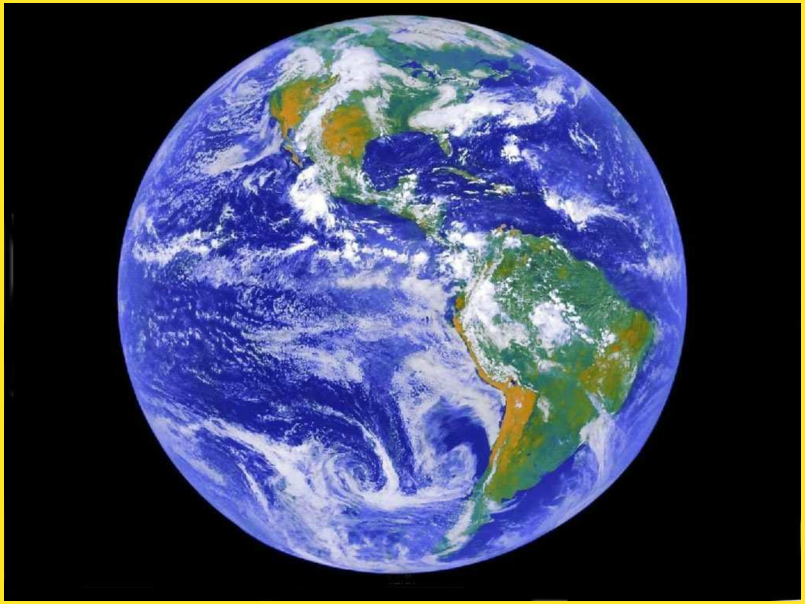
ABOUT A CIRCLE AND BEYOND

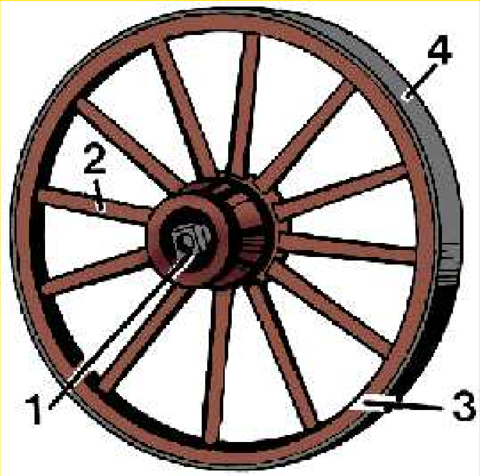
Kiryl Tsishchanka

kit@knox.edu

Department of Mathematics

Knox College

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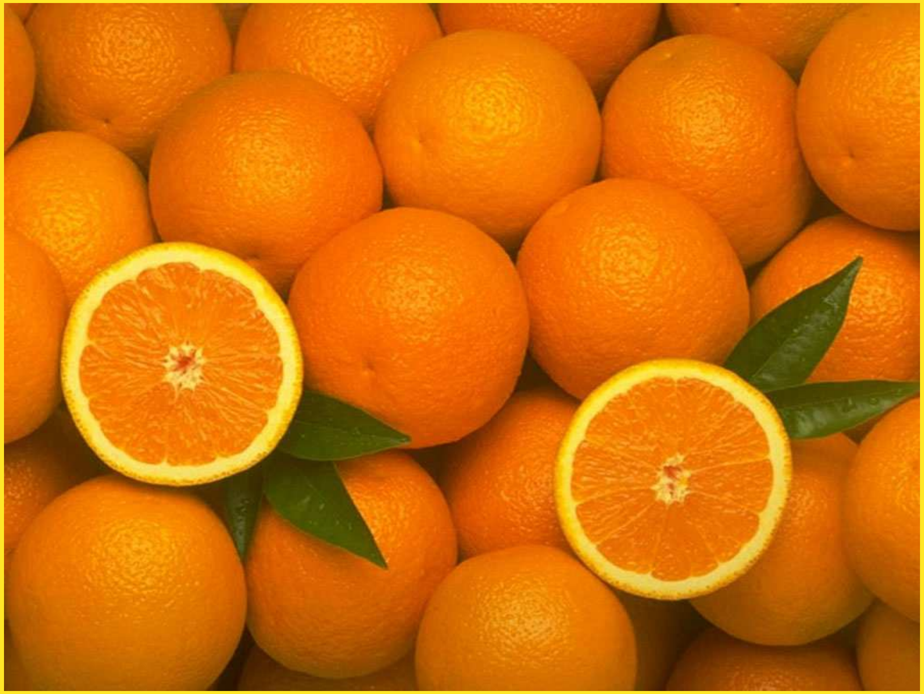

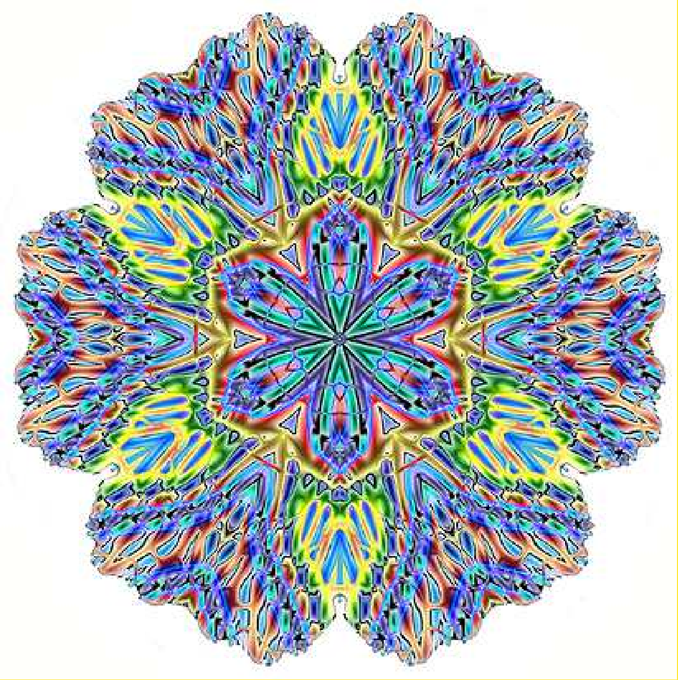

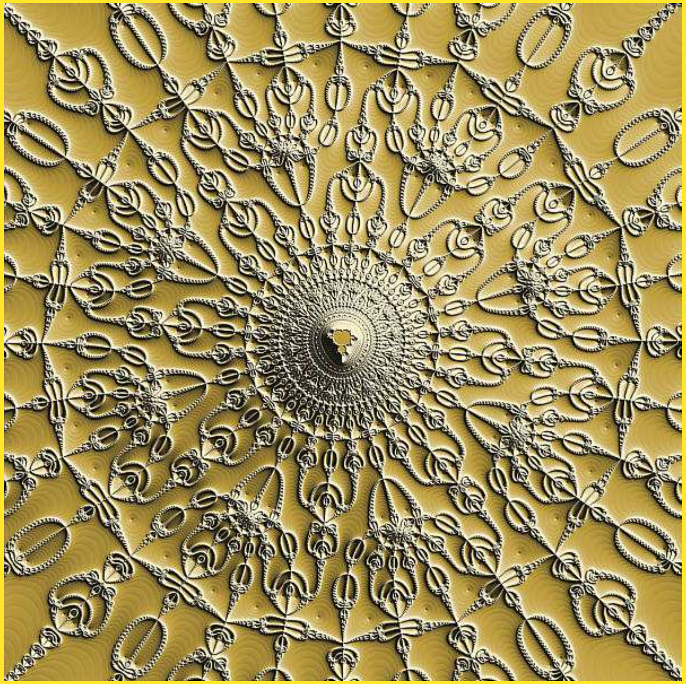

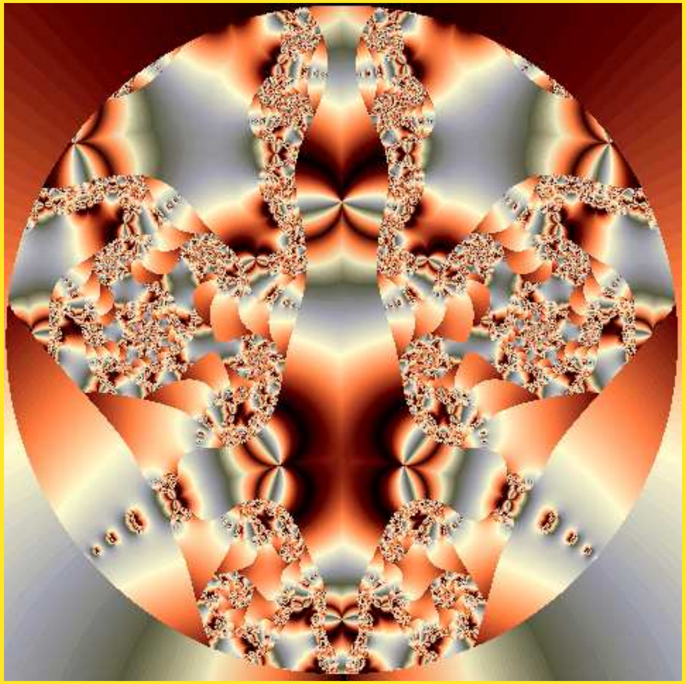

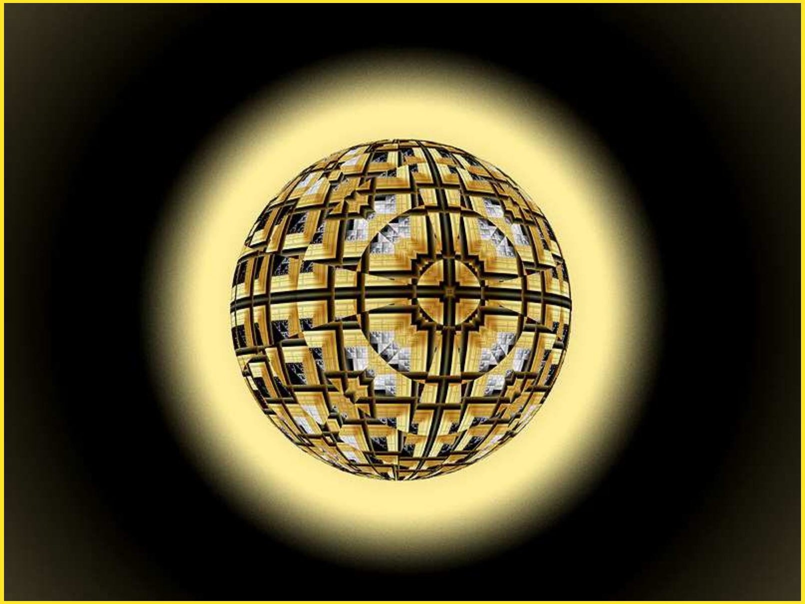

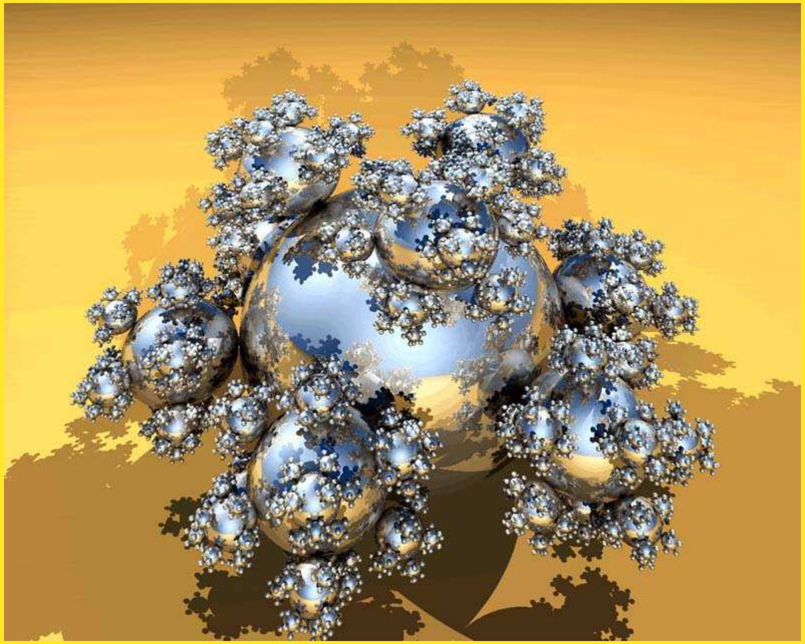

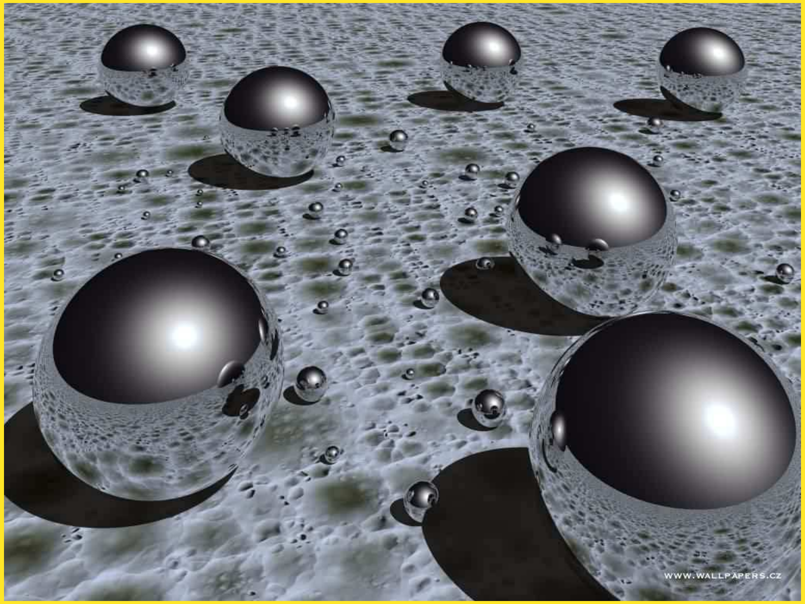
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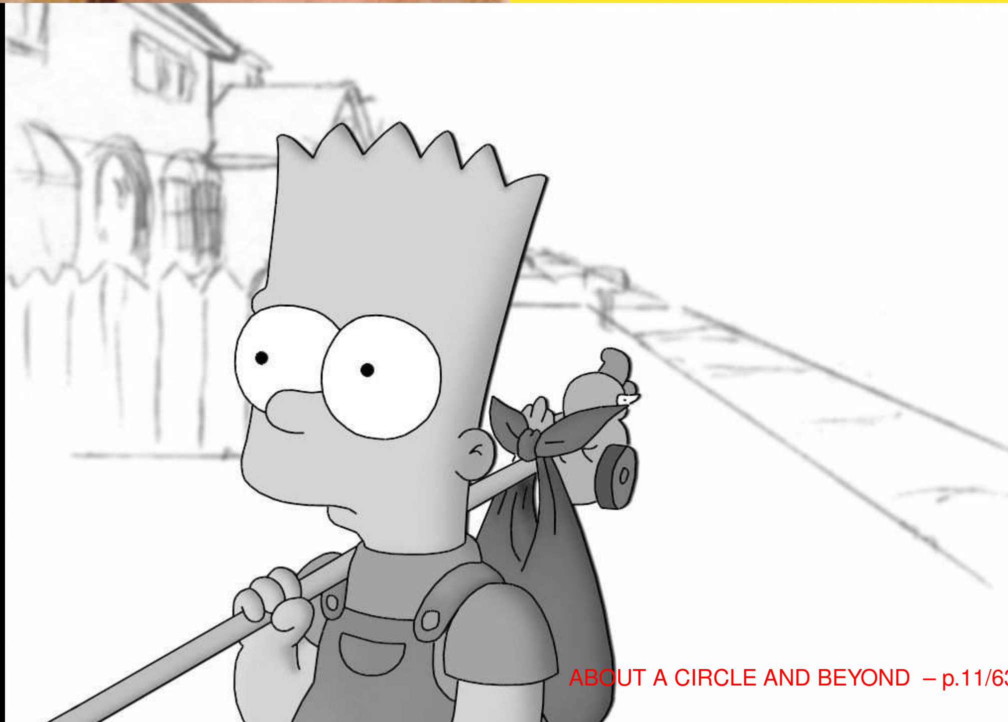

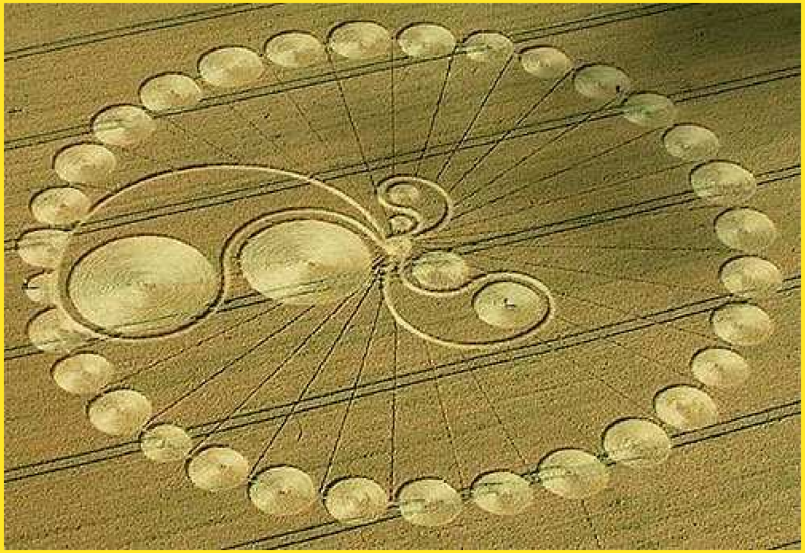

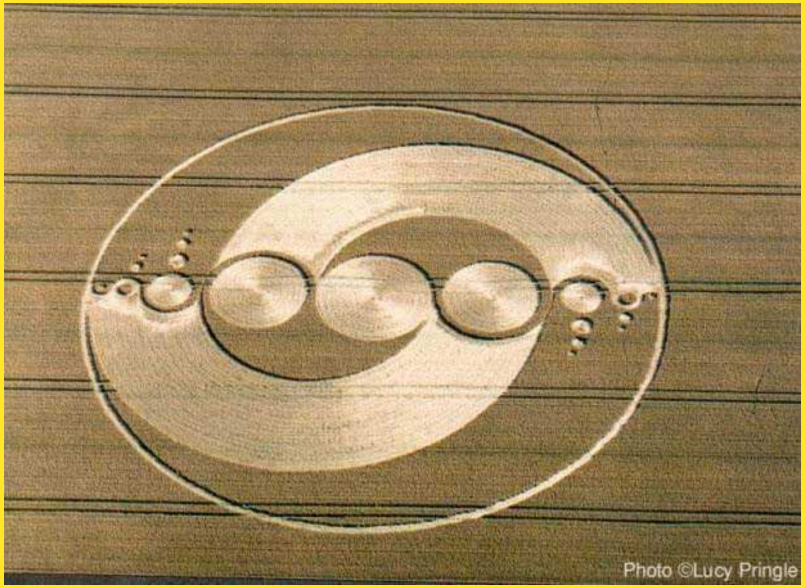
Photo ©Lucy Pringle

<u>DEFINITION</u>:

A circle is the set of points in a plane that are equidistant from a given point.

