

# Class Information

- Klaus Bichteler's web page:  
[www.ma.utexas.edu/users/kbi](http://www.ma.utexas.edu/users/kbi)  
There you find my office hours, my email address, etc.

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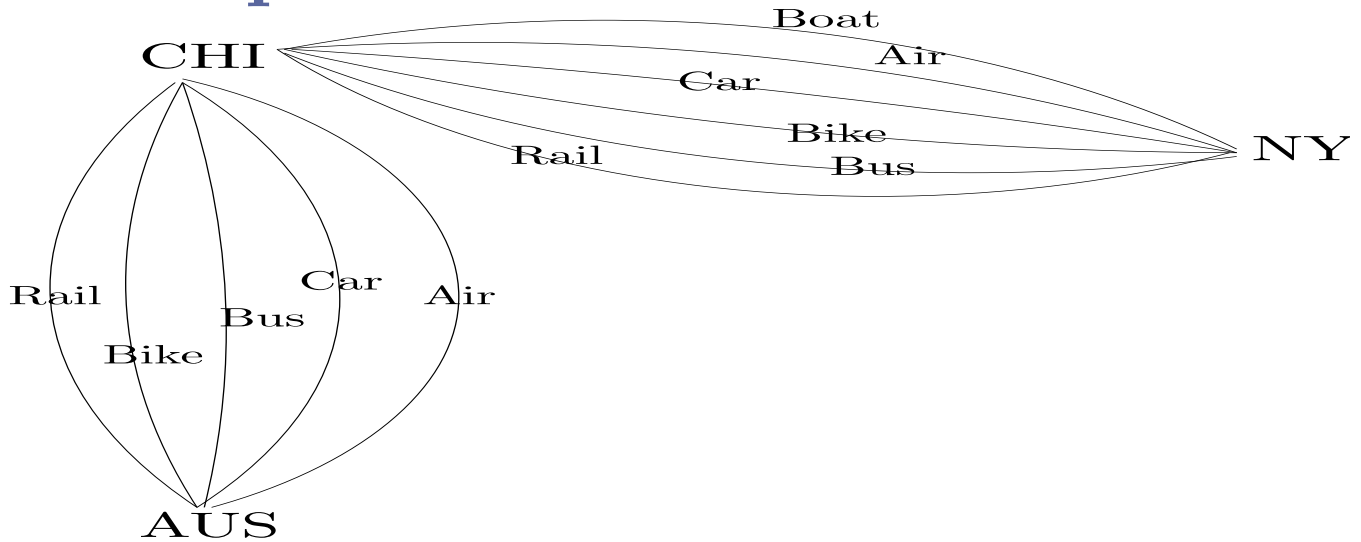
From there click your way to this link:

- Class web page: [325K](#)  
This [page](#) contains the grading scheme for this class and the test dates, and it lists the homework.

# Introduction

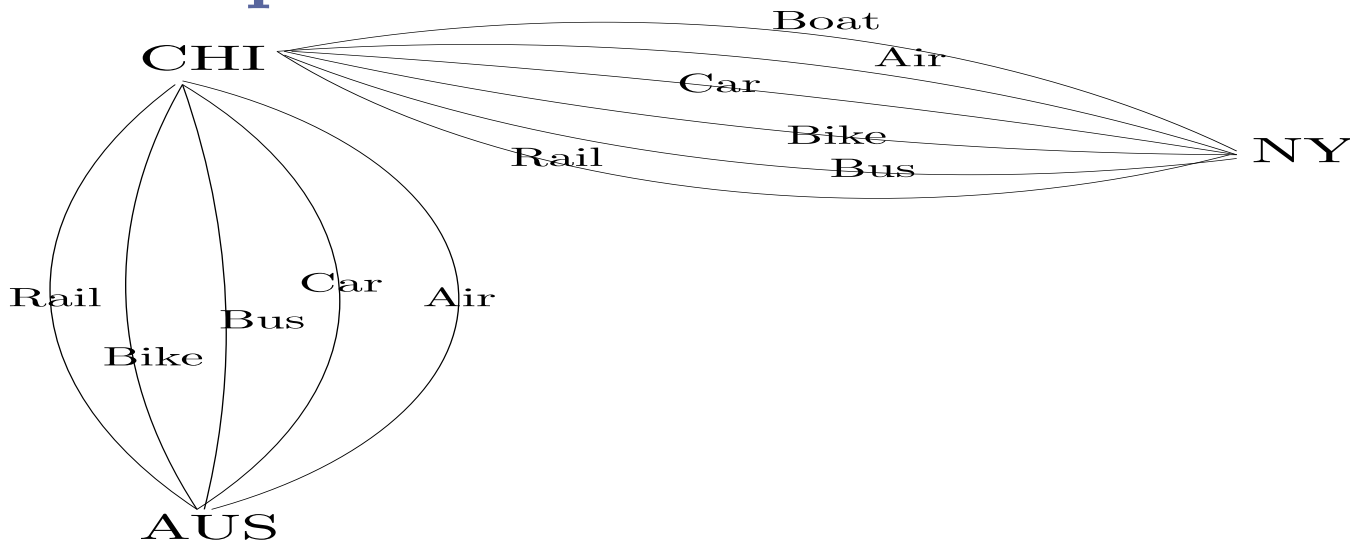
This course is about learning to make a cogent argument [“proof” for those who don’t start shaking at that word] in various areas of mathematics, and to learn a few facts on the side. Altogether easy fun.

