Laws of Set Theory

Let $A, B \subseteq \mathcal{U}$

$\overline{\overline{A}} = A$	**Law of double complement**
$\overline{A \cup B} = \overline{A} \cap \overline{B}$	**de Morgan's law**
$\overline{A \cap B} = \overline{A} \cup \overline{B}$	**de Morgan's law**
$A \cup B = B \cup A$	**Commutative Law**
$A \cap B = B \cap A$	**Commutative Law**
$A \cup (B \cup C) = (A \cup B) \cup C$	**Associativity**
$A \cap (B \cap C) = (A \cap B) \cap C$	**Associativity**
$A \cup A = A, A \cap A = A$	**Idempotent Laws**
$A \cup \emptyset = A, A \cap \mathcal{U} = A$	**Identity Laws**
$A \cup \overline{A} = \mathcal{U}, A \cap \overline{A} = \emptyset$	**Inverse Law**

$A \cup \mathcal{U} = \mathcal{U}, A \cap \emptyset = \emptyset$	**Domination**
$A \cap (A \cup B) = A$	**Absorption**
$A \cup (A \cap B) = A$	**Absorption**
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	**Distributive**
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	**Distributive**

Example, Proof of de Morgan's law:

To prove:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad\qquad A, B \subseteq \mathcal{U}\ .$$

$$
\begin{aligned}
& x \in \overline{A \cup B} \\
& \neg[x \in A \cup B] \\
& \neg[x \in A \quad \vee \quad x \in B] \\
& \neg[x \in A] \quad \wedge \quad \neg[x \in B] \\
& x \in \overline{A} \ \wedge \ x \in \overline{B} \\
& x \in \overline{A} \cap \overline{B}\ .
\end{aligned}
$$

Example, Proof of de Morgan's law:

To prove:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad\qquad A, B \subseteq \mathcal{U}\ .$$

$$x \in \overline{A \cup B}$$
$$\neg[x \in A \cup B]$$
$$\neg[x \in A \quad \vee \quad x \in B]$$
$$\neg[x \in A] \quad \wedge \quad \neg[x \in B]$$
$$x \in \overline{A} \ \wedge \ x \in \overline{B}$$
$$x \in \overline{A} \cap \overline{B}\ .$$

Example, Proof of de Morgan's law:

To prove:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad\qquad A, B \subseteq \mathcal{U}\ .$$

Let $A, B \subseteq \mathcal{U}$**. To prove:**

$$\forall\, x \in \mathcal{U} \quad x \in \overline{A \cup B} \iff x \in \overline{A} \cap \overline{B}\ .$$