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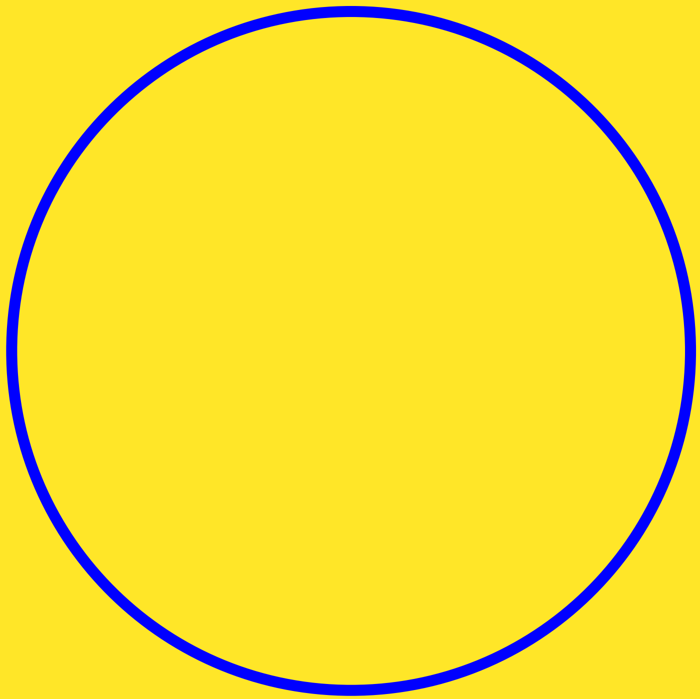

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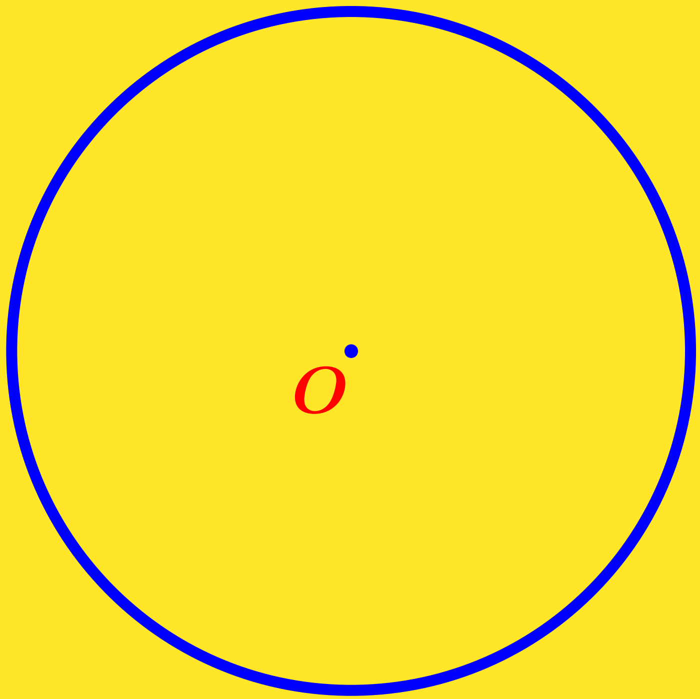

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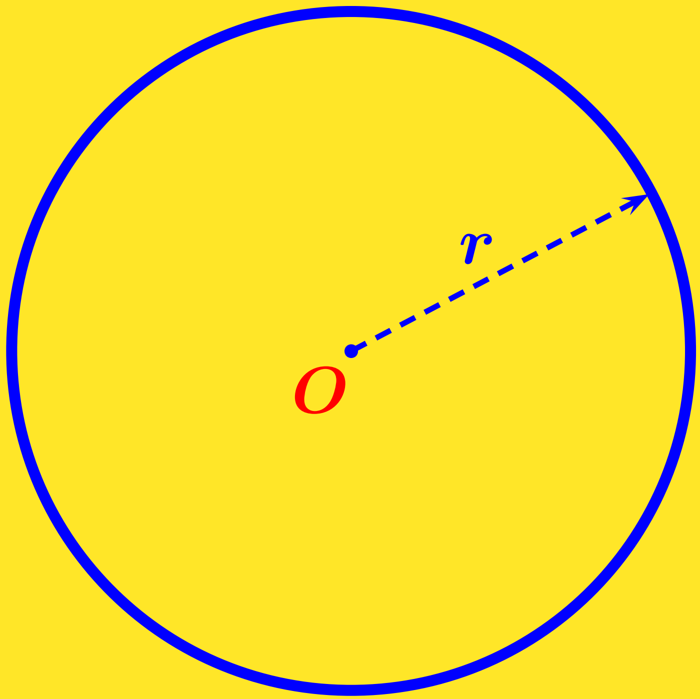

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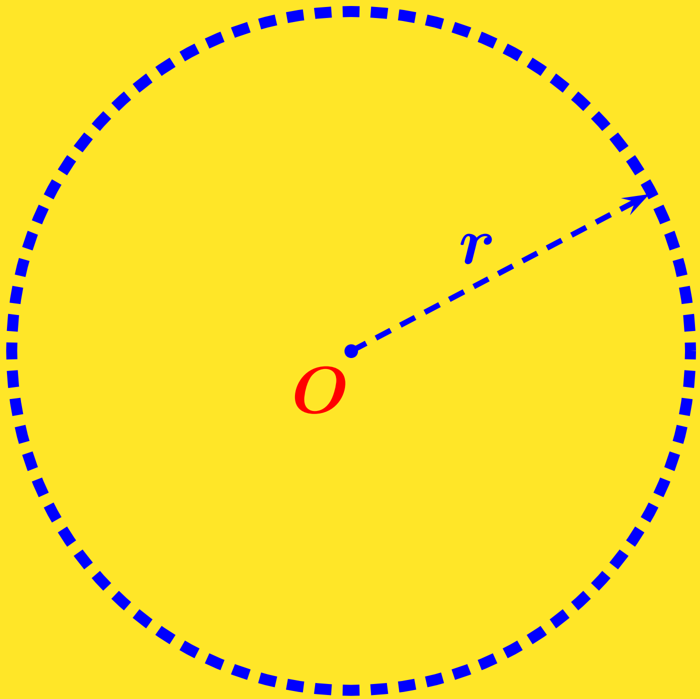

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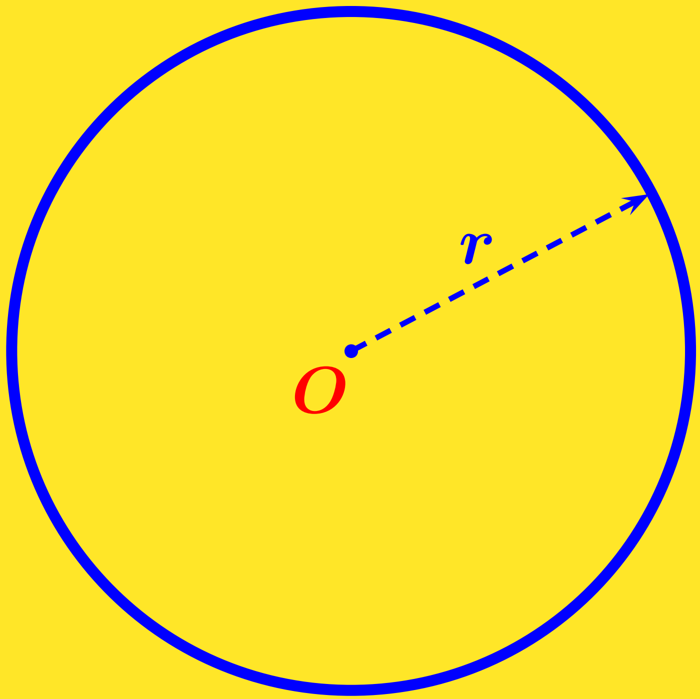

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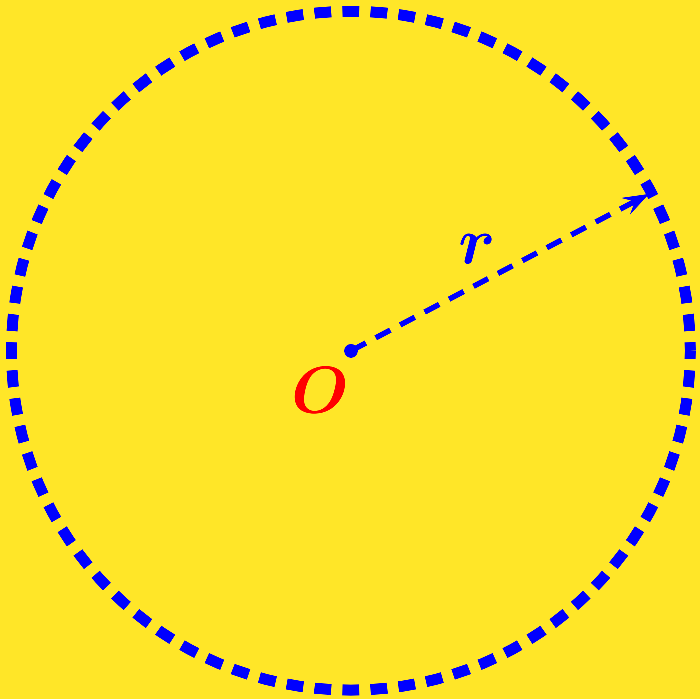

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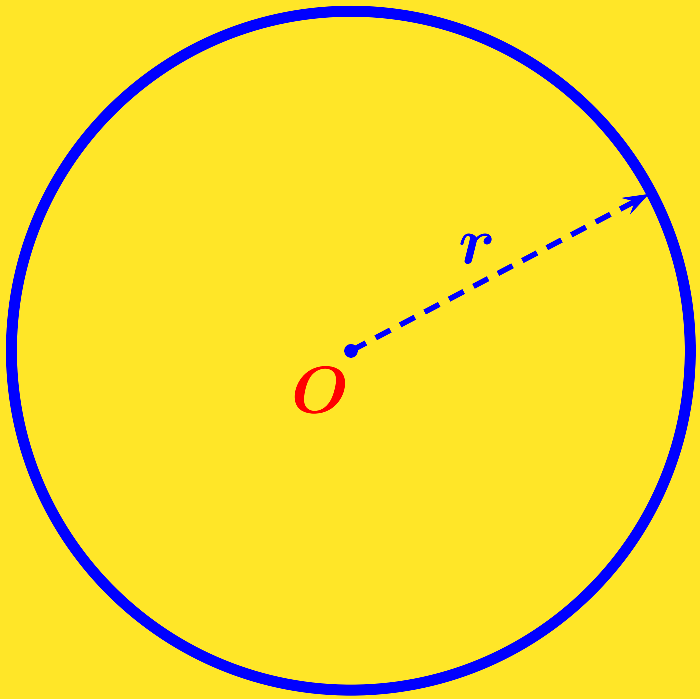

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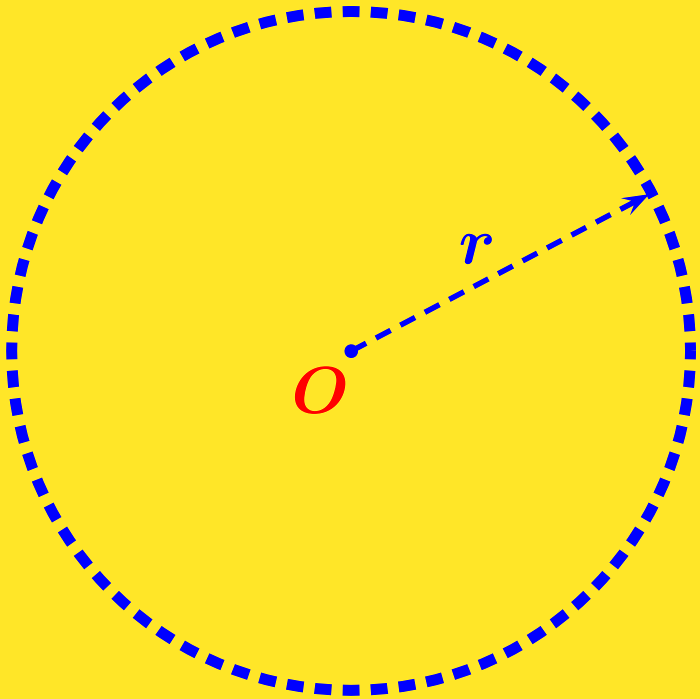

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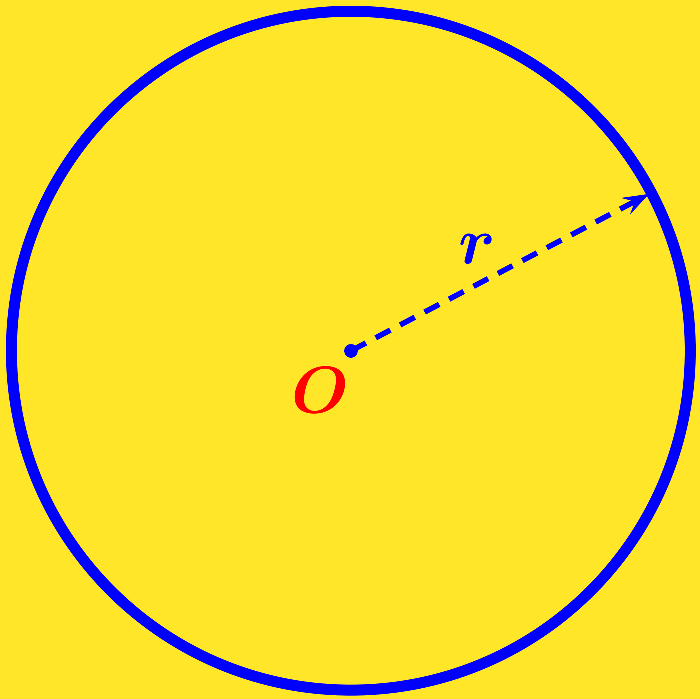

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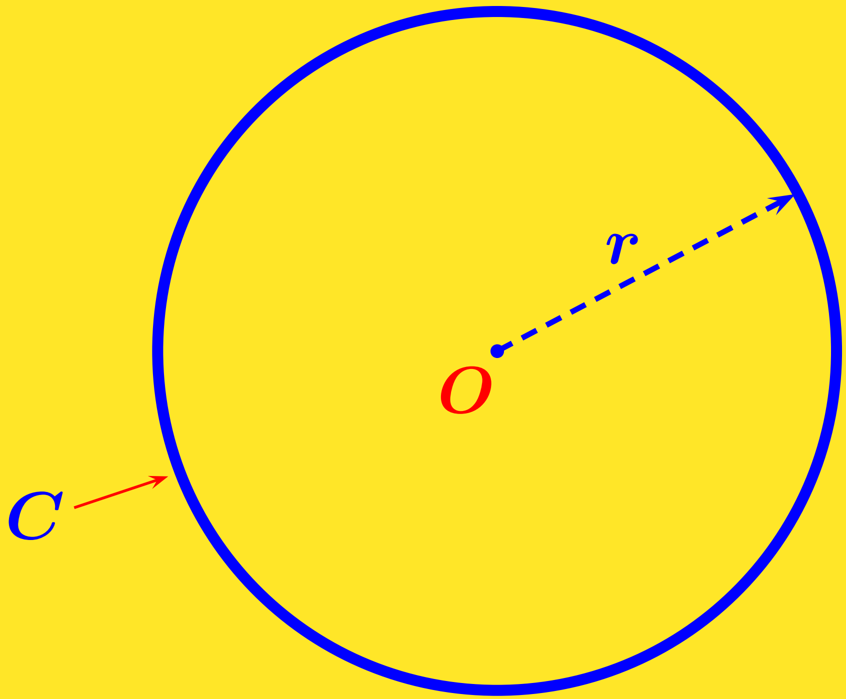

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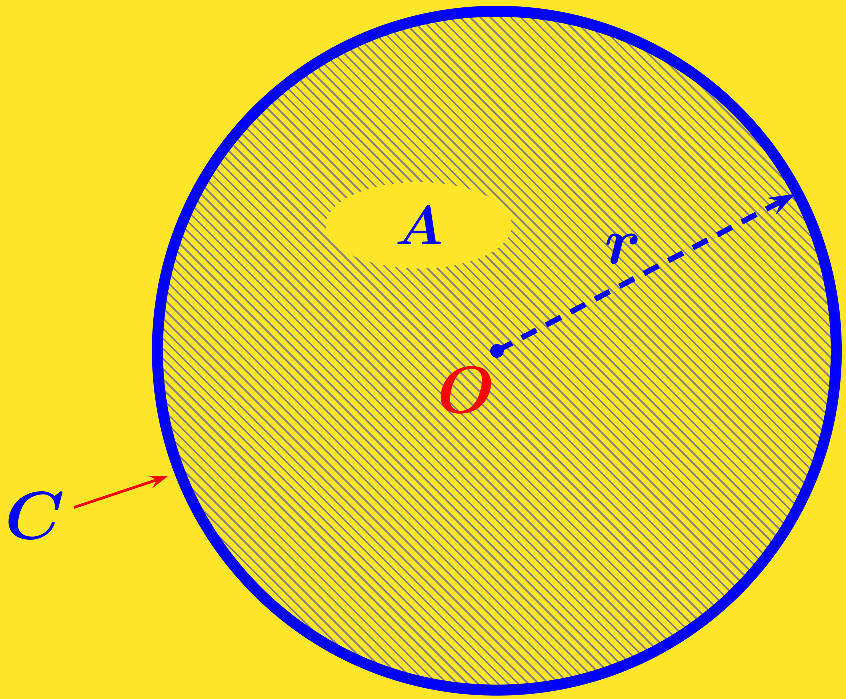

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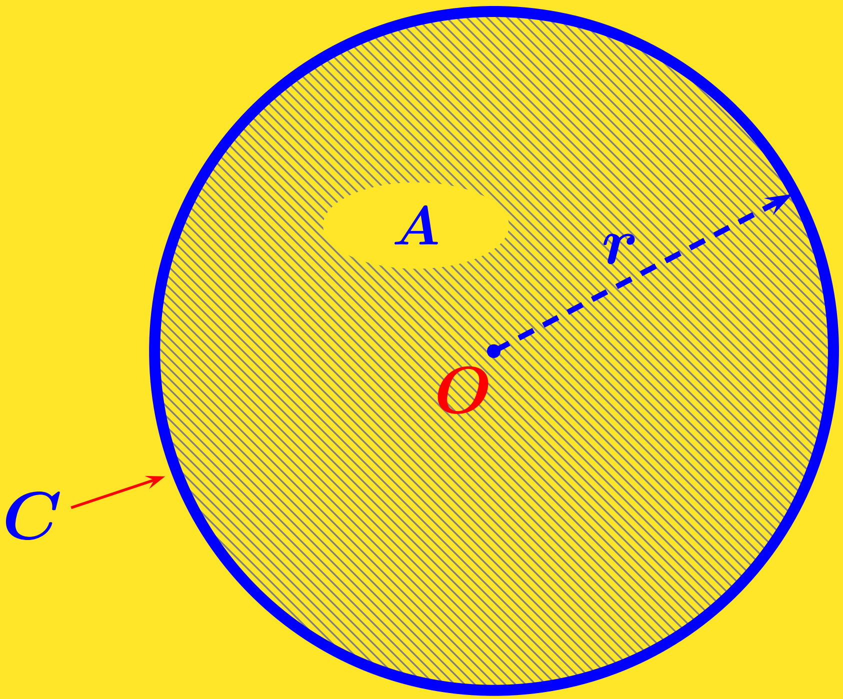