# Example:



How many ways are there to travel from Austin to New York City, visiting aunt Elinor in Chicago?

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How many ways are there to travel from Austin to New York City, visiting aunt Elinor in Chicago?

**Answer:** There are  $5 \times 6 = 30$  ways.

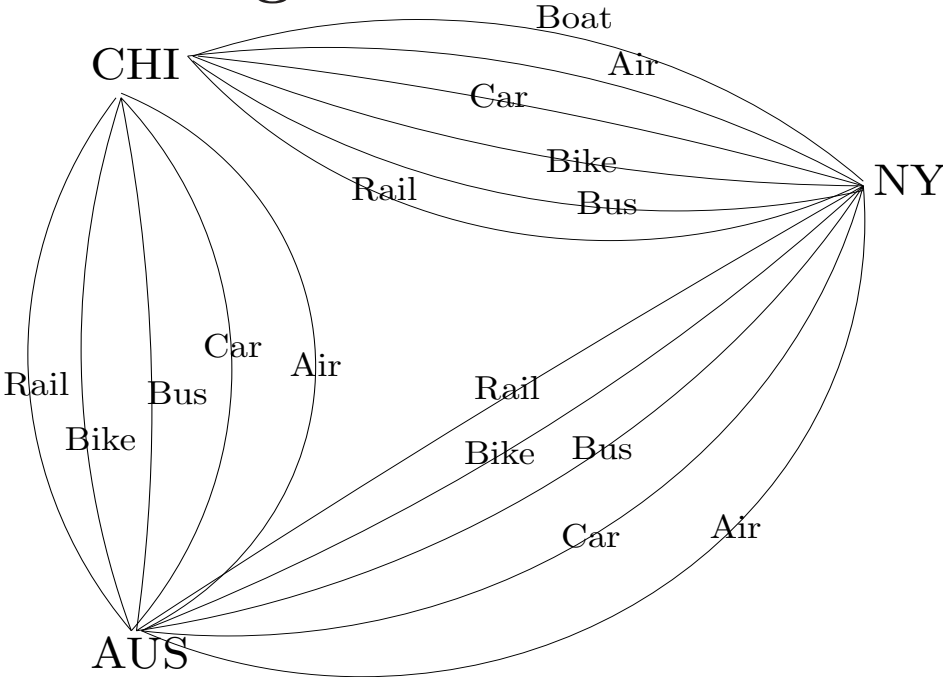
When you came up with this answer, you did use, consciously or not, Counting's

**Rule of Product:** If a task  $T$  can be performed by performing two subtasks  $T_1, T_2$  consecutively that can be done in  $N_1, N_2$  ways, respectively, then  $T$  can be performed in  $N_1 \cdot N_2$  ways.

