB	non-negligible	$\ \cdot\ ^*$ -non-negligible	
78	1	RB	integrable	integrable (for both means)	
78	7	RB	by $f.$	by $f,$ provided the mean is σ-finite!	
78	1	RB	non-negligible	$\ \cdot\ ^*$ -non-negligible	
78	8	MR	integrable set	integrable sets	
175	11	RB	Replace the answer by the	hint to apply 3.5 (iii)	
175	12	RB	C_i	C_k	
78	14	RB	Complex Valued	Complex-Valued	
78	-6	MR	almost every convergent	almost everywhere convergent	same on p79 1-6
79	6	RB	$\bar{\mathcal{E}} \otimes \mathbb{C}$	$\bar{\mathcal{E}} \otimes \mathbb{C} = \{\phi + i\psi : \phi, \psi \in \bar{\mathcal{E}}\}$	
79	8	RB	und	and	
79	-14	RB	Banach Space Valued	Banach Space-Valued	
79	-11	RB	add: $\ f\ _E : x \mapsto \ f(x)\ _E$		
80	8	RB	complex valued	complex-valued	
80	9	RB	und	and	
175	-20	RB	After "there." insert: Namely, $\ f\ _E \leq \sum \ \xi\ _E \int[\phi = \xi] = \int \ \phi\ _E = \ \phi\ ^*.$		
80	-4	RB	Extend: [Hint: use exercise II.5.16 on page 57.]		
175	-15	RB	\mathcal{E}	$\bar{\mathcal{E}}$	
175	-14	RB	measurable.	measurable (use 5.16).	
80	-18	RB	$\ f\ ^*$	$\ \phi\ ^*$	
81	-14	RB	by the	by the permanence properties	
82	13	RB	lattice	vector lattice	
83	4	MR	$S \in \mathcal{A}$	$S \in \mathcal{A}[\mathbb{R}]$	
83	-5	MR	$[f - f_n > \delta]$	$[f - f_n > \delta]$	
84	-13	RB	delete ", say $x \in A_k,$ "		
84	-11	RB	for i	for all i	
85	8	RB	insert: ((M) reads here $\rho_0(\lambda \sum_k \phi_k) \xrightarrow{\lambda \rightarrow 0} 0 \implies \rho_0(\phi_k) \xrightarrow{k \rightarrow \infty} 0$ for $\phi_k \in \mathcal{E}_+.$)		
86	-7	RB	.	. (delete spurious right parenthesis)	

Chapter III Continued

Pa-ge	Li-ne	Au-thor	Old Text	▷ New Text	Com ment
86	-0	RB	◁ (Also MR)At the very end of the page insert this paragraph:		
So far we have not specified in which sense the pointwise limits are to be taken, as limits of real-valued or of numerical functions, i.e. in \mathbb{R}^S or in $\overline{\mathbb{R}}^S$, S being the ambient space. Both make sense, and we shall correspondingly talk about the real sequential closure or the numerical sequential closure where the distinction needs to be stressed. We are generally interested in numerical functions, so \mathcal{E}^Σ denotes the sequential closure of \mathcal{E} in $\overline{\mathbb{R}}^S$, the numerical sequential closure. Unless the meaning is clear from the context we shall write $\mathcal{E}_\mathbb{R}^\Sigma$ for the real sequential closure, taken in \mathbb{R}^S , and $\mathcal{E}^\Sigma = \mathcal{E}_\mathbb{R}^\Sigma$ if it must be emphasized that the numerical \mathcal{E} -Baire functions are meant. To make the distinction we also simply talk about real-valued or numerical Baire functions; this is justified by the following simple observation:					
Exercise 5.0* If \mathcal{E} consists of real-valued functions then $\mathcal{E}_\mathbb{R}^\Sigma$ contains the real-valued functions of \mathcal{E}^Σ . (Answer: The real-valued functions in \mathcal{E}^Σ form a real-sequentially closed family.)					
These conventions require that the notation \mathcal{E}^Σ be changed to $\mathcal{E}_\mathbb{R}^\Sigma$ where the real sequential closure is intended, e.g. in Proposition 5.3, Corollary 5.6 (i), etc.					
87	8	RB	◁ \mathcal{E}^Σ	▷ $\mathcal{E}_\mathbb{R}^\Sigma$	Same in lines 15 — 25
87	15	RB	◁ anyone	▷ any one	
87	-19	MR	◁ to $\phi * f$.	▷ to $\phi * f$ for all $\phi \in \mathcal{E}$.	
88	15	KB	◁ chopping.	▷ chopping of bounded functions	
88	16	RB	◁ \mathcal{E}^Σ	▷ $\mathcal{E}_\mathbb{R}^\Sigma$	Also in lines -14 — -11
88	-20	RB	◁ Delete: strictly		Also on line -18
88	-7	RB	◁ from 5.4	▷ from 5.4 and 5.3	
89	5	KB	◁ quite an	▷ a real	
89	9	RB	◁ Replace this and the next line with:		
(ii): Let f be a function integrable for the mean $\ \cdot \ _* + \ \cdot \ ^\#$. If (ϕ_n) converges to f in this mean then it converges both in $\ \cdot \ _*$ -mean and in $\ \cdot \ ^\#$ -mean to f . Thus ...					
89	15	RB	◁ Theorem II.5.5	▷ Theorems II.5.5 and II.7.5	
90	2	MR	◁ (i) [$f > r$]	▷ (i) [$f < r$]	
90	3	MR	◁ (v) [$f > d$]	▷ (i) [$f < d$]	
90	7	MR	◁ $\mathcal{M}(\mathcal{F})$	▷ the family $\mathcal{M}(\mathcal{F})$	
90	8	RB	◁ $\bigcap_N \bigcup_{n \geq N} [f_n > r]$	▷ $\bigcup_{N,k} \bigcap_{n > k} [f_n > r + 1/n]$	
90	16	MR	◁ Σ -algebras are	▷ a σ -algebra is	
90	-14	MR	◁ Delete: separable		
91	4	MR	◁ Delete: Since for a ... thus		
91	15	MR	◁ One the	▷ On the	
92	8	RB	◁ negligible	▷ negligible	
92	16	RB	◁ delete \mathcal{E}^Σ		
92	-2	RB	◁ Delete the whole sentence up to (Exercise II.6.9). See p61, 4		
93	16	RB	◁ $< r$.	▷ $\leq r$.	
93	20	RB	◁ Proposition ... (ii)	▷ Proposition 5.7 (ii) on page 88	
93	-6	RB	◁ Clearly	▷ Since $\ \cdot \ ^\dagger \in \mathfrak{M}$	