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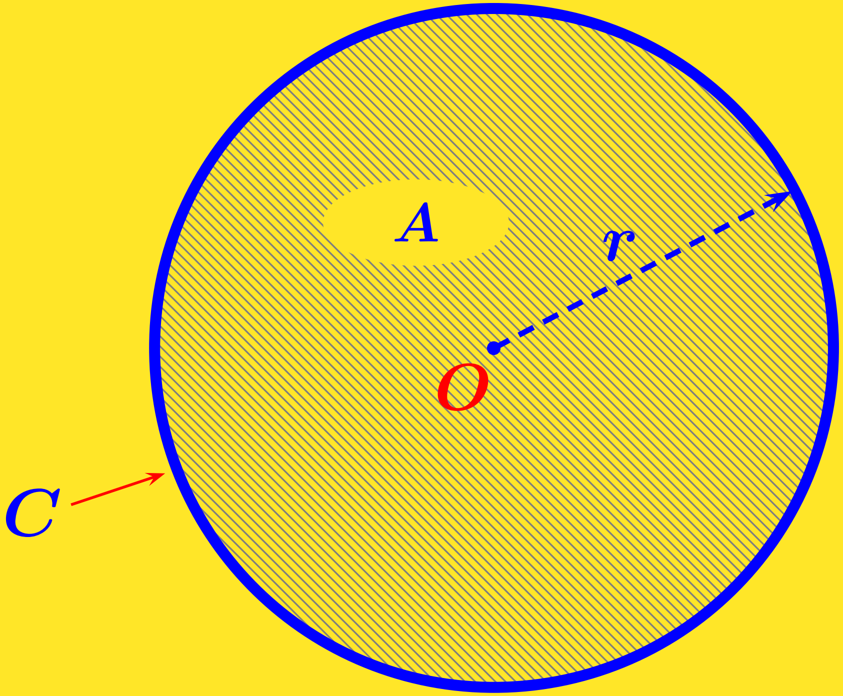

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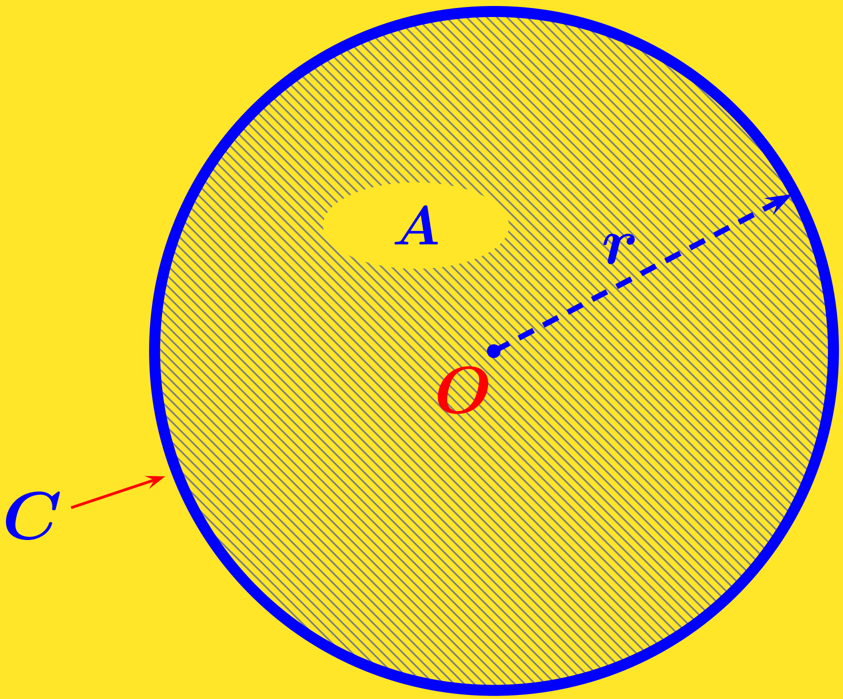

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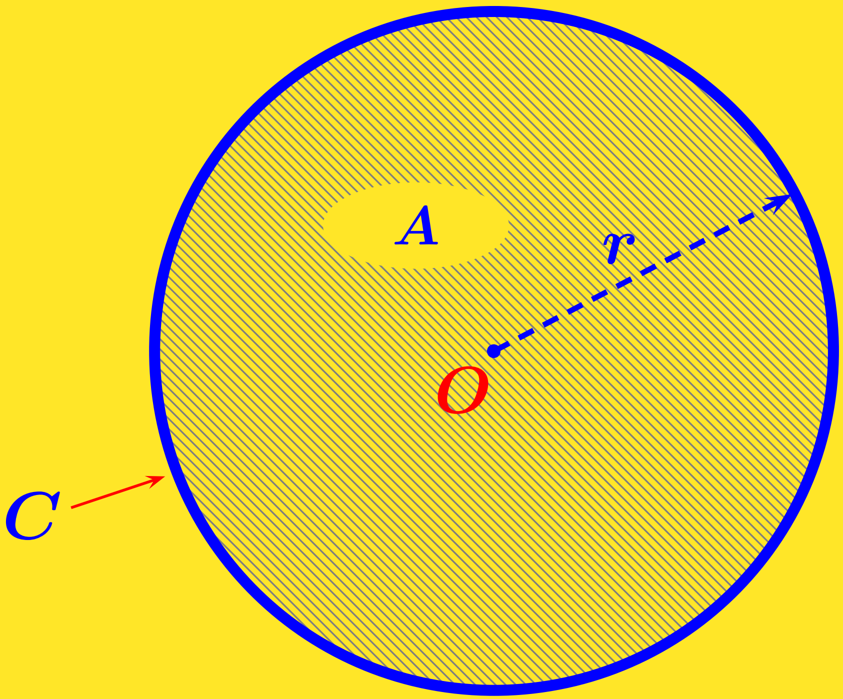

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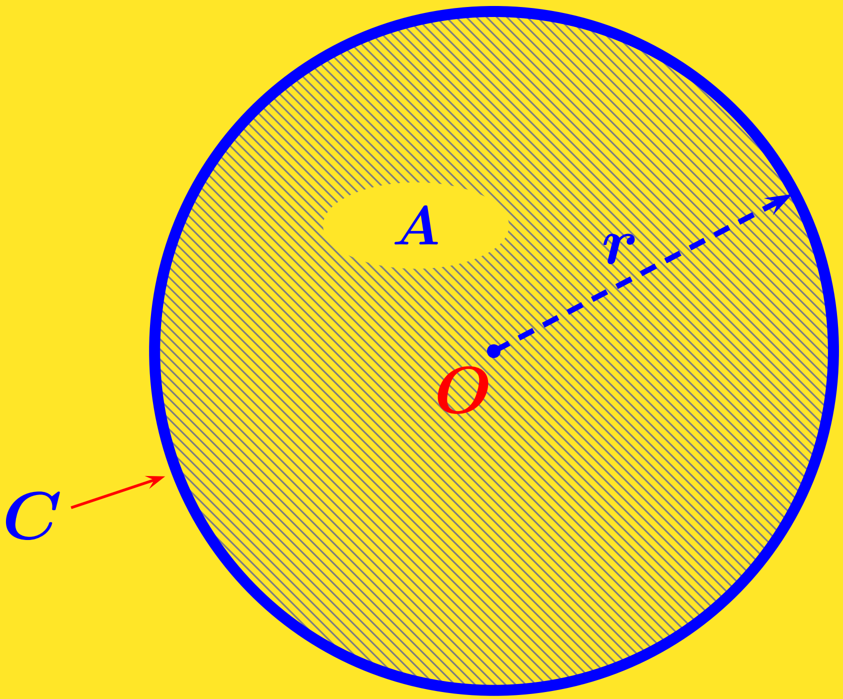

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Among all planar shapes with the same perimeter the circle has the largest area.

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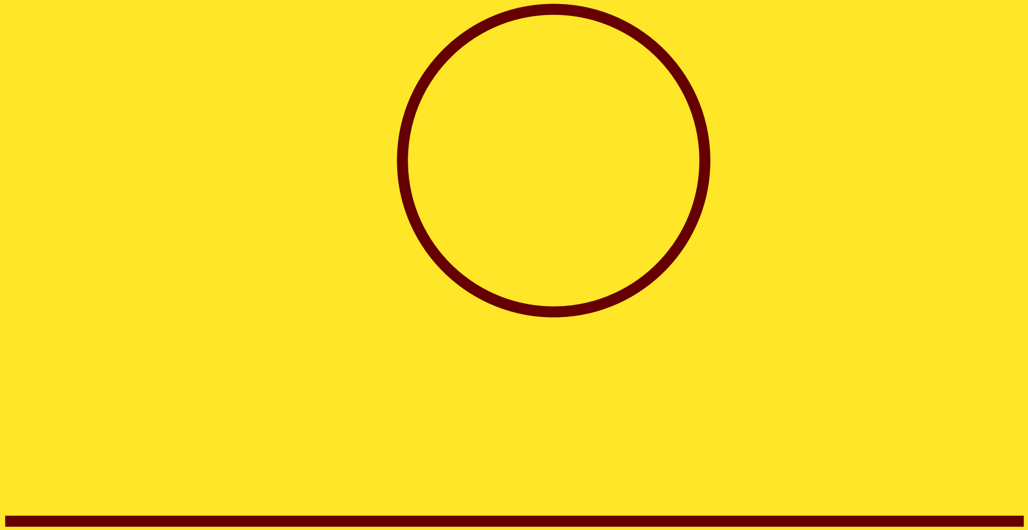

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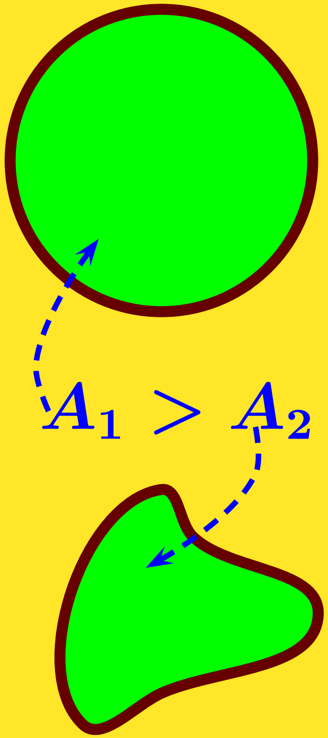

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Is it possible to construct a square equal in area to a circle using only a straight-edge and compass?

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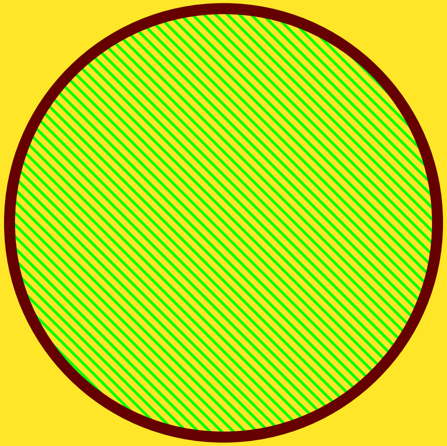

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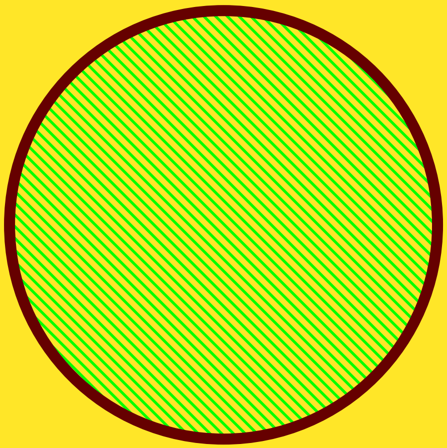

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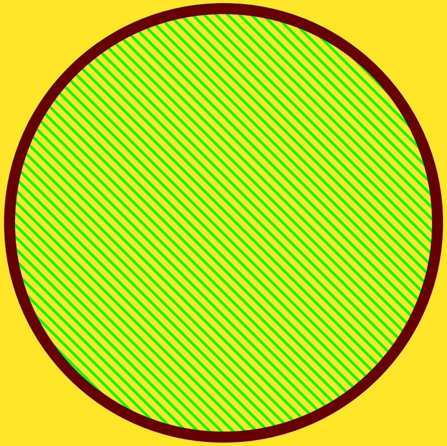

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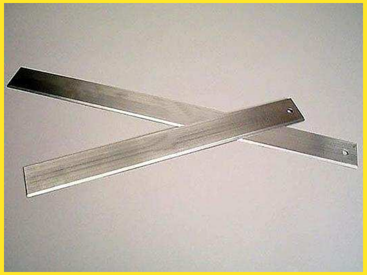

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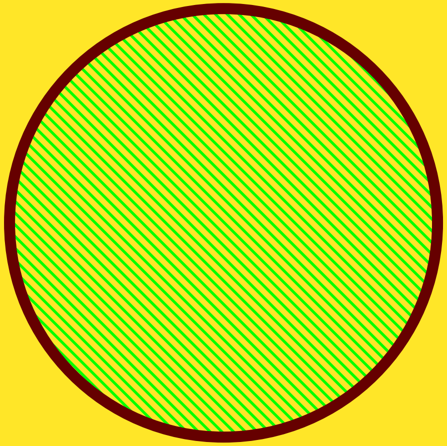

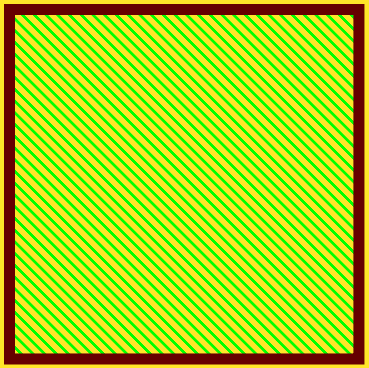

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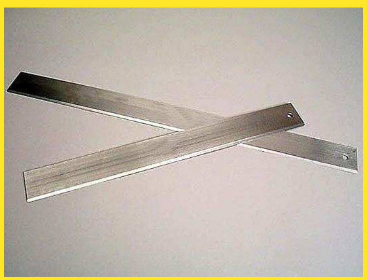

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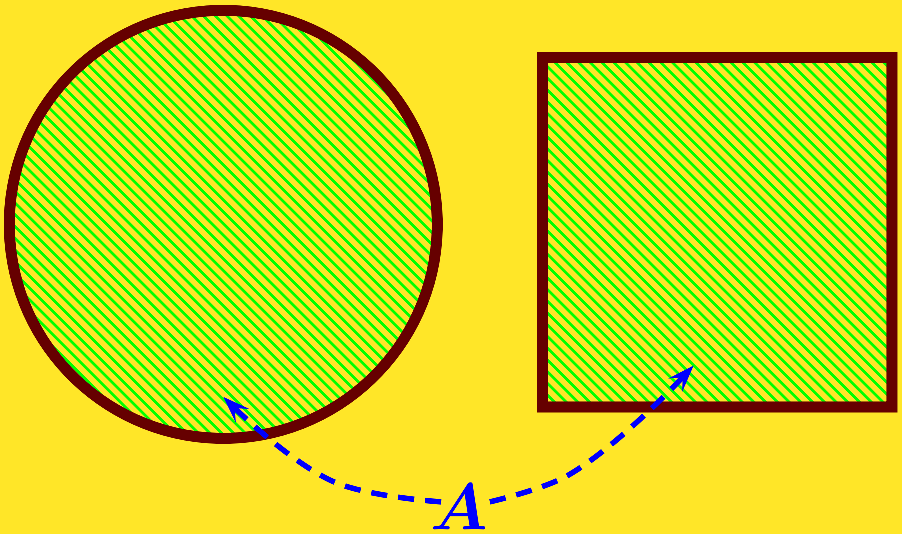

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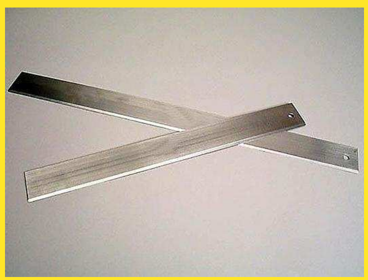

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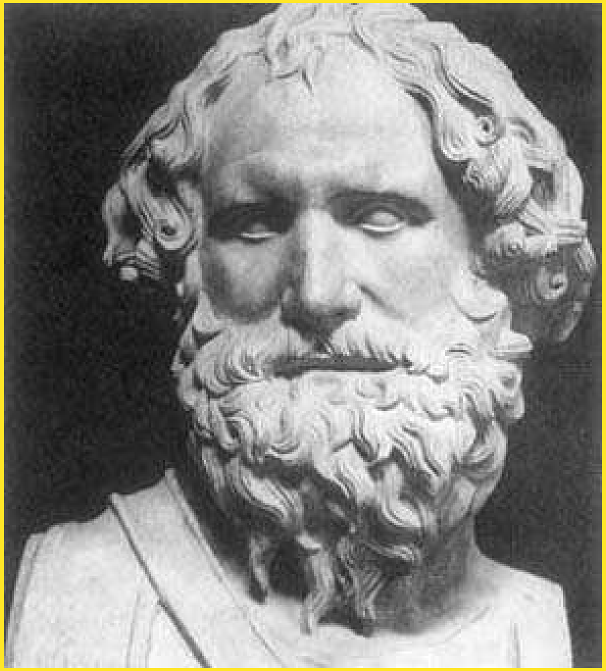

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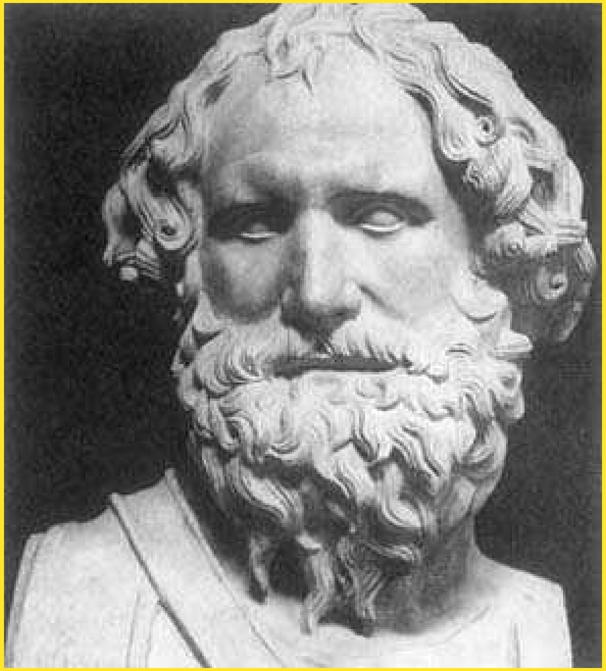

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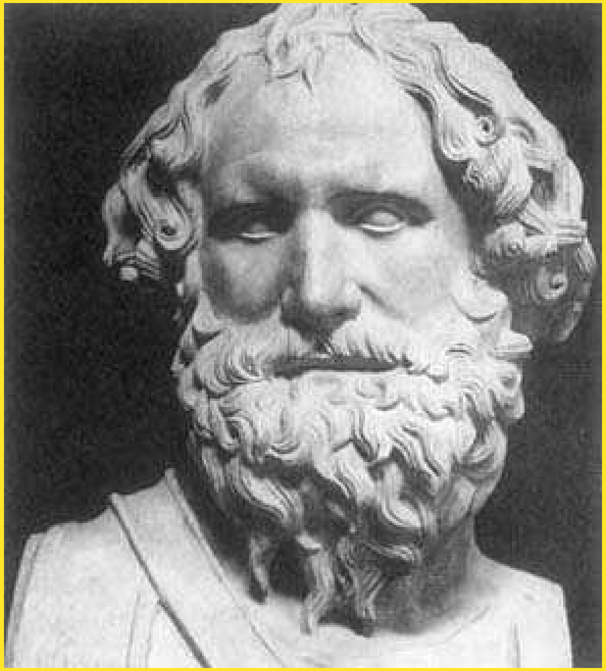

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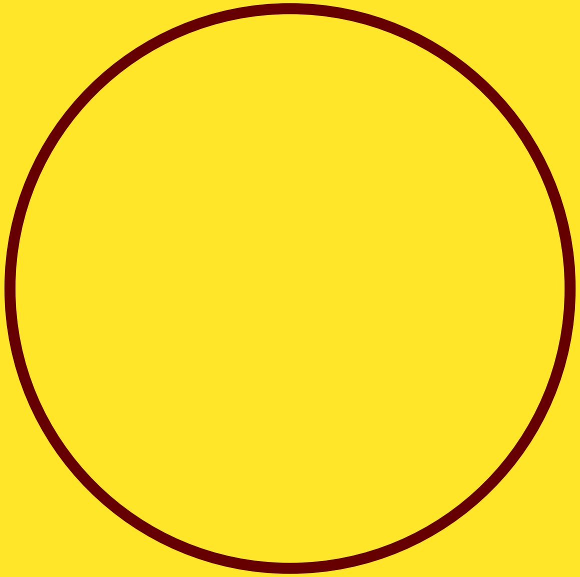

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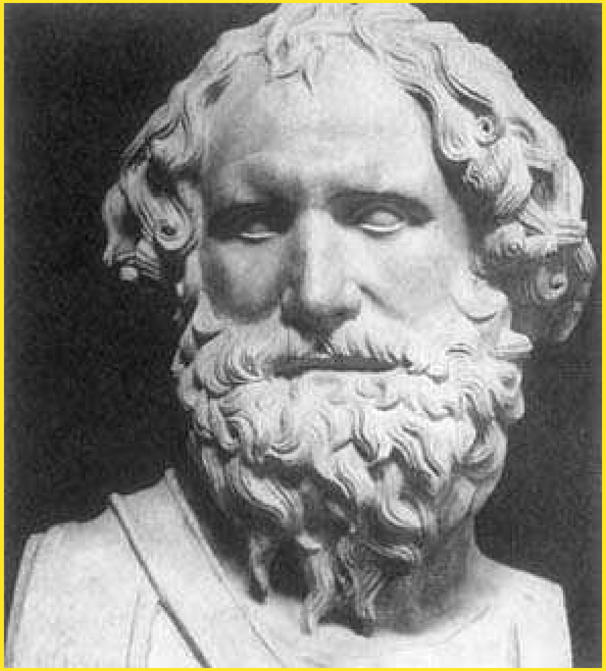

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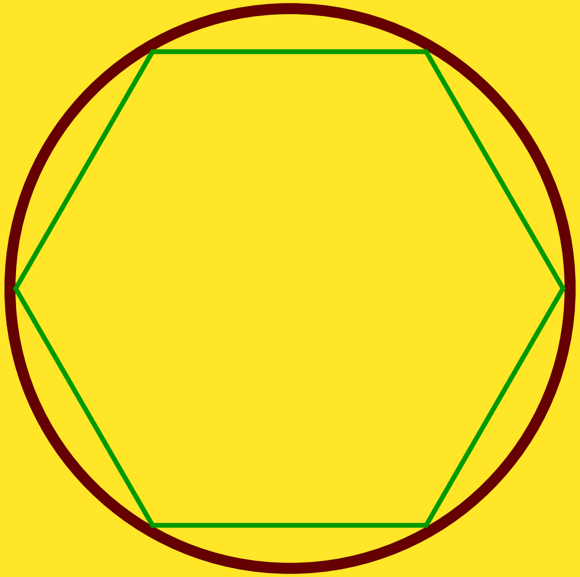

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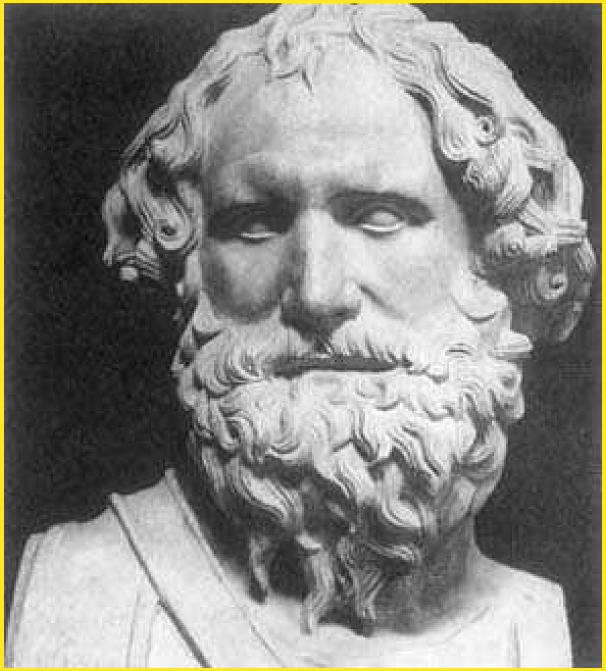

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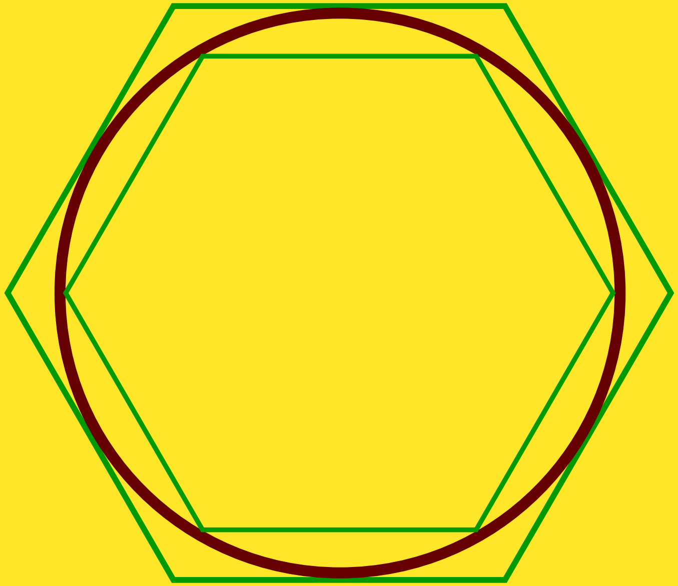

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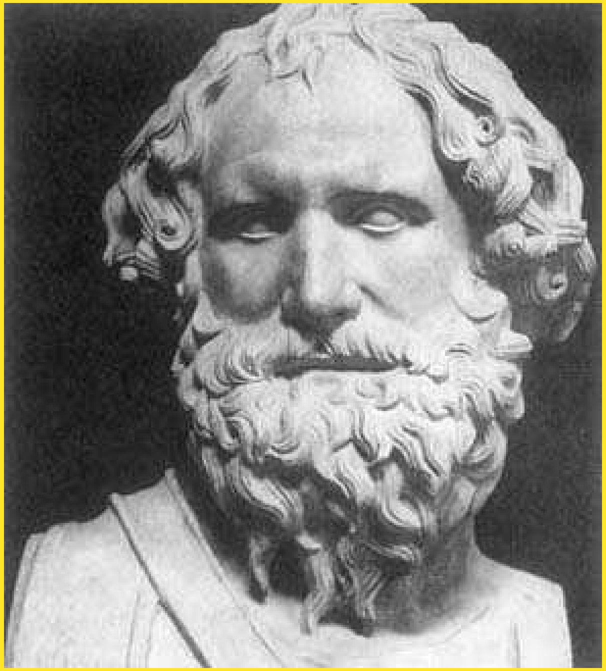

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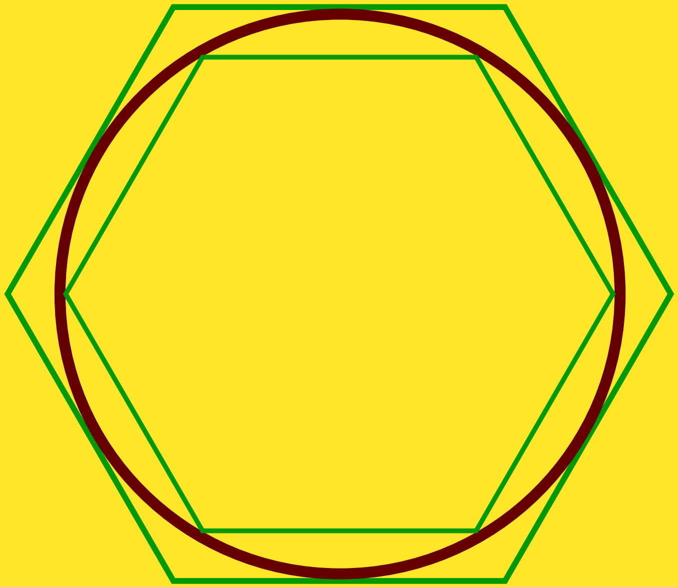

$$3 < \pi < 2\sqrt{3}$$

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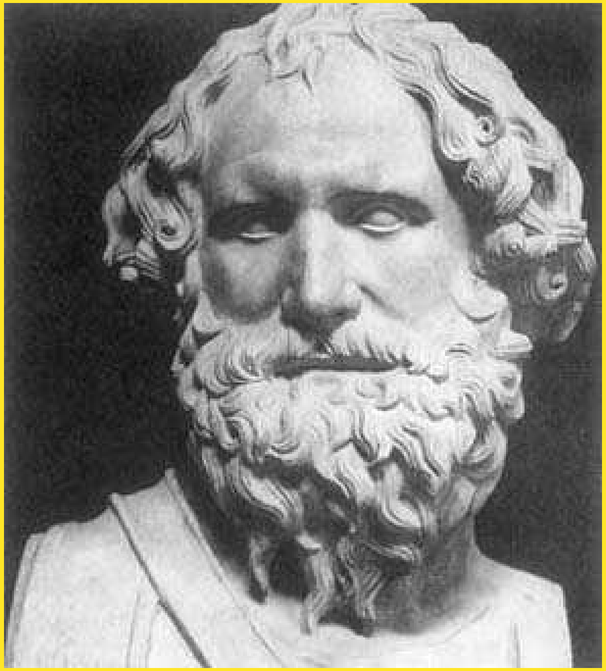

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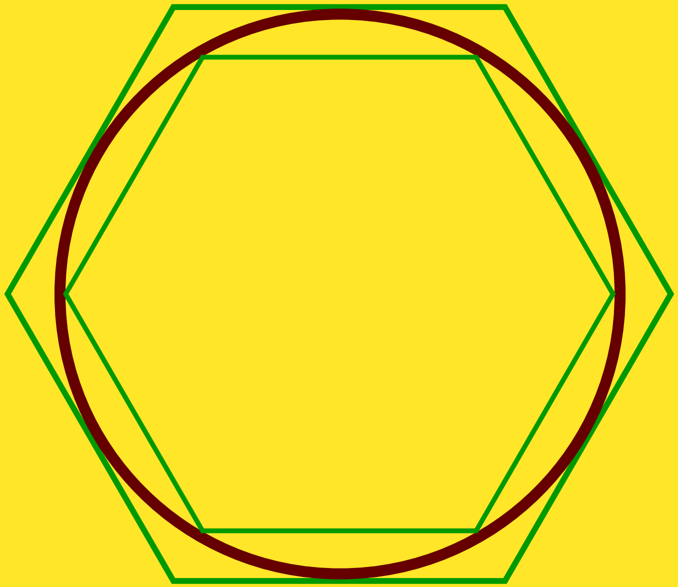

$$3 < \pi < 2\sqrt{3} \approx 3.46$$

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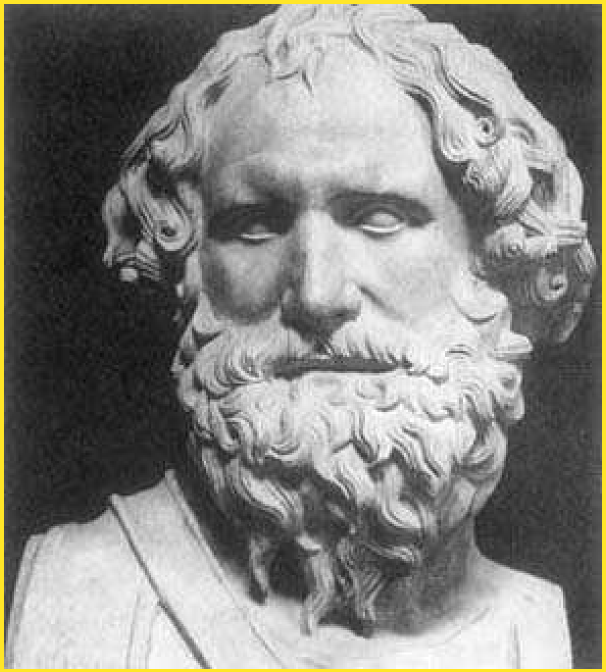

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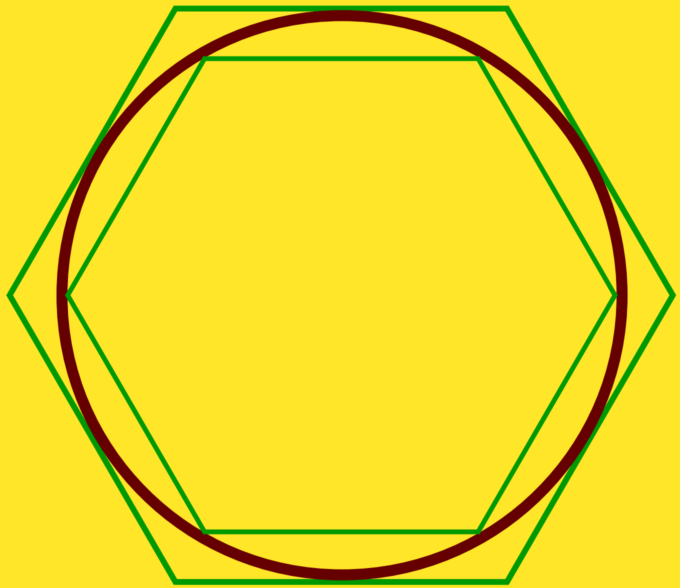

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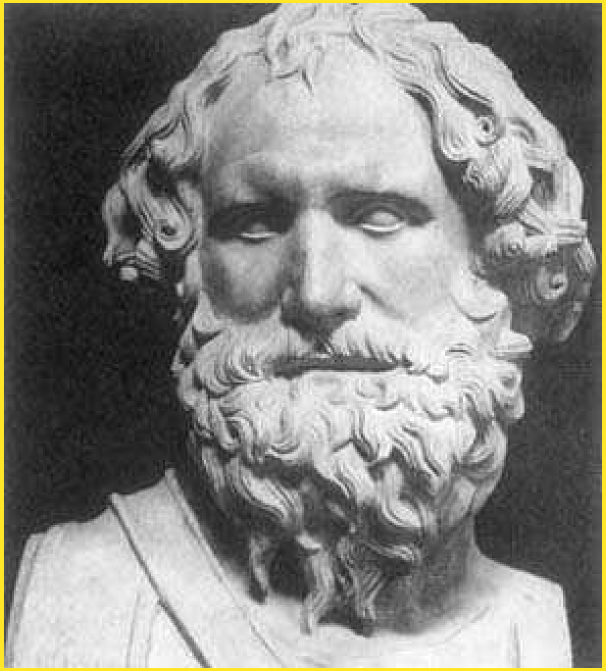

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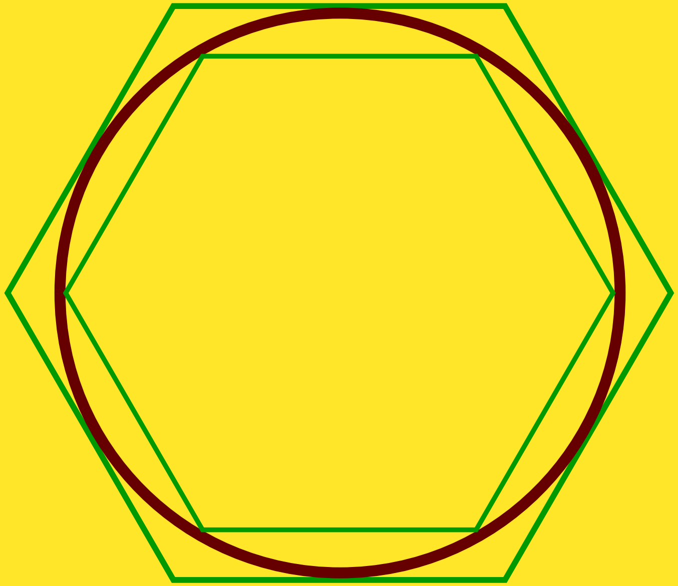

$$3.14084 \approx \frac{223}{71} < \pi < \frac{22}{7} \approx 3.14285$$

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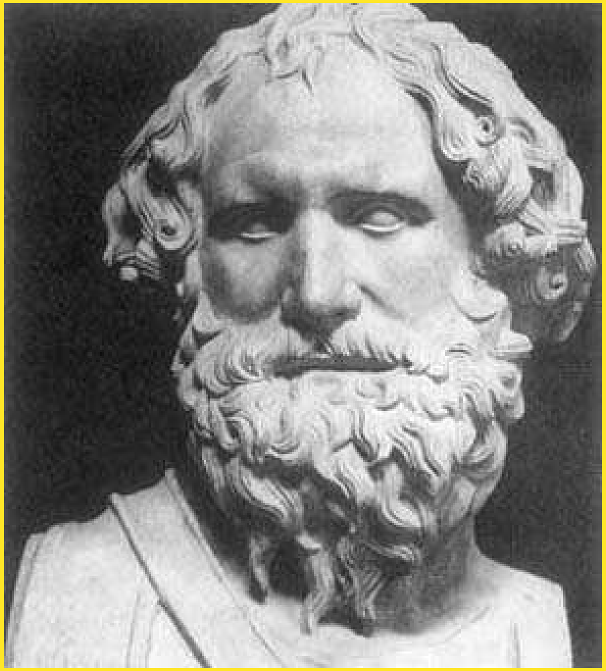

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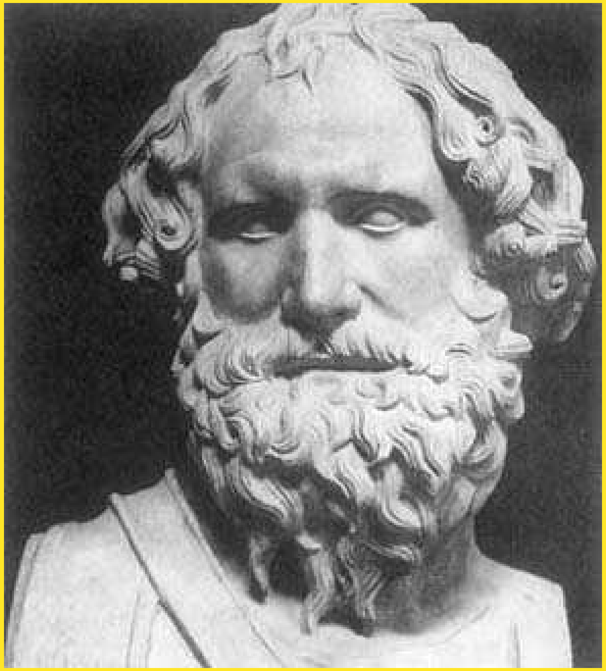
$$C = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} r\, d\theta = 2\pi r$$

DEFINITION:

A circle is the set of points in a plane that are equidistant from a given point.



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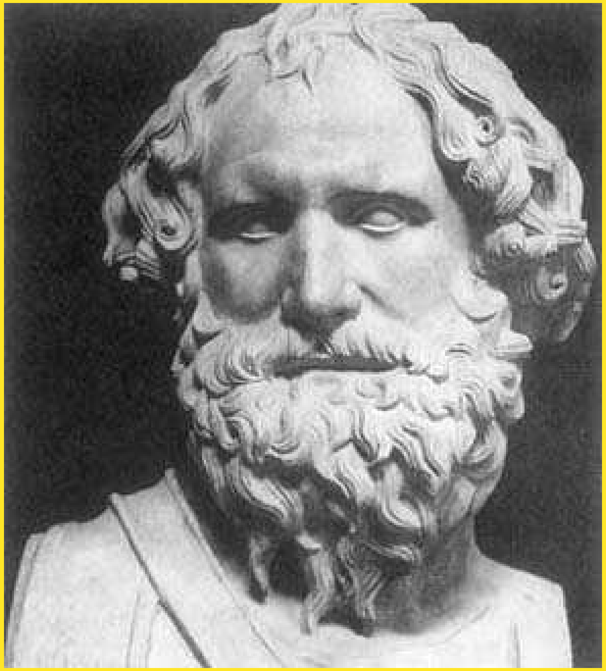

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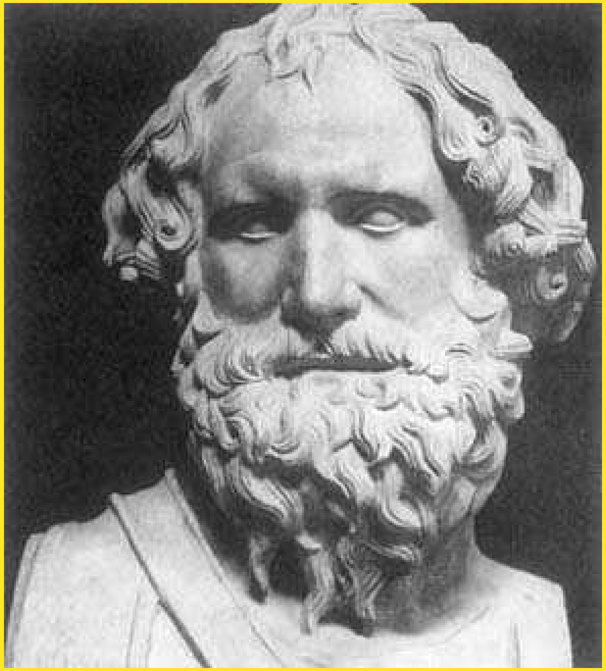

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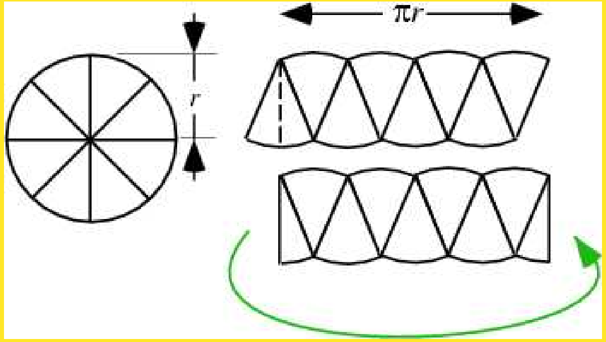

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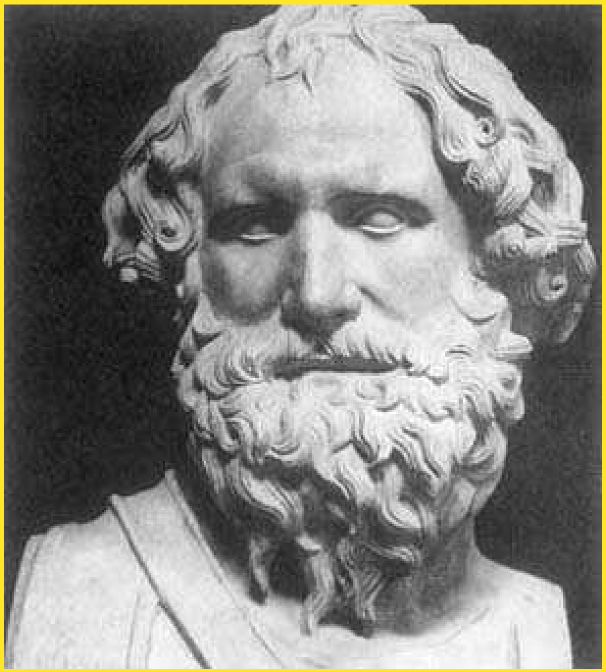

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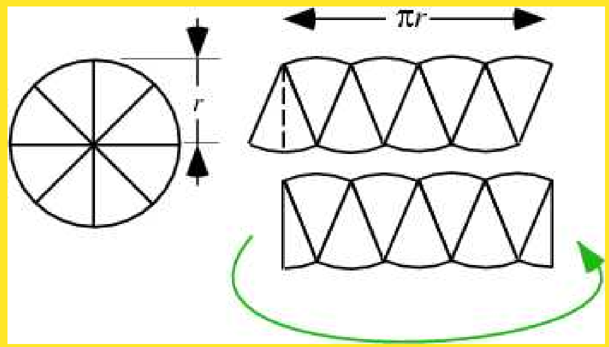

$$A = (\pi r)r = \pi r^2$$

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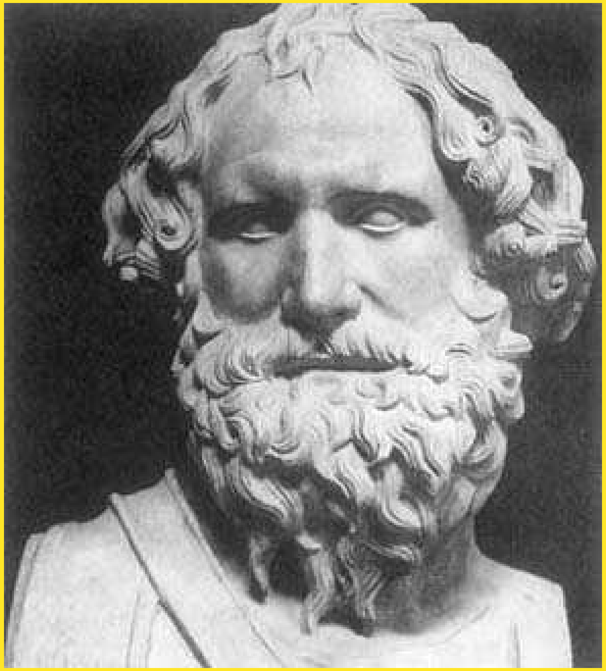

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$$\begin{aligned} A &= \int_0^{2\pi} d\theta \int_0^{r} r\,dr \\ &= (2\pi)\left(\frac{1}{2}r^2\right) \\ &= \pi r^2 \end{aligned}$$

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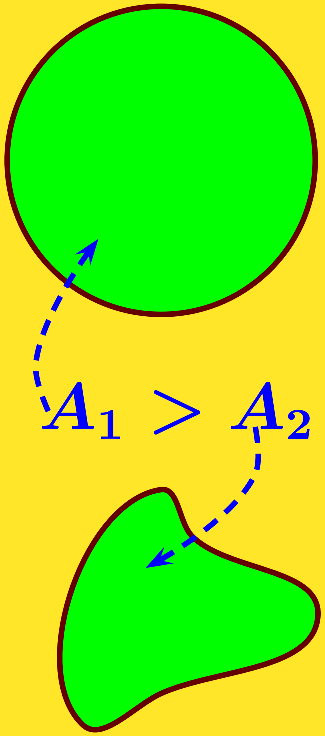

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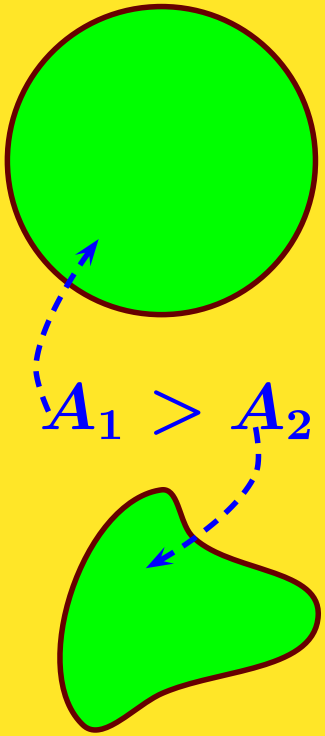

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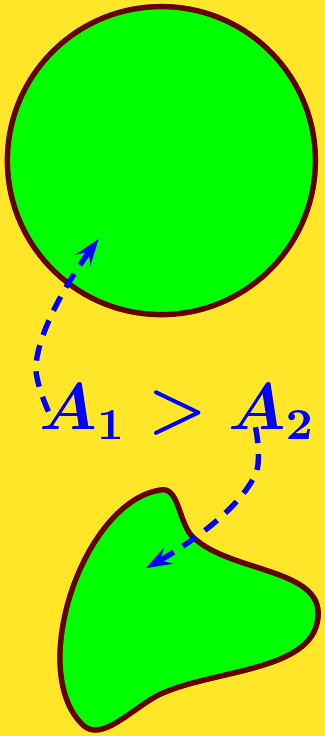

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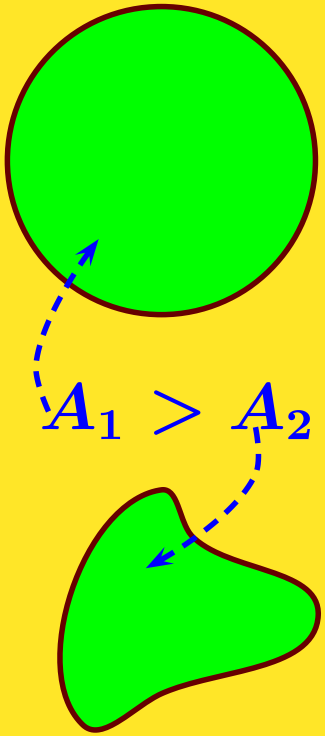

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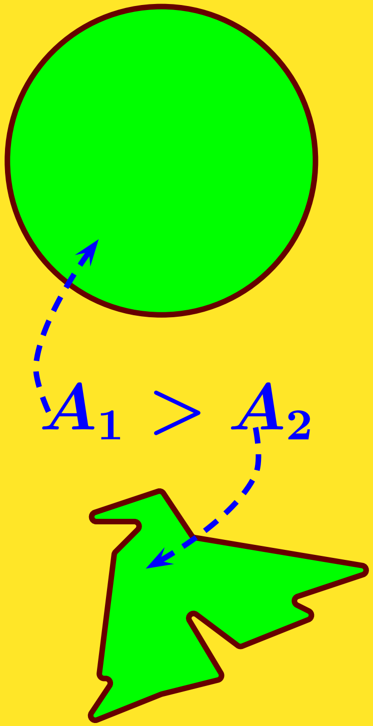

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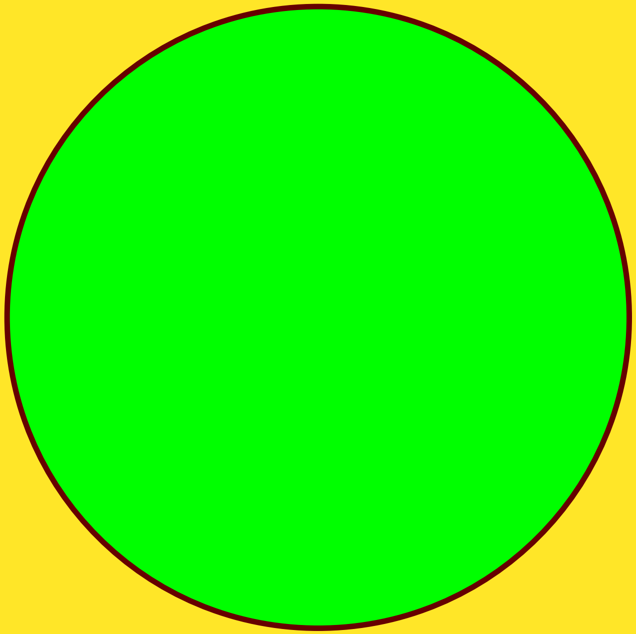

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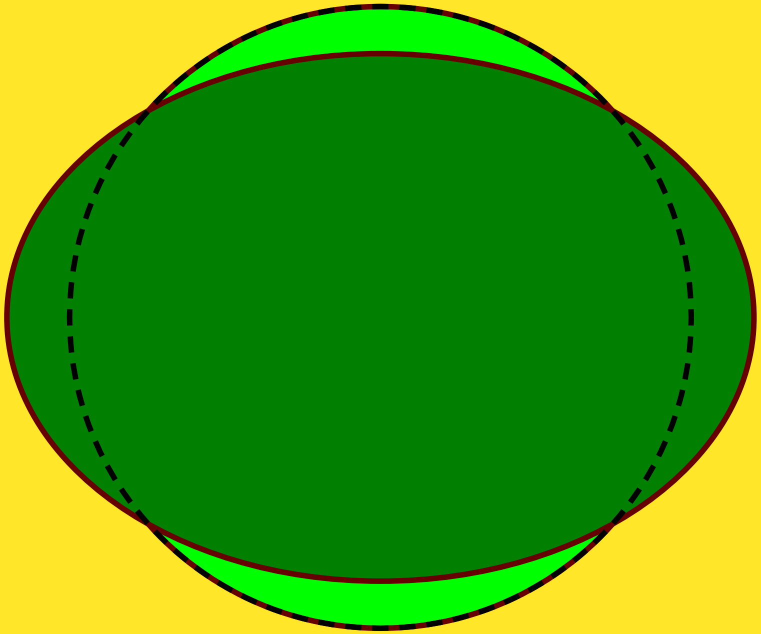

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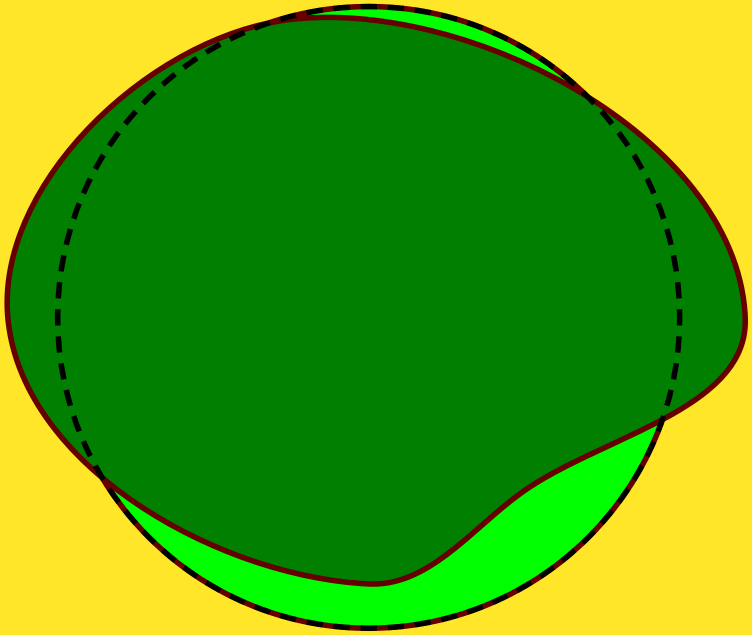

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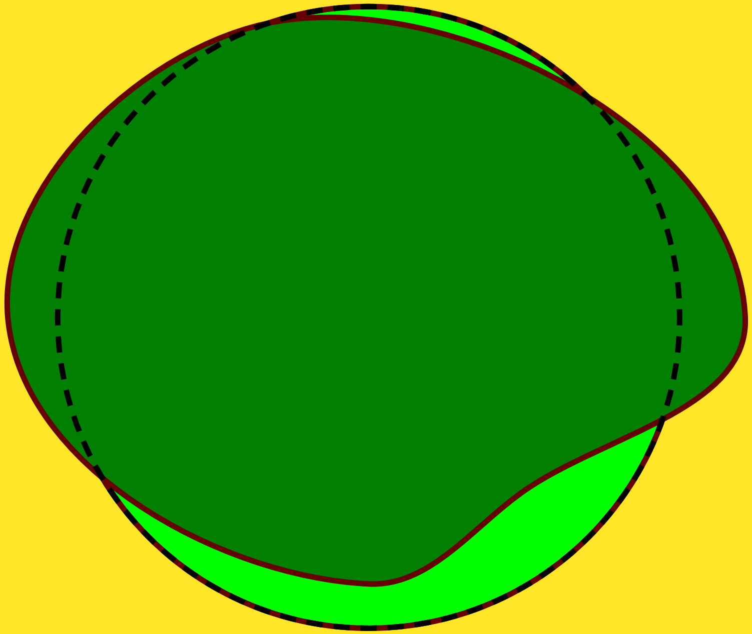

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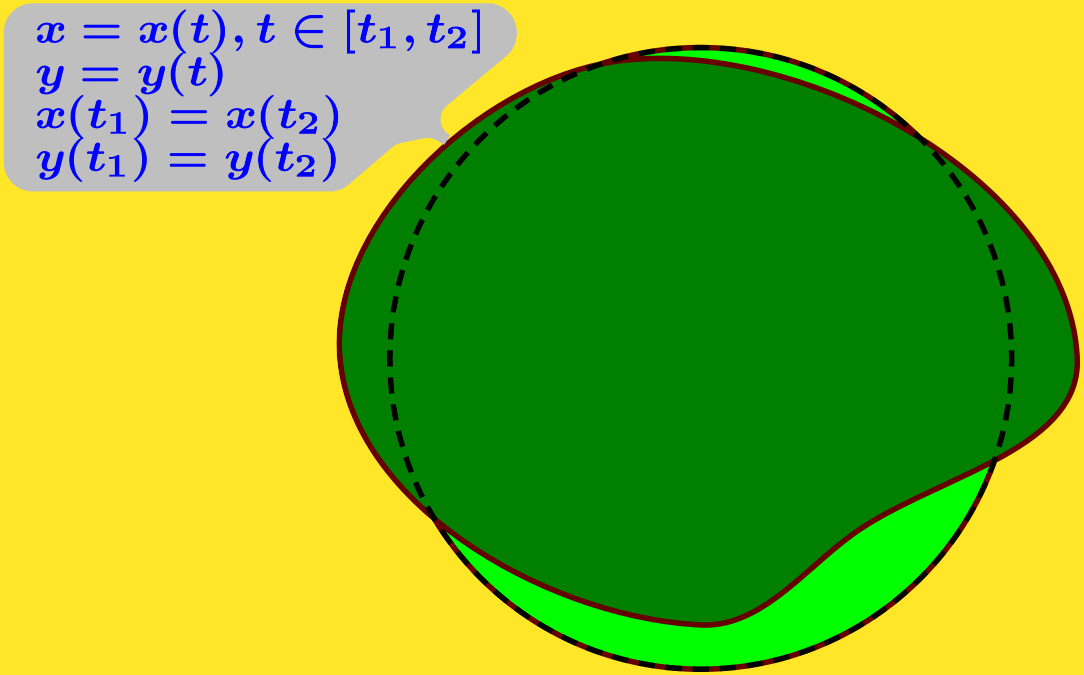

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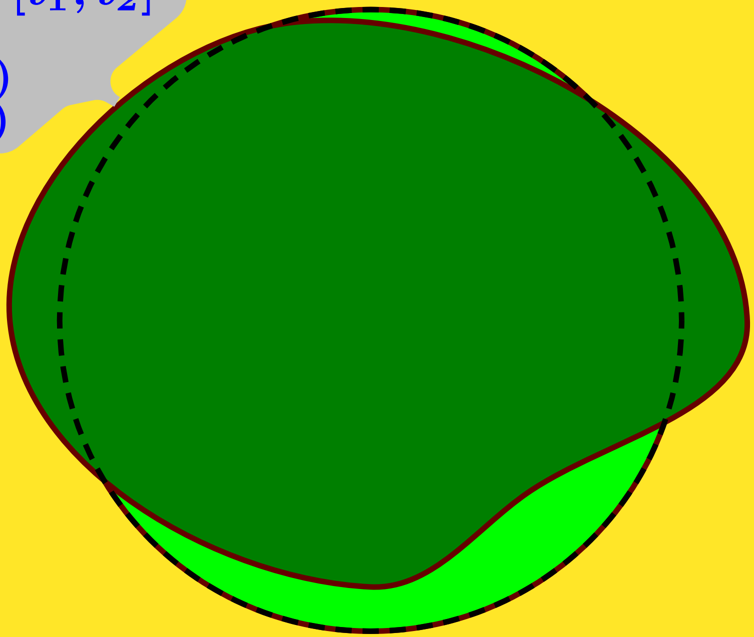
$x = x(t), t \in [t_1, t_2]$
$y = y(t)$
$x(t_1) = x(t_2)$
$y(t_1) = y(t_2)$



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$$\ell = \int_{t_1}^{t_2} \sqrt{x'^2 + y'^2}\, dt, \quad A = \frac{1}{2} \int_{t_1}^{t_2} (xy' - x'y)\, dt$$

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Jakob Steiner
(1796 - 1863)

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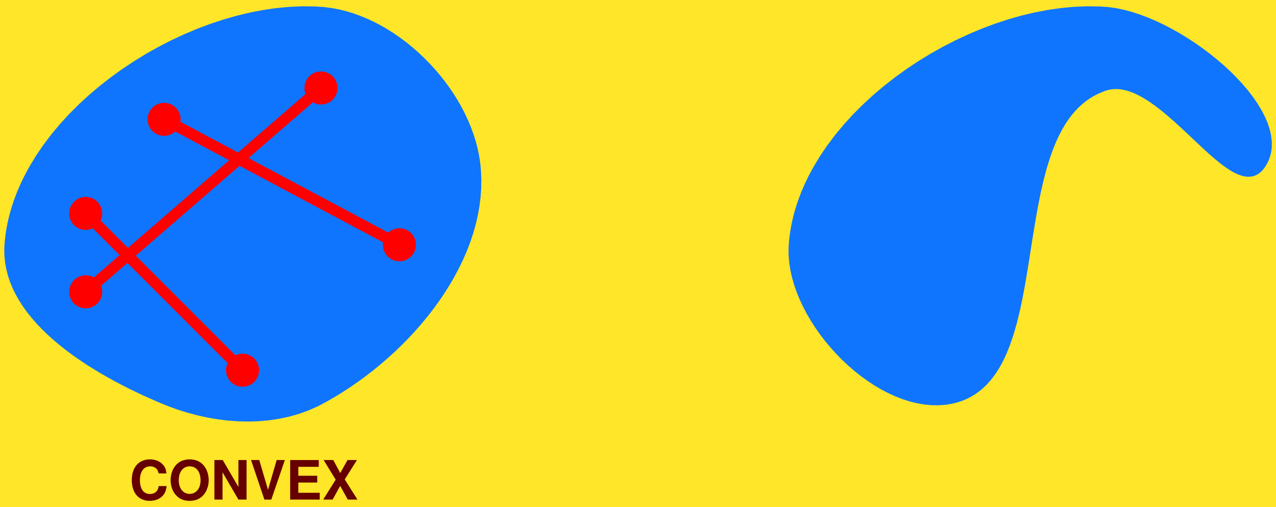

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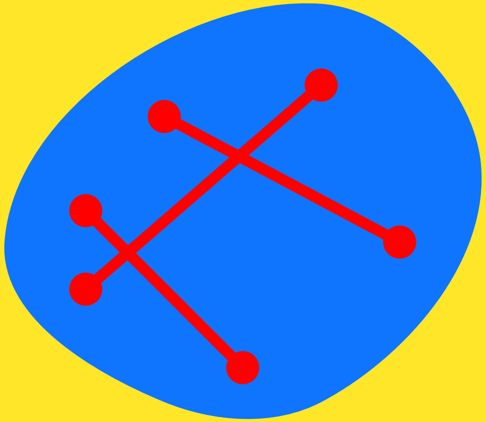

CONVEX

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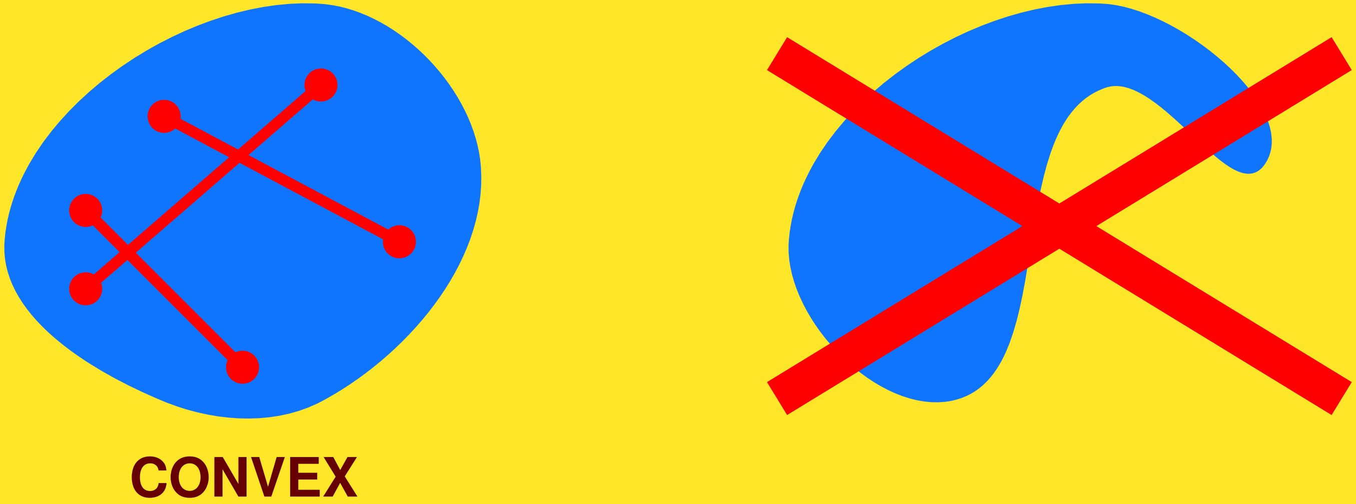

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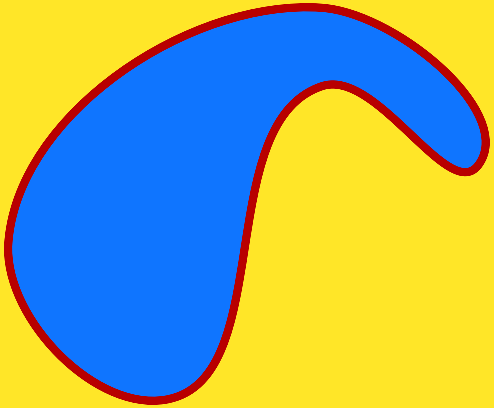

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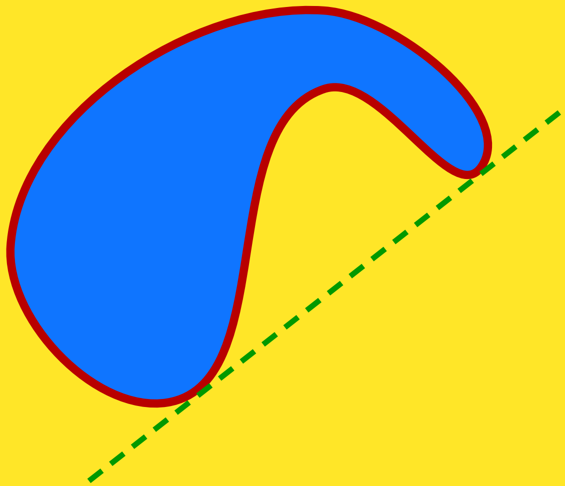

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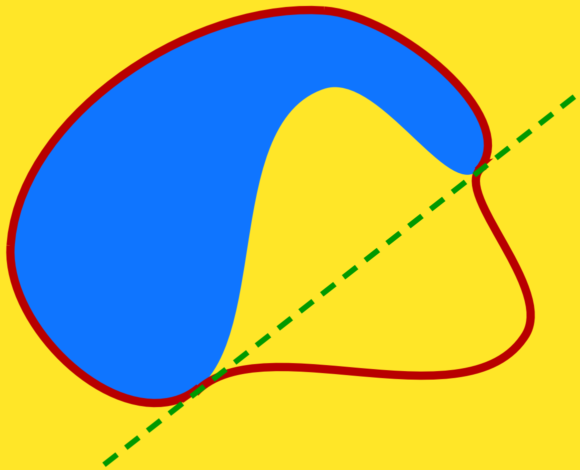

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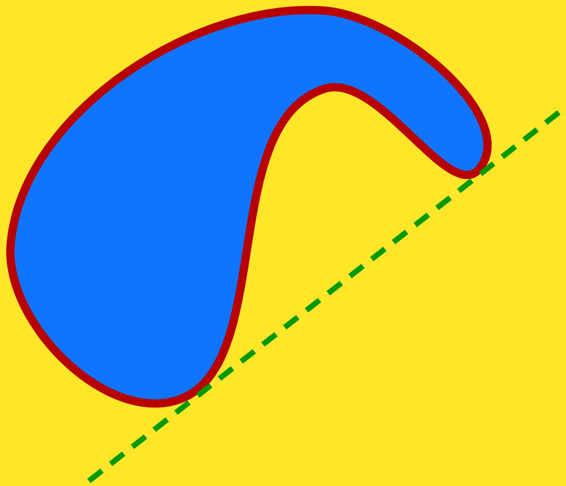

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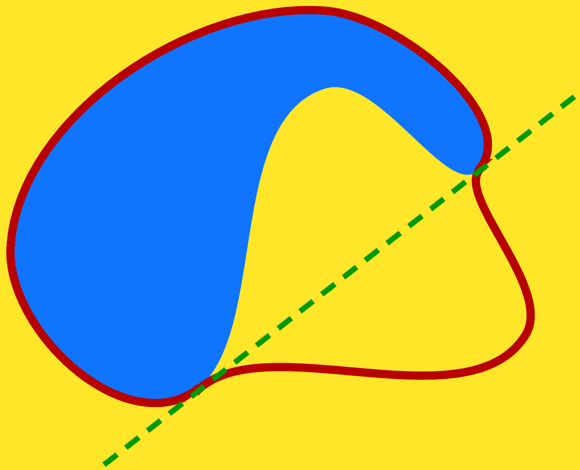

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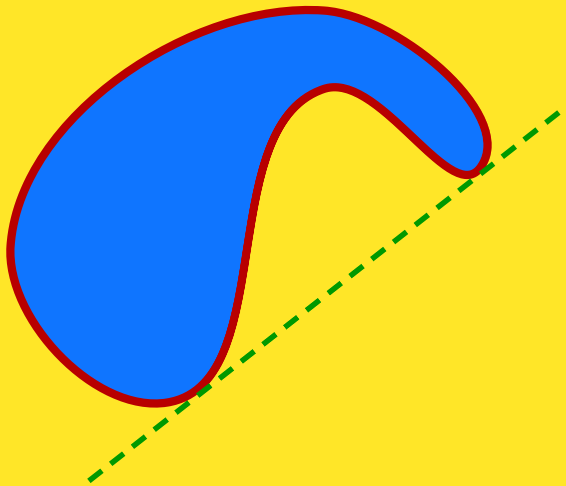

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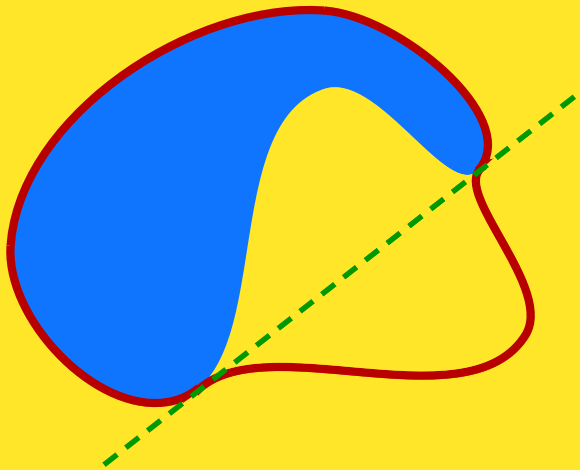

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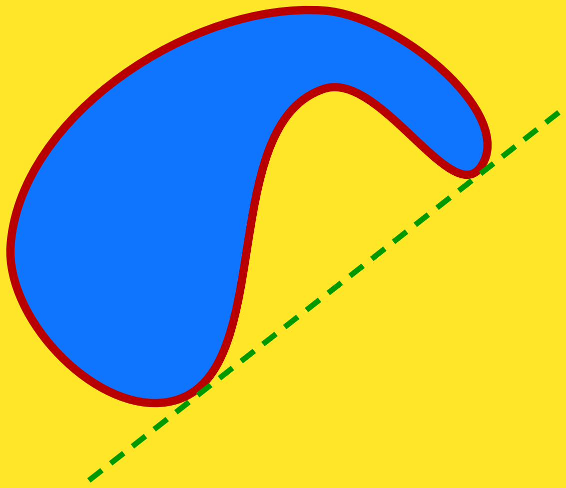

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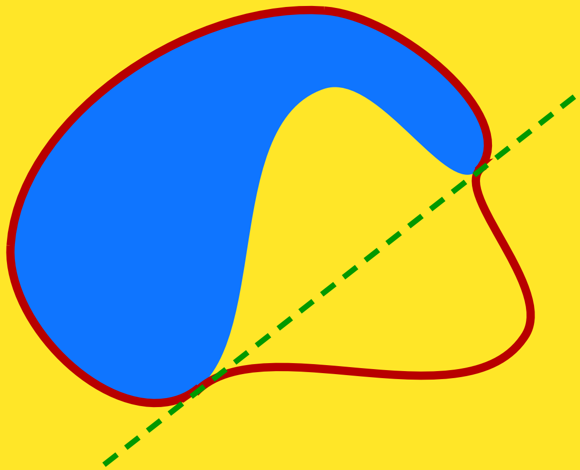

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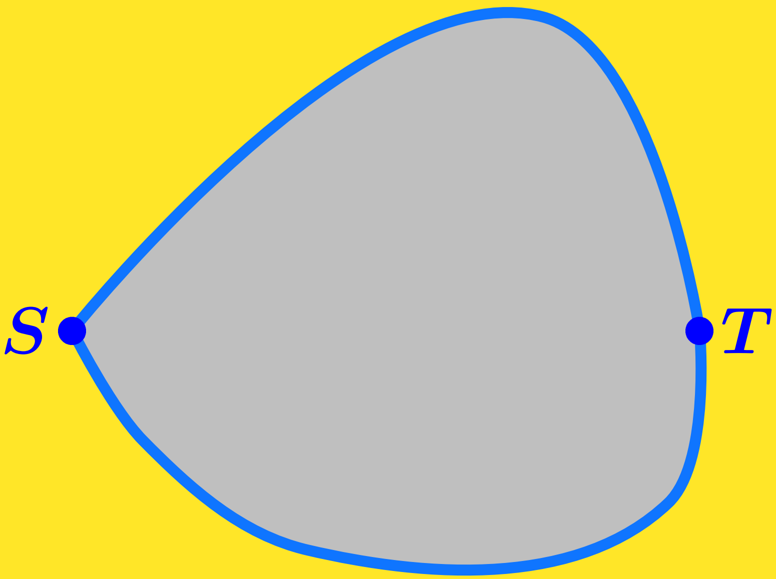
LEMMA 2:

Choose points S and T on the curve that the points divide the perimeter into two equal parts.



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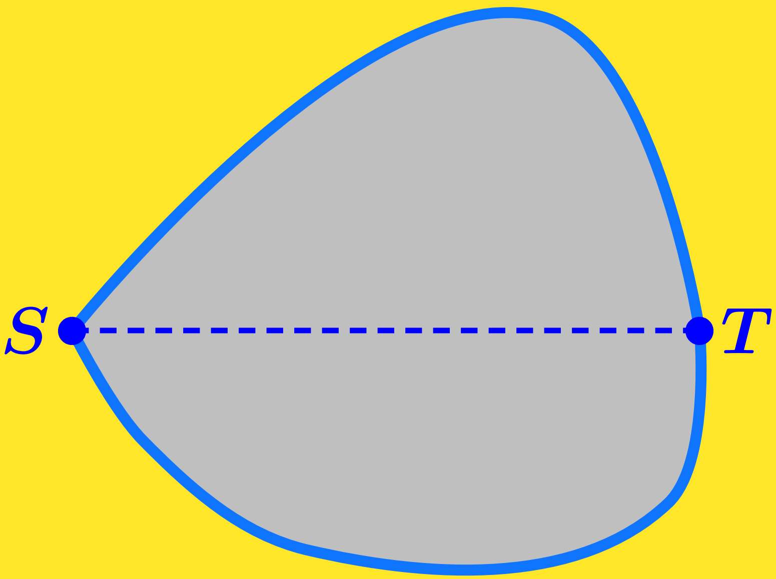

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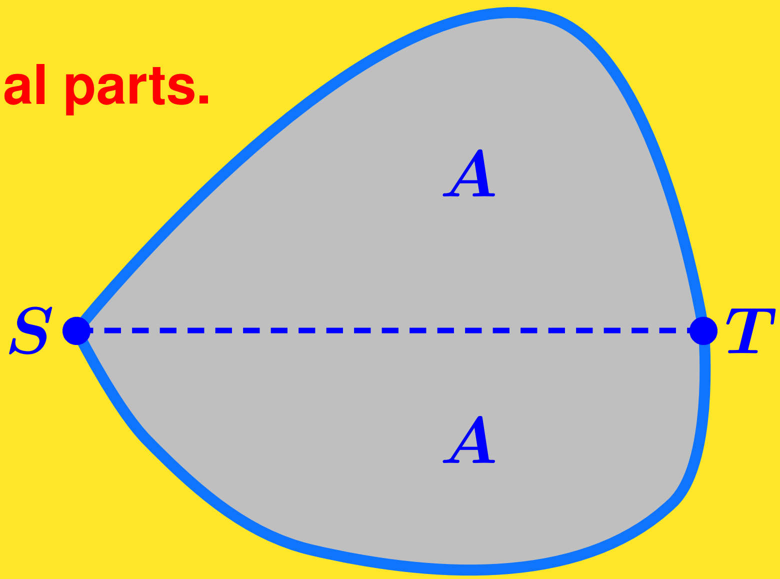

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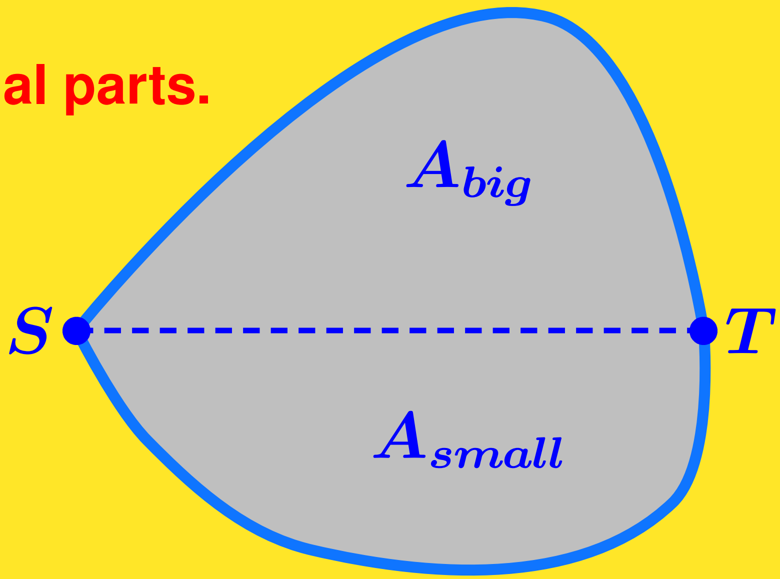

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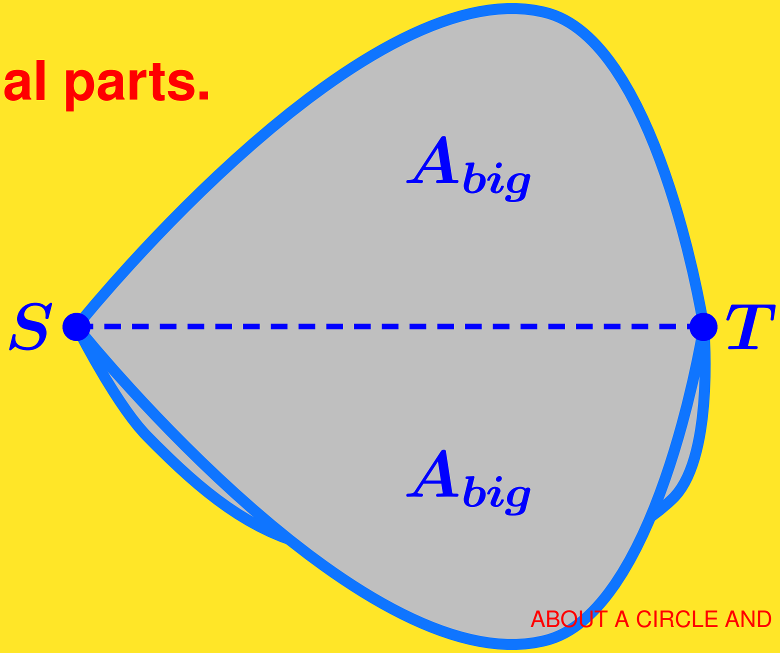

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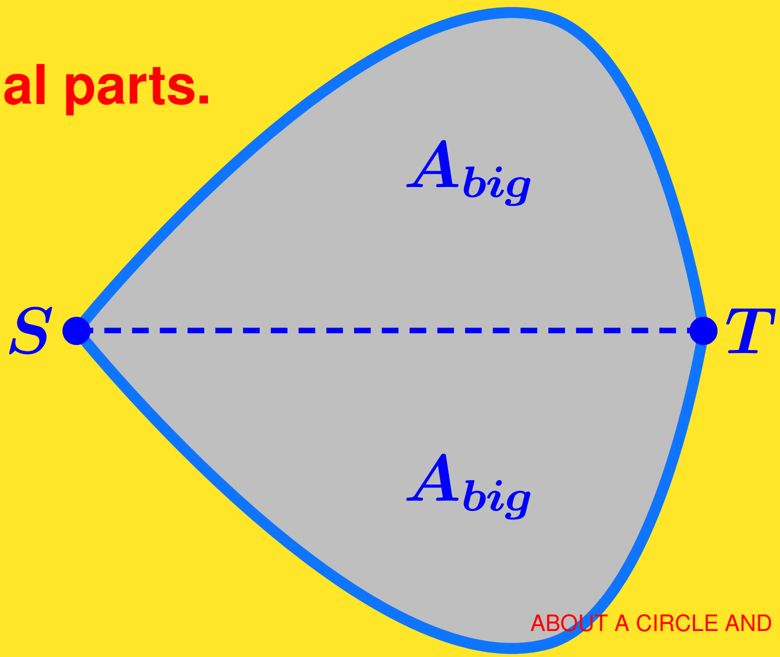

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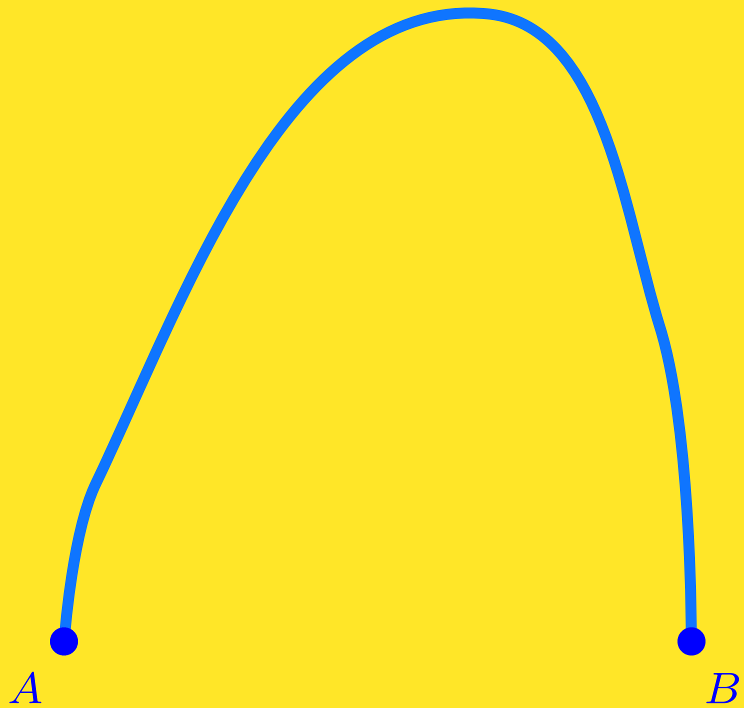
LEMMA 3:

Consider all arcs with a given length and endpoints on a line. The curve that encloses the maximum area between it and the line is a semicircle.


A
B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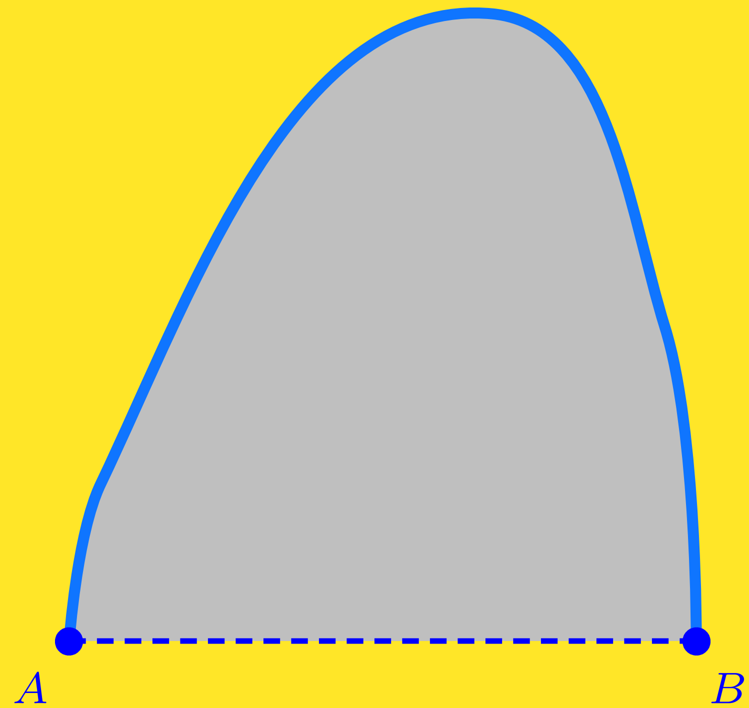
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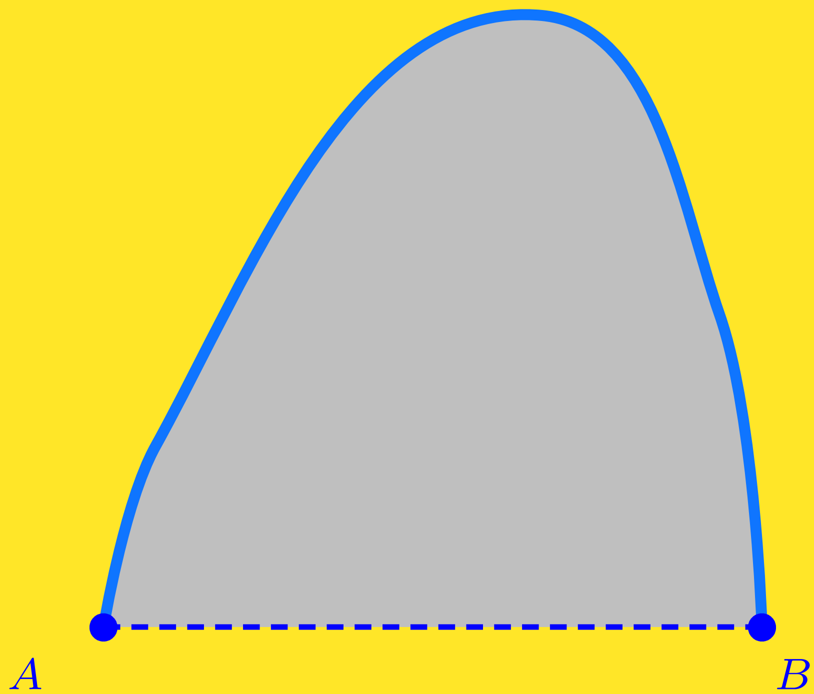

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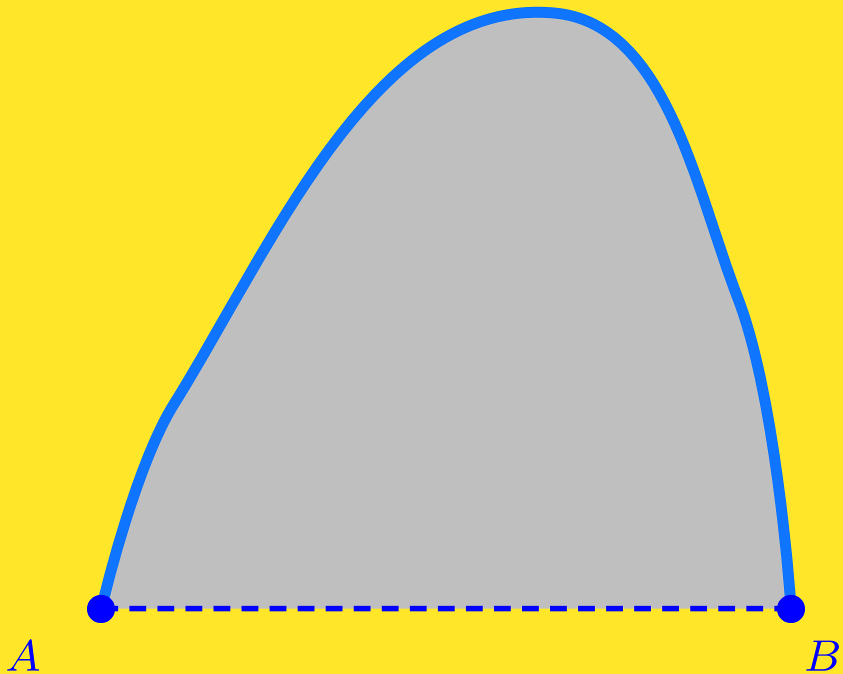

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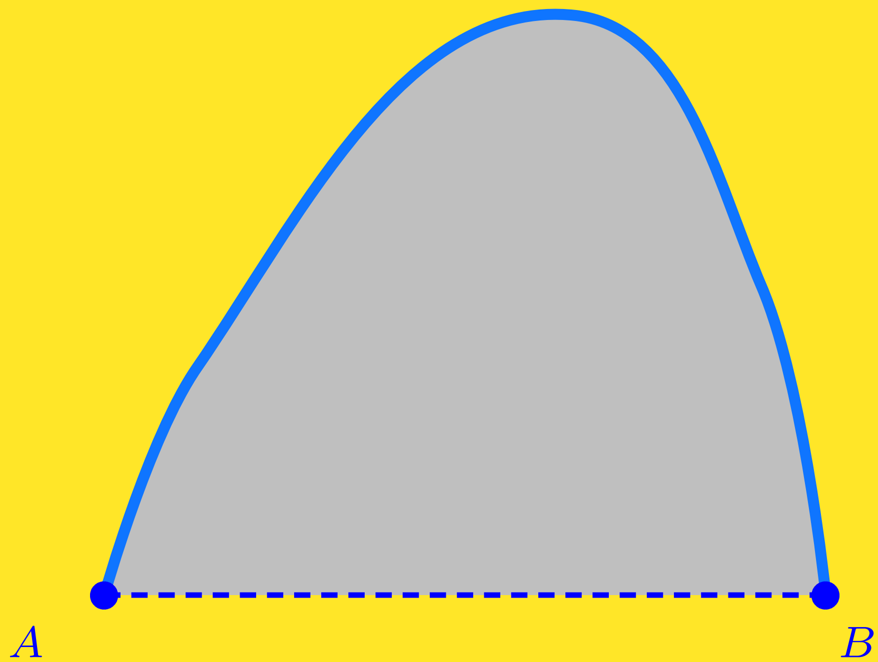
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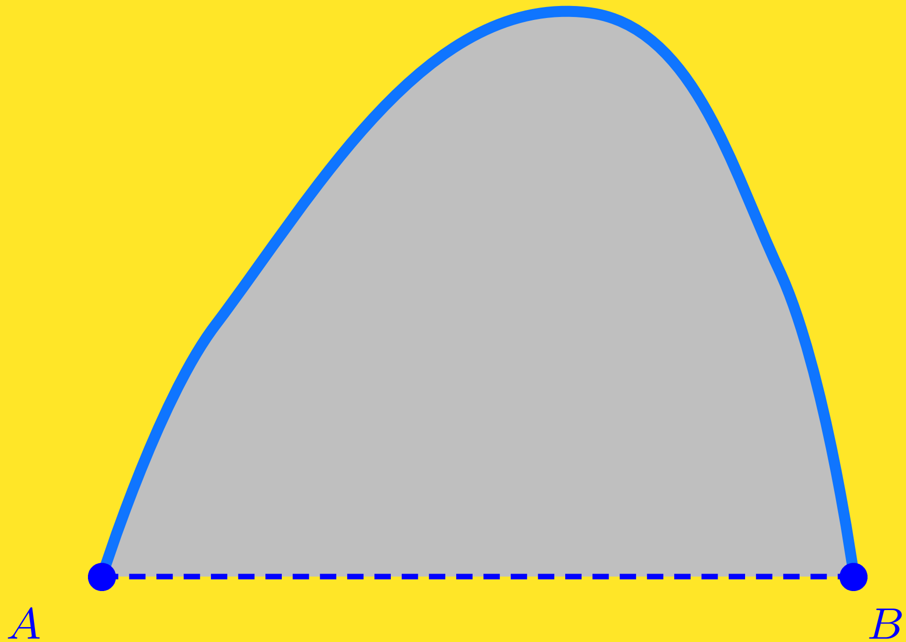
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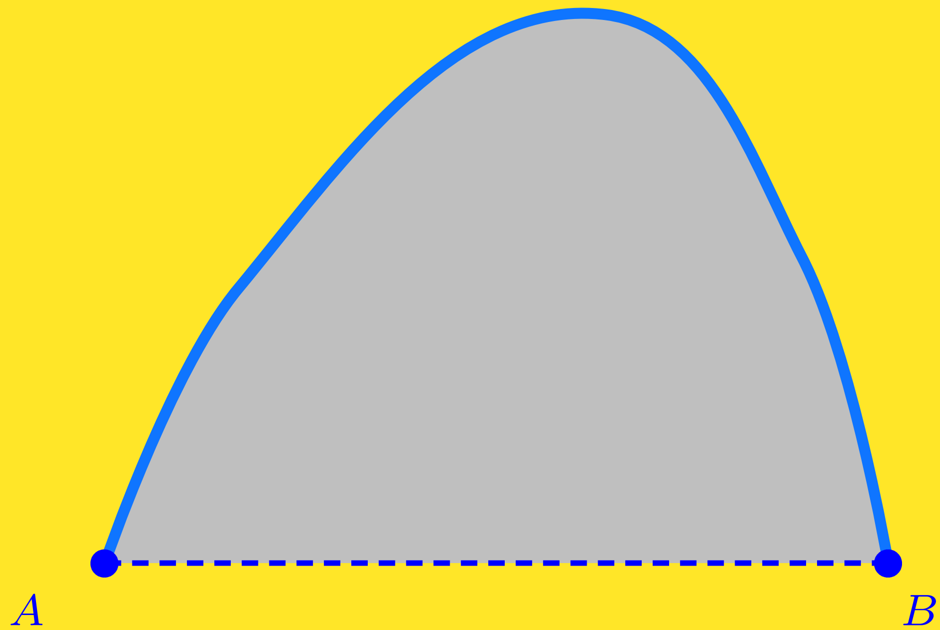
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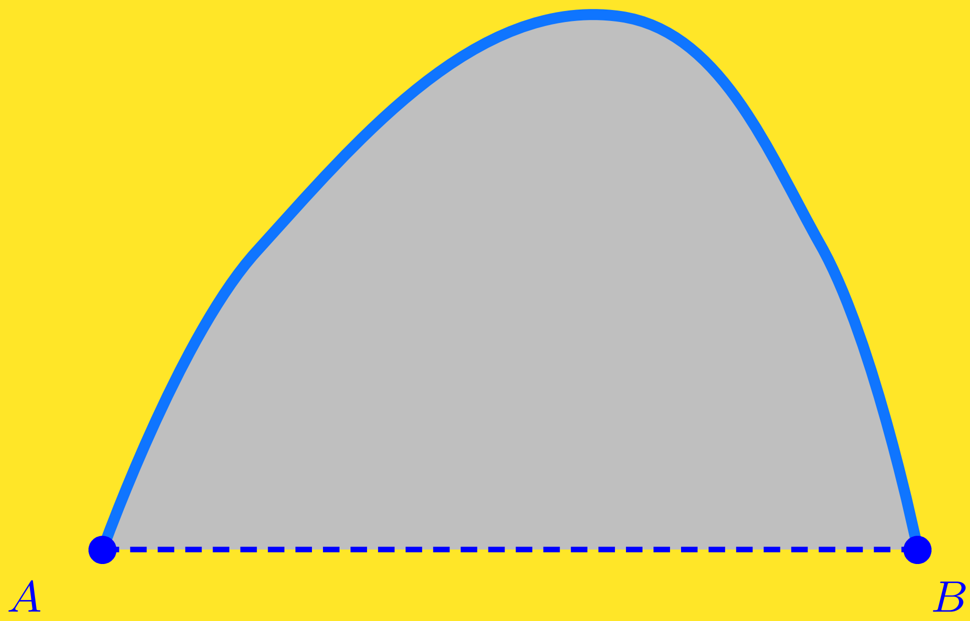
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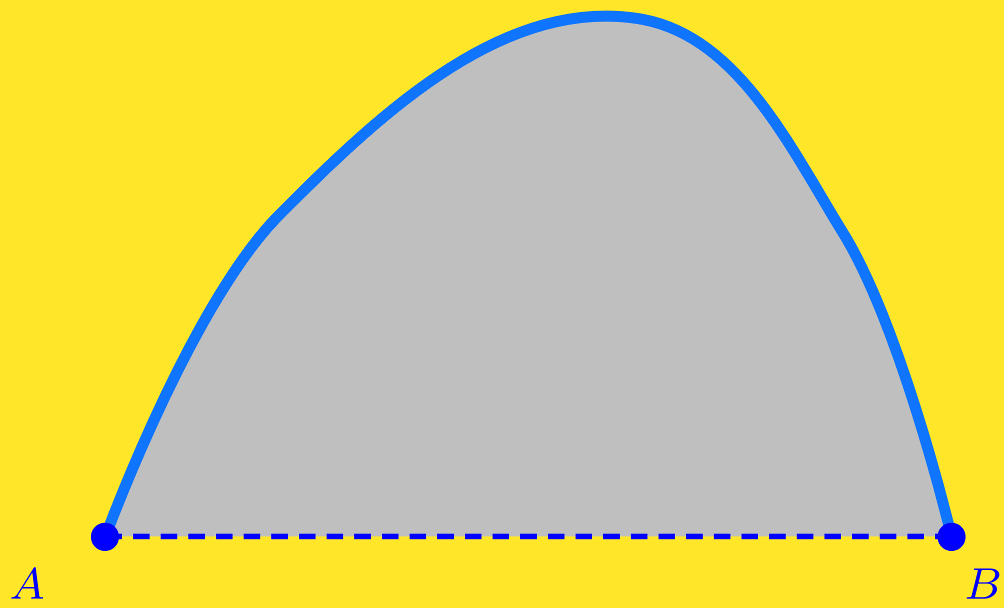
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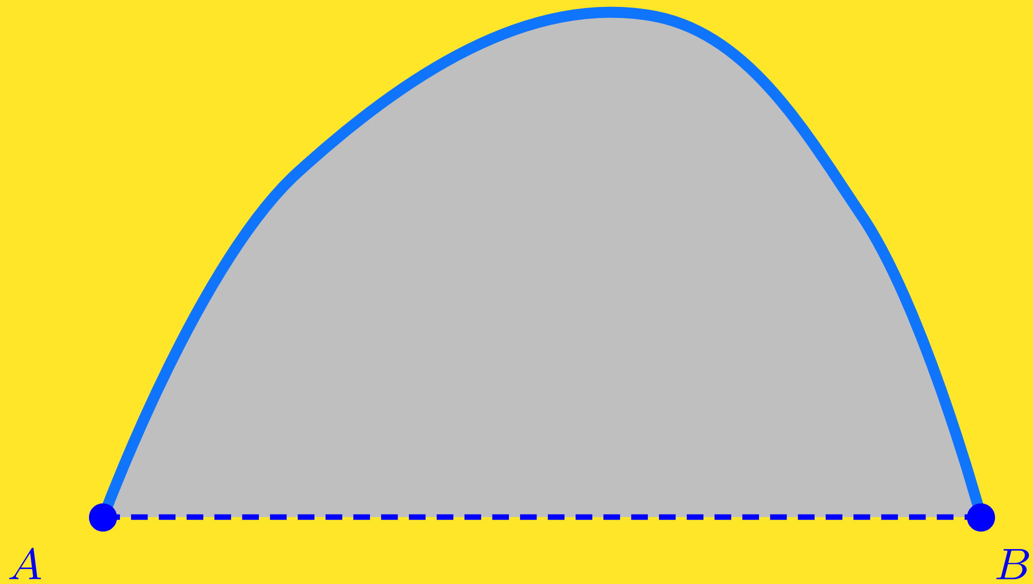
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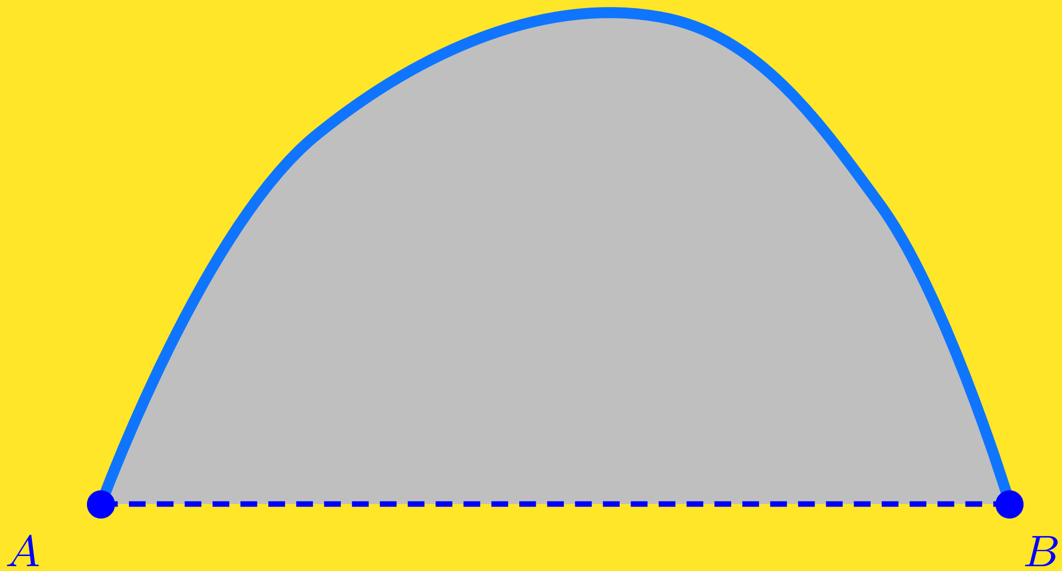
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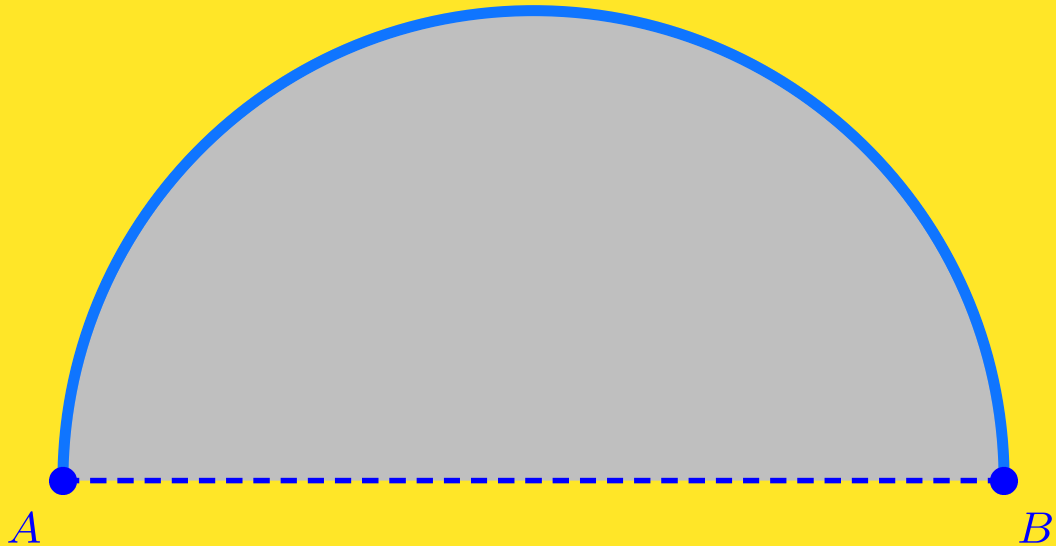

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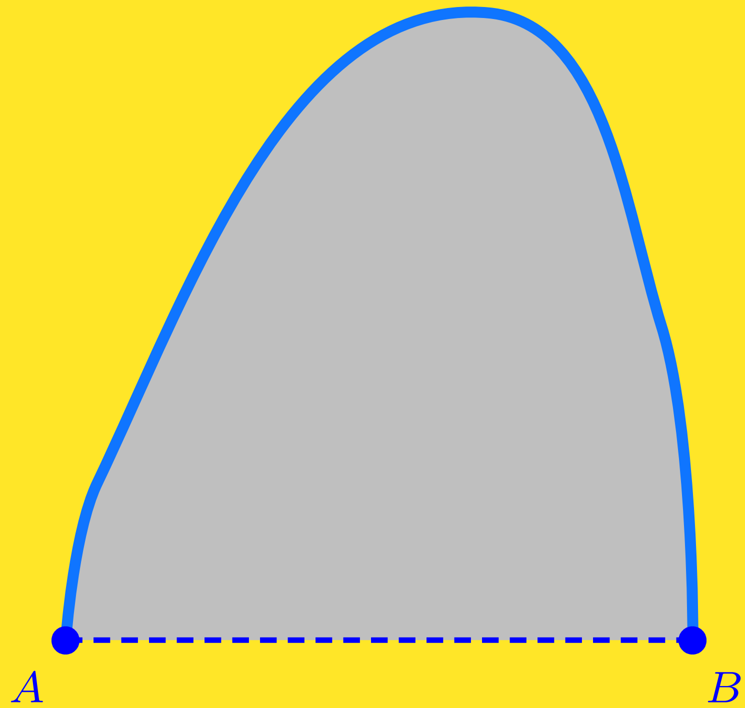
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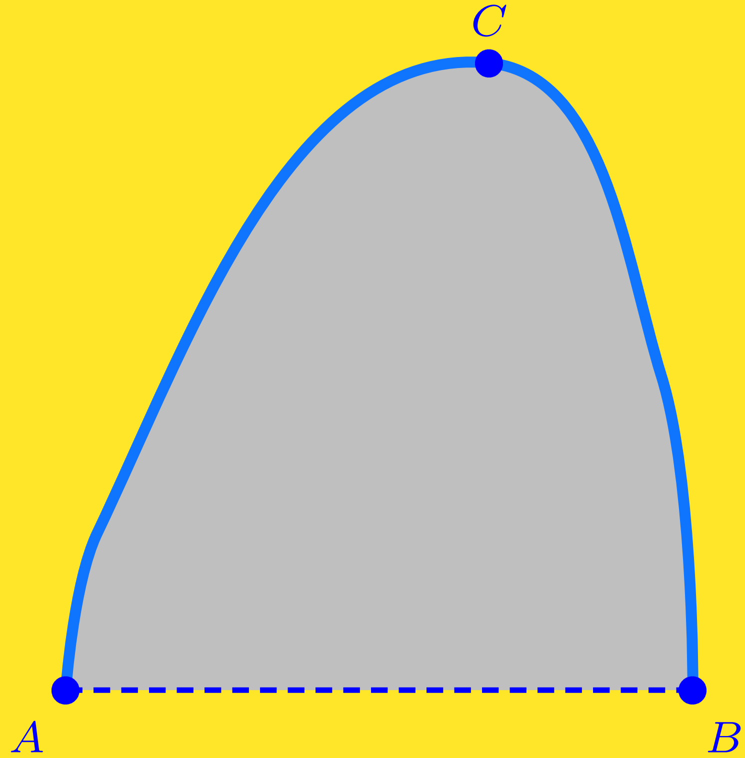
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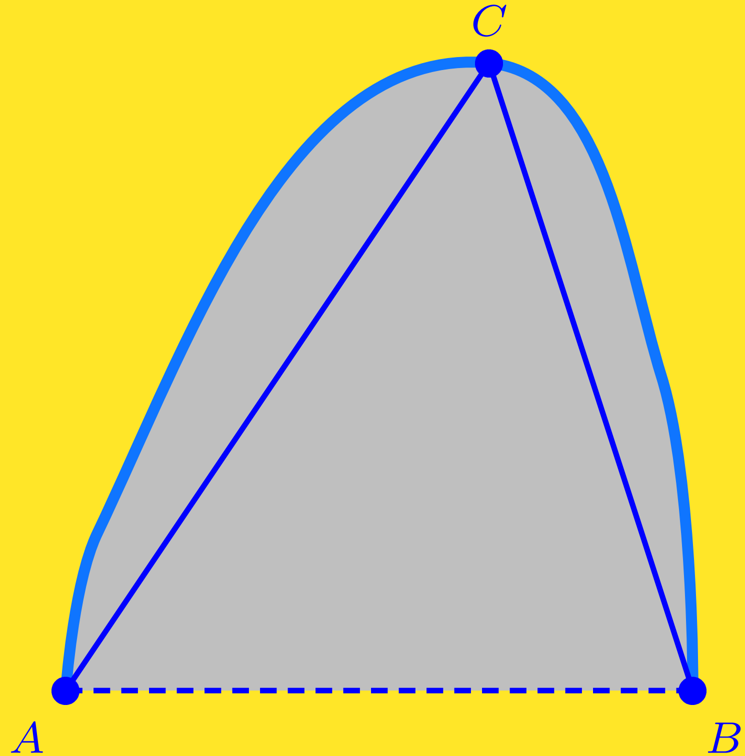
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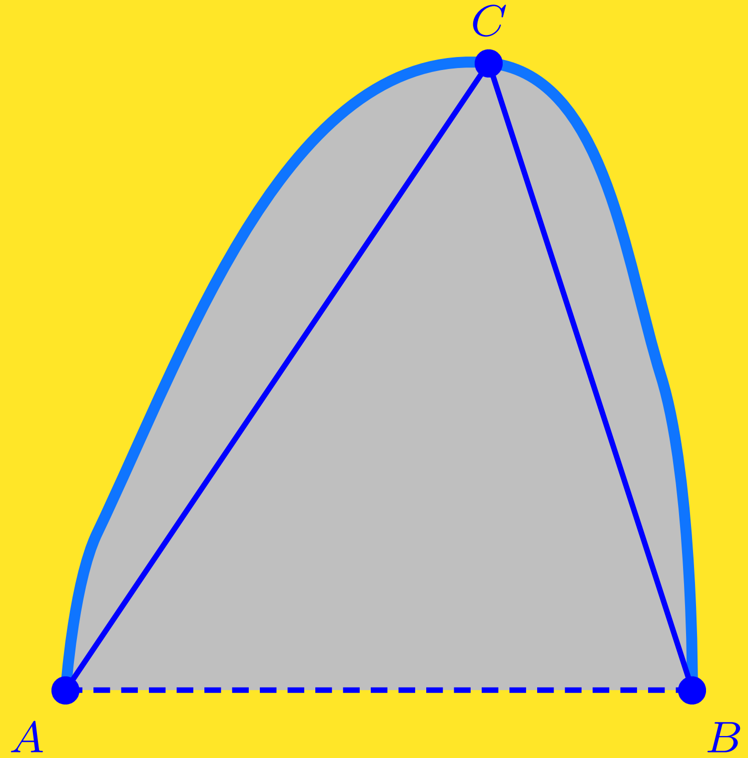
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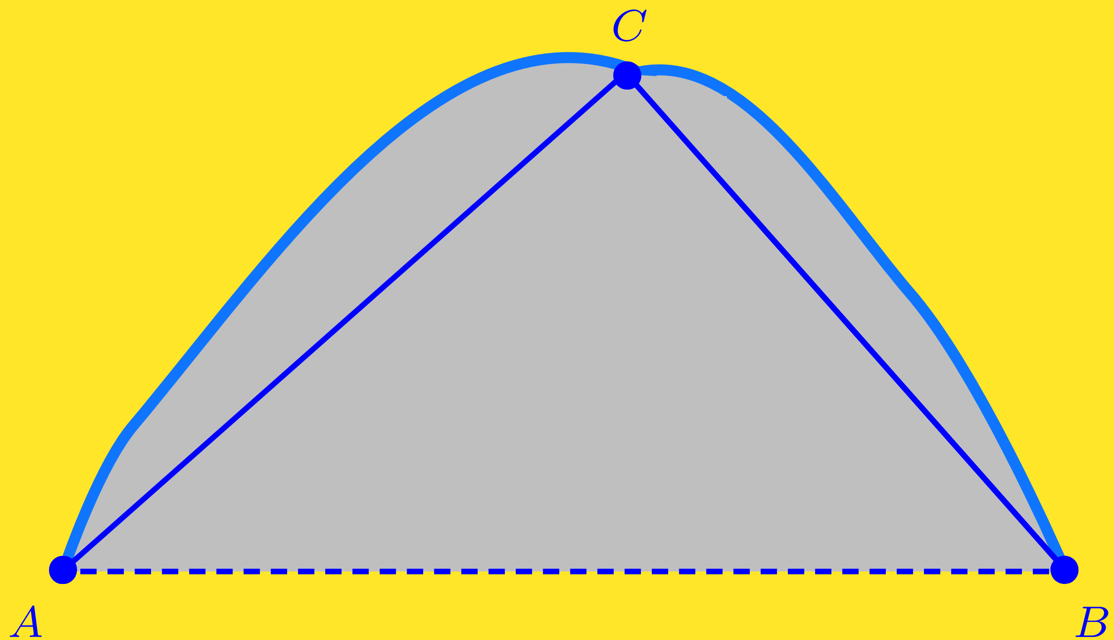
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