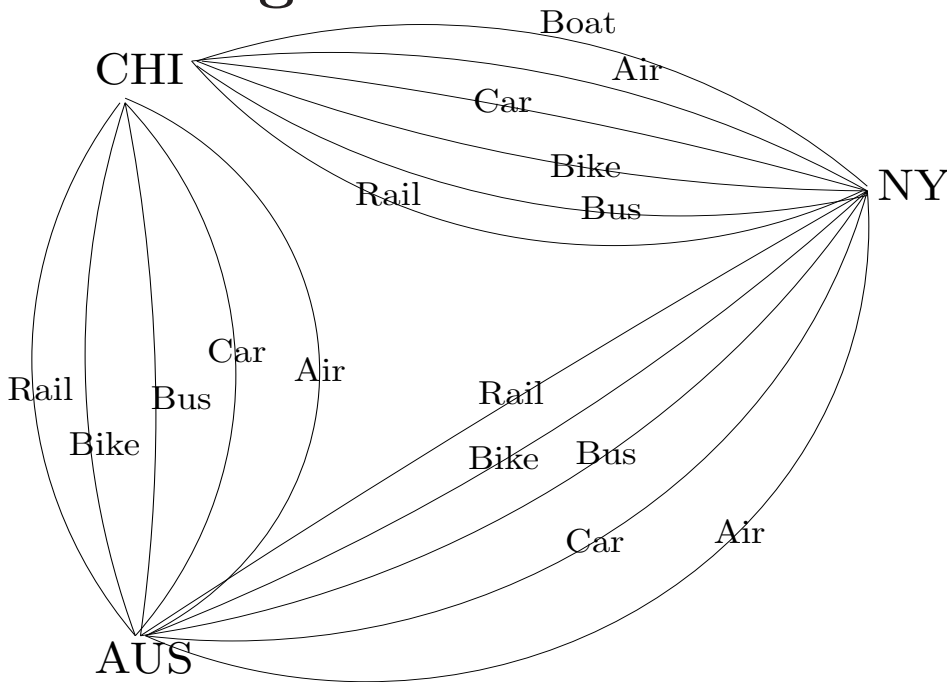
Equally obvious to you is Counting's

**Rule of Sum:** If a task  $T$  can be performed by performing either of two mutually exclusive subtasks  $T_1, T_2$  that can be done in  $N_1, N_2$  ways, respectively, then  $T$  can be performed in  $N_1 + N_2$  ways.

**Example:** How many ways are there to travel to New York City along one of the following routes?



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**Answer:** You can travel either via Chicago, in 30 ways, or directly in 5 ways, for a total of 35 ways.

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**Principle 0 of Life: Know what you are talking about.**

**Principle 1 of Counting: There are as many widgets as you can make.**

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**Answer:** As many as you can make. See later.

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**Answer:** As many as you can make. See later.

**Example:** How many subsets does a set  $S$  of size  $|S| = n$  have?

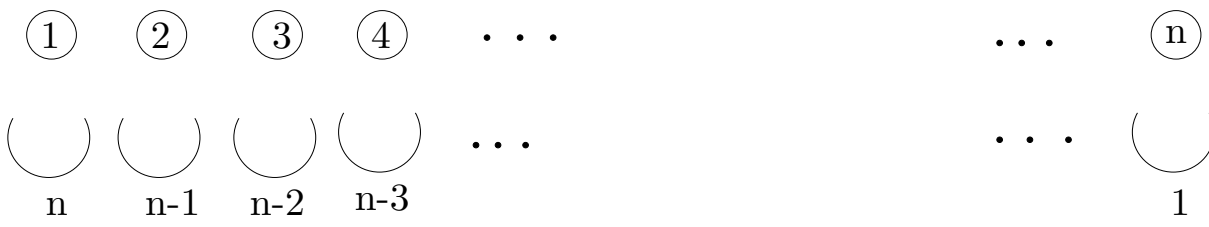
**Answer:** As many as you can make.

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So how do you make a subset of  $S$ ? You get yourself a basket. For each of the  $n$  elements of  $S$  you decide whether it goes into the basket or not. When you are done you have a subset of  $S$  in the basket. You have made

$$\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{n \text{ factors}} = 2^n$$

decisions:  $S$  has  $2^n$  subsets.

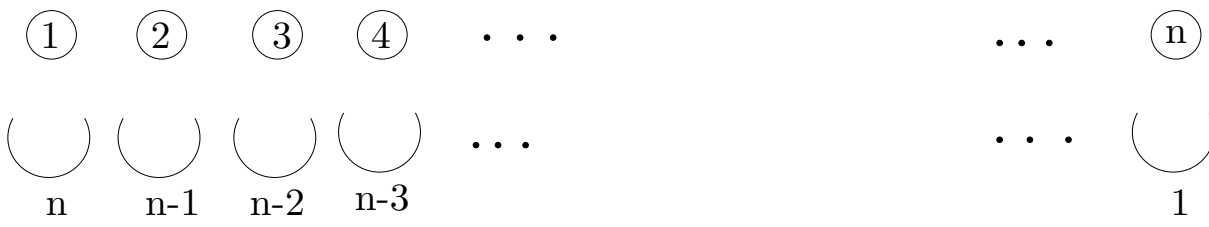


Number of permutations of  $n$  widgets:

$$n(n-1)(n-2)(n-3) \dots 1 = n! \text{ [pronounce "n factorial"]}$$

= number of ways of distributing  $n$  distinguishable widgets  
 over  $n$  distinguishable urns in such a way that  
 every urn contains exactly one widget.