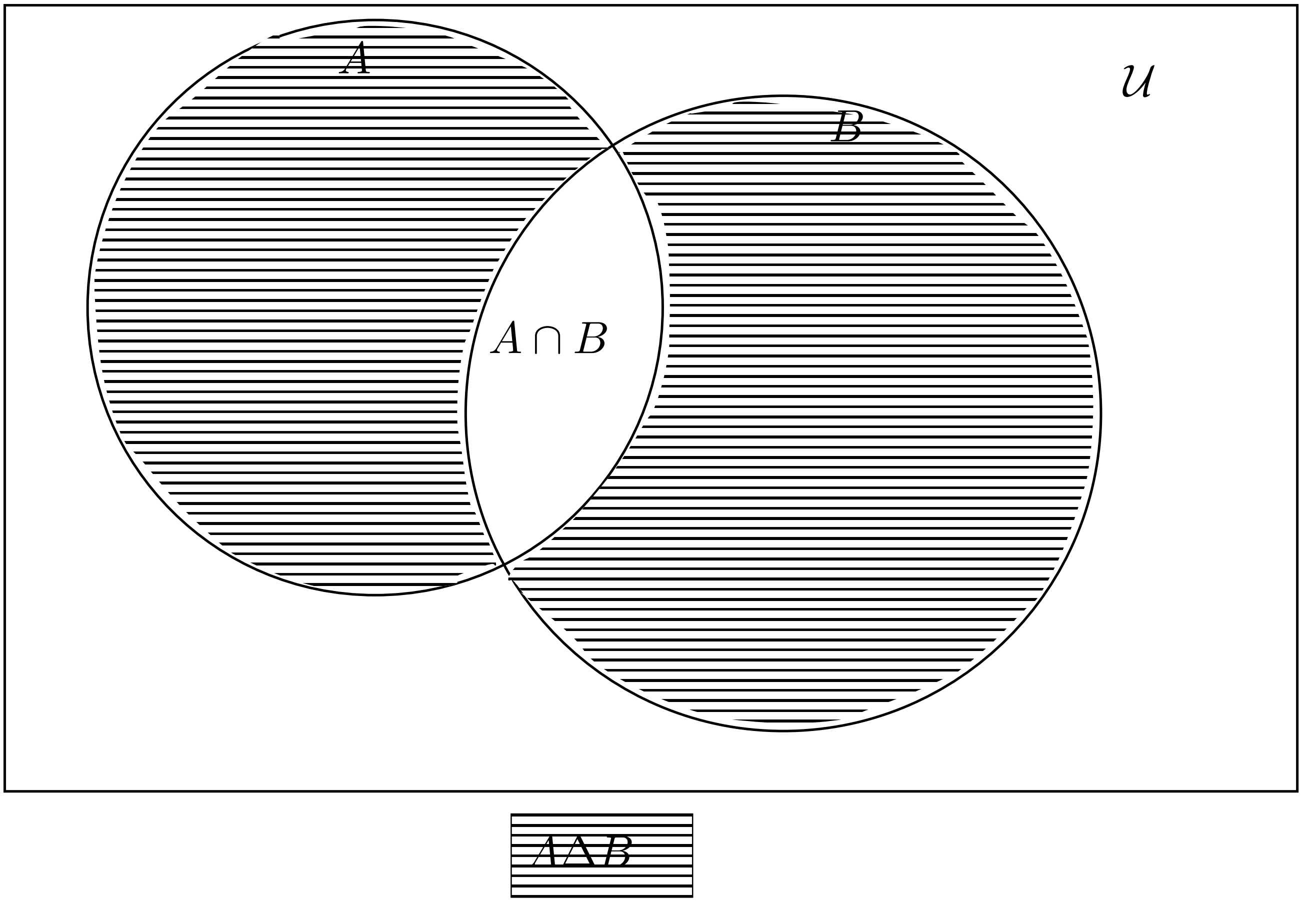
Let $x \in \mathcal{U}$**.**

$$\begin{aligned} & x \in \overline{A \cup B} \\ \iff & \neg[x \in A \cup B] \\ \iff & \neg[x \in A \ \lor\ x \in B] \\ \iff & \neg[x \in A] \ \land\ \neg[x \in B] \\ \iff & x \in \overline{A} \ \land\ x \in \overline{B} \\ \iff & x \in \overline{A} \cap \overline{B}\ . \end{aligned}$$

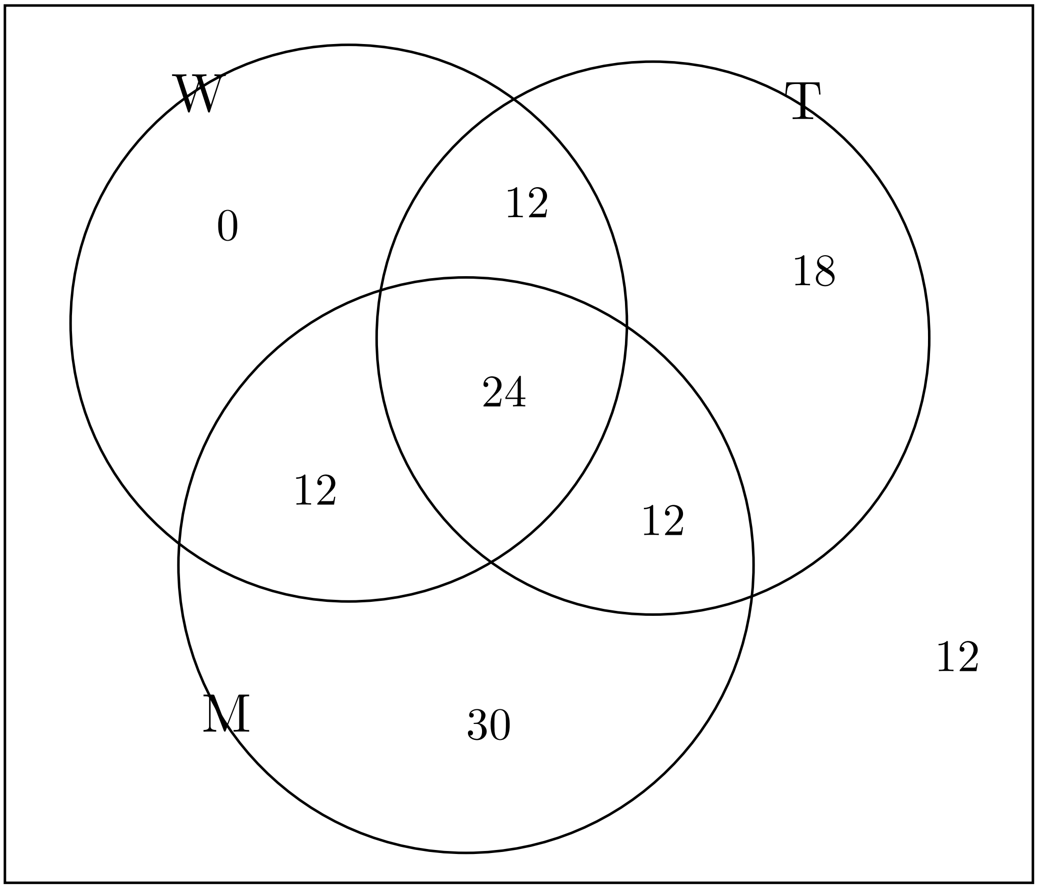
Thus $x \in \overline{A \cup B} \iff x \in \overline{A} \cap \overline{B}$**.** **QED**

Venn Diagrams

Venn diagrams help to visualize and suggest laws of set theory, and to help counting sets.



Example: Of 120 airplane passengers 48 ordered wine, 78 ordered mixed drinks, 66 wanted tea. In addition, 36 wanted any given pair of beverages, and 24 wanted all three. How many drank only tea? only wine? nothing?


W
T
0
12
18
24
12
12
M
30
12