Practice Final, M325K, 07/10/09 **PRINTED** NAME:____________________

Every problem is worth an equal number of points for a total of 100 points.
You must show your work; answers without substantiation do not count.
Answers must appear in the box provided!

This does not aim nor claim to be exhaustive! Use this as a guide of what to study and not of what not to study! Do not expect to find every test problem listed here! Sigh.

If you scramble the letters of the word SOCIOLOGICAL and then put them down randomly in linear order, **what is the probability** p you will spell SOCIOLOGICAL?

Answer: $p =$

State the multinomial formula for $(a+b+c)^{10}$:

$$(a+b+c)^{10} = \quad \sum$$

(b) How many summands are there in this sum?

Answer: There are summands.

Write the converse, inverse, and contrapositive of the implication "If the moon is made of yellow cheese then the pope is catholic." For this statement and its converse, inverse, and contrapositive determine its truth value.

Truth value of "If the moon is made of yellow cheese then the pope is catholic.":

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converse:

truth value:

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contrapositive:

truth value:

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inverse:

truth value:

Prove that for every $n \in \{1, 2, 3...\}$ $\quad \displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}.$

The XYZ Company has a dinner for all its employees. A place setting costs \$45, except for the women, who get an extra little vase with a flower for an additional \$2. The company lays out a total of \$1613 for this dinner. How may male and female employees does XYX company have?

Answer: XYZ company has ______ male and ______ female employees.

The magician says "Select seven distinct integers between 1 and 24 and denote by S the set of numbers you got. For every non–void subset $A \subseteq S$ of your selection S compute the sum of the numbers in it and call it $s(A)$. In other words $s(A) \stackrel{\text{def}}{=} \sum_{i \in A} i$. You will find that there are two distinct subsets $A, A' \subseteq S$ with $s(A) = s(A')$." How does the magician know that?

Let $|A| = m$. (a) How many binary operations are there on A? (b) How many of them have a neutral element? (c) How many of them are commutative? (d) How many commutative ones have a netral element?

Answer: (a) (b) (c) (d)

Let A, B be sets. In which cases is $A \times B = B \times A$? **Prove your assertion!**

Answer: In summary, $A \times B = B \times A$ **if and only if** OR OR .

Let A, B be finite sets with $|A| = m$ and $|B| = n$.
(a) How many relations from A to B are there?
(b) How many functions $f : A \to B$ are there?
(c) How many injective functions $f : A \to B$ are there?
(d) How many surjective functions $f : A \to B$ are there?

Answer: (a) (b) (c) (d)

Let A, B be sets and $f : A \to B$ a function. Show that f is injective if and only if $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ for any pair of subsets $A_1, A_2 \subseteq A$.

Let $\mathcal{R}_1$, $\mathcal{R}_2$ be relations on a set A. Prove or disprove the following

a) If $\mathcal{R}_1$ and $\mathcal{R}_2$ both are reflexive then $\mathcal{R}_1 \cup \mathcal{R}_2$ is reflexive. True False
The reason:

b) If $\mathcal{R}_1$ and $\mathcal{R}_2$ both are symmetric then $\mathcal{R}_1 \cap \mathcal{R}_2$ is symmetric. True False
The reason:

c) If $\mathcal{R}_1$ and $\mathcal{R}_2$ both are antisymmetric then $\mathcal{R}_1 \cap \mathcal{R}_2$ is antisymmetric. True False
The reason:

d) If $\mathcal{R}_1$ and $\mathcal{R}_2$ both are transitive then $\mathcal{R}_1 \cup \mathcal{R}_2$ is transitive. True False
The reason:

How many (a) reflexive, (b) symmetric, (c) antisymmetric, (d) reflexive and symmetric, (e) symmetric and antisymmetric, (f) reflexive, symmetric, and non–transitive, relations are there on a set of size n?

Answer: (a) (b) (c) (d) (e) (f) .

(VIIc) Let $A \stackrel{\text{def}}{=} \{1, 2, 3, 4\}$ and $\mathcal{R} \stackrel{\text{def}}{=} \{(1,1), (1,2), (2,3), (3,3), (3,4), (4,4)\}$.
Find the relation matrix and the directed graph of $\mathcal{R}$:

$$M(\mathcal{R}) = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

. Graph:

(b) With A and $\mathcal{R}$ as in II), find two relation $\mathcal{S}, \mathcal{T}$ on A such that $\mathcal{S} \neq \mathcal{T}$ yet $\mathcal{R} \circ \mathcal{S} = \mathcal{R} \circ \mathcal{T} = \{(1,1), (1,2), (1,4)\}$.

On $A \stackrel{\text{def}}{=} \{1,2,4,6,10,12\}$ let $\preceq$ be the relation defined by $a \preceq b \iff a|b$.
(a) Draw the directed graph of $\preceq$ and its incidence matrix.

(b) Show that $\preceq$ is a partial order on A.

(c) Draw its Hasse diagram.

(d) Toplogically sort $(A, \preceq)$ in two different ways and draw the resulting Hasse diagrams.

How are reflexivity ***etc.*** of the relation $\mathcal{R} \subseteq A \times A$ reflected in the incidence matrix $M(\mathcal{R})$?

Set $A := \{3,6,12,18,21\}$ and define the relation $\mathcal{R}$ on A by $a\mathcal{R}b \iff a|b$.
b) Show that $\mathcal{R}$ is a partial order.
c) Find the incidence matrix of $\mathcal{R}$.
d) Topologically order $\mathcal{R}$.

On $\mathbb{Z}$ define the relation $\simeq$ by $x \simeq y$ if a and b have the same remainder modulo 37. Show that $\simeq$ is an equivalence relation.

How many equivalence relations are there on the set $A \stackrel{\text{def}}{=} \{1,2,4,6,10,12\}$?

Answer: There are   equivalence relations on A.

Let $f, g : \mathbb{R} \to \mathbb{R}$, $f : x \mapsto ax + b$, $g : x \mapsto cx + d$, where a, b, c, d are constants in $\mathbb{R}$. What relationship(s) must these four constants satisfy in order that $f \circ g = g \circ f$? How are reflexivity ***etc.*** of the relation $\mathcal{R} \subseteq A \times A$ reflected in the incidence matrix $M(\mathcal{R})$?

Set $A := \{3,6,12,18,21\}$ and define the relation $\mathcal{R}$ on A by $a\mathcal{R}b \iff a|b$.
b) Show that $\mathcal{R}$ is a partial order.
c) Find the incidence matrix of $\mathcal{R}$.
d) Display the directed graph and the Hasse diagram of $\mathcal{R}$.
e) Topologically order $\mathcal{R}$.

$\mathbb{Z}$ define the relation $\simeq$ by $x \simeq y$ if a and b have the same remainder modulo 37. Show that $\simeq$ is an equivalence relation.