

# Test Review

**Counting: Rules of Sum and Product.**

**Principles Zero and One.**

**Count Poker hands.**

**The numbers  $P_n, P_{n,k}, C_{n,k} = \binom{n}{k} \cdot \binom{n}{n_1 n_2 \dots n_r}$ .**

**The Binomial and Multinomial Theorems.**

**How many summands appear in the Multinomial Theorem?**

**Why is  $\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$ .**

**Distributing  $n$  indistinguishable widgets over  $k$  distinguishable bins.**

**$\#\{x_1 + \dots + x_k = n ; x_i > r\}$ .**

**Elementary Logic: Statements.**

**Connectives. Truth Tables.**

**Negate  $P \rightarrow Q$ .**

**Contrapositive...Inverse.**

**Contradiction and Tautology.**

**Logical Equivalence and Laws of Logic.**

**Substitution Rules.**

**Rules of Inference.  $(P_1, \dots, P_n) \implies Q$  is also written, clumsily,**

$$\begin{array}{c} P_1 \\ \vdots \\ P_n \\ \hline \therefore Q \end{array}$$

**Open Statements and Quantifiers.**

**Negation of statements containing quantifiers.**

**Set Theory: Subsets. Pascals's Triangle. Complement, Union, Intersection, Symmetric Difference, cartesian Product, Power set. The Laws of Set Theory. Venn Diagrams.**

**Example:** Let  $A, B \subseteq \mathcal{U}$ . Then

$$A \times B = B \times A \iff [A = \emptyset \vee B = \emptyset \vee A = B].$$

**Proof of  $\Leftarrow$ :** Obvious.

**Proof of  $\Rightarrow$ :**

$$A \times B = B \times A \rightarrow [A = \emptyset \vee B = \emptyset \vee A = B]$$

is logically equivalent to

$$[A \times B = B \times A \wedge A \neq \emptyset \wedge B \neq \emptyset] \rightarrow A = B,$$

so we prove that this is a tautology.

**To prove:** Under assumptions  $[\dots]$ ,  $A \subseteq B$  and  $B \subseteq A$ .

**To prove:**  $\forall a \in \mathcal{U} a \in A \implies a \in B$  etc.

Let  $a \in A$ . Since  $B \neq \emptyset$ ,  $\exists b \in B$ . Then  $(a, b) \in A \times B$ . Since  $A \times B = B \times A$ ,  $(a, b) \in B \times A$ . Thus  $a \in B$ , and  $A \subseteq B$ .  $B \subseteq A$  is proved similarly. Hence  $A = B$ . **QED**

**Probability:** State Space  $S$ , outcomes, events, and Probability  $\mathbb{P} : \mathcal{P}[S] \rightarrow [0, 1]$ . **Axioms.** Probability function  $p : S \rightarrow [0, 1]$ . **Meaning of  $\mathbb{P}$ . A priori Probability.**

**Conditional Probability  $\mathbb{P}[A|B]$ .**

**Bayes' Theorem.**

**Application to testing for a condition.**

**Independence. Bernoulli Trials. Binomial Distribution. Random Variables  $X : S \rightarrow \mathbb{R}$ . Expectation. Variance. Tchebysheff's Inequality. Weak Law of Large Numbers.**