Chapter III Continued

Page	Line	Author	Old Text	New Text	Comment
94	7	MR	\mathcal{E} -Baire functions	numerical \mathcal{E} -Baire functions	same in whole proof
94	16	MR	Replace: (ii): According to (6.1) there are $h_n \in \mathcal{E}^\Sigma$ with $ f \leq h_n$ and $\ h_n\ ^* \leq \ f\ ^* \dots$		
94	17	MR	(Exercise II.7.7)	as the limit of $h_n \wedge k$ (Theorem 3.1 and MCT)	
94	-16	RB	Proposition ... (i)	Proposition 5.7 (i) on page 88	
94	-11	MR	the upper the	the upper	
95	14	RB	Carathéodory	Carathéodory	same in headers
96	6	RB	set function	set-function	
97	7	RB	analogue	analogous	
97	11	RB	(F_n)	$\lambda^*(F_n)$	
97	12	RB	is increasing along	is continuous along	
97	17	RB	Proposition II.3.2	Lemma II.3.1 and Proposition II.3.2	
97	-8	RB	restrictions...	restrictions to sets of known facts about functions.	
98	2	RB	with $\int^* h < r$	with $h \geq 1_F$ and $\int^* h < r$	
99	-12	RB	inequality	inequalities	
100	3	RB	II.2.17	II.7.1	
100	22	RB	page 99	page 98	
176	-14	RB	measure;	measure — such exists due to σ -finiteness	
106	-6	RB	This is	By Corollary II.5.6 on page 88 this is	

Chapter IV

Page	Line	Author	Old Text	New Text	Comment
101	-5	RB	Replace “that ... do so.” by		
			that \mathcal{E} is σ -finite and $\ \cdot\ ^*$ is continuous along arbitrary increasing sequences, and we shall do so.		
176	-20	RB	delete: ≤ 0		
176	-5	KB	Delete: II.6.9		see p61, 14
171	14	KB	\mathcal{P} and thus ...	\mathcal{P} , and thus ...	insert a comma

Chapter V

Page	Line	Author	Old Text	New Text	Comment
103	-10	SB	Delete: <i>Minkowski's inequality</i>		
103	-5	RB	$< < b$	$\triangleright \leq b$	
105	9	RB	$<$ is maximal	\triangleright is maximal if $\ \cdot \ $ is	
105	-6	RB	$<$ it	\triangleright its conclusions	
106	-7	RB	$< \phi_n$	$\triangleright \phi_k$	twice
1646	9	SB	$<$ Using Hölder's...	\triangleright Using Hölder's...	insert a space

Indices

Page	Line	Author	Old Text	New Text	Comment
181	16	RB	$<$ Insert: \mathcal{E}_+ the positive elements of \mathcal{E}		3
182	1		Insert: \mathcal{F}^{\sharp} the functions f with $\int^{\sharp} f < \infty$		17
182	-7	RB	$<$ insert: $\mathcal{R}(\mathcal{E})$ the σ -ring generated by \mathcal{E}		88
187	19	RB	$<$ insert the line: Fatou's lemma, 56		
188	-4	RB	$<$ measurable 72	\triangleright measurable 72, 92	
189	-3	RB	$<$ MCT, 106	\triangleright MCT, 54, 106	
190	3	RB	$<$ mean, 48, 90	\triangleright mean, 48, 80, 90	
191	13	RB	$<$ Radon measure 140	\triangleright Radon measure 80, 140	