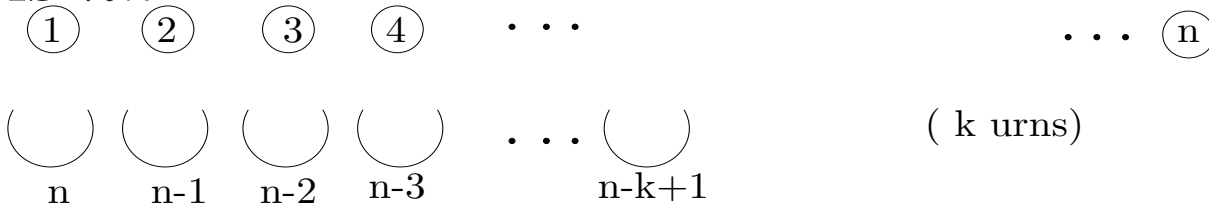
**The number of permutations of  $n$  widgets  
 is  $n!$ .**



Number of permutations of  $n$  widgets:

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 = number of ways of distributing  $n$  distinguishable widgets  
 over  $n$  distinguishable urns in such a way that  
 every urn contains exactly one widget.

## The number of permutations of $n$ widgets is $n!$ .



Number  $C(n,k)$  of permutations of  $n$  widgets taking  $k$  at a time:  
 $n(n-1)(n-2)(n-3) \dots (n-k+1) = n!/(n-k)!$   
 = number of ways of distributing  $n$  distinguishable widgets over  
 $k$  distinguishable urns in such a way that every urns contains  
 exactly one widget.

## The number of permutations of $n$ widgets taking $k$ at a time is $n!/(n - k)!$ .

# Combinations

The number of permutations of  $n$  widgets taking  $k$  at a time is the number of ways we can make such a permutation [principle 1]. Here is another way: we first select a subset of size  $k$  from the  $n$  widgets, then distribute these  $k$  widgets over the  $k$  urns ( $k!$  ways). If  $C(n, k)$  is the number of ways we can select a subset of size  $k$  from a set of size  $n$ , we get

$$\frac{n!}{(n-k)!} = C(n, k) \cdot k! , \quad \text{whence}$$

$$C(n, k) = \frac{n!}{k!(n-k)!} .$$

$C(n, k)$  is written  $\binom{n}{k}$ ,

pronounced “ $n$  choose  $k$ ”:

$$\binom{n}{k} \stackrel{\text{def}}{=} \begin{cases} 0 & \text{if } k > n, \text{ of course,} \\ \frac{n!}{k!(n-k)!} & \text{if } k \leq n. \end{cases}$$

pronounced “n choose k”:

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**Theorem:** There are

$$\binom{n}{k} \stackrel{\text{def}}{=} \frac{n!}{k!(n-k)!}$$

ways to choose a subset of size  $k$  from a set of size  $n$ .

**Example:** How many full houses are there in 5-card poker?

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**Answer:** As many as you can make.

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How do you make a full house? First, you choose one of the 13 denominations for the three cards of equal denomination:  $\binom{13}{1} = 13$ ; then you choose 3 of the 4 cards of that denomination:  $\binom{4}{3}$ ; then you choose 1 of the remaining 12 denominations for the pair:  $\binom{12}{1}$ ; finally you choose which of the 4 cards in that denomination shall make up the pair:  $\binom{4}{2}$ . Altogether there are

$$\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2} = 13 \cdot 4 \cdot 12 \cdot 6$$

ways to build a full house.

**Example:** Count other poker hands.

**Example:** A curious boy walks to school.

**Example: Pascal's triangle** Because a set  $S = \{s_0, s_1, \dots, s_n\}$  has  $\binom{n+1}{k}$  subsets of size  $k$ ,  $\binom{n}{k-1}$  of which contain  $s_0$  and  $\binom{n}{k}$  of which don't,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}.$$

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				$\binom{0}{0}$						
			$\binom{1}{0}$	$\binom{1}{1}$						
		$\binom{2}{0}$	$\binom{2}{1}$	$\binom{2}{2}$						
	$\binom{3}{0}$	$\binom{3}{1}$	$\binom{3}{2}$	$\binom{3}{3}$						
$\binom{4}{0}$	$\binom{4}{1}$	$\binom{4}{2}$	$\binom{4}{3}$	$\binom{4}{4}$						
$\binom{5}{0}$	$\binom{5}{1}$	$\binom{5}{2}$	$\binom{5}{3}$	$\binom{5}{4}$	$\binom{5}{5}$					
			1							
			1	1						
		1	1	2	1					
	1	1	3	3	1					
	1	4	6	4	1					
1	5	10	10	5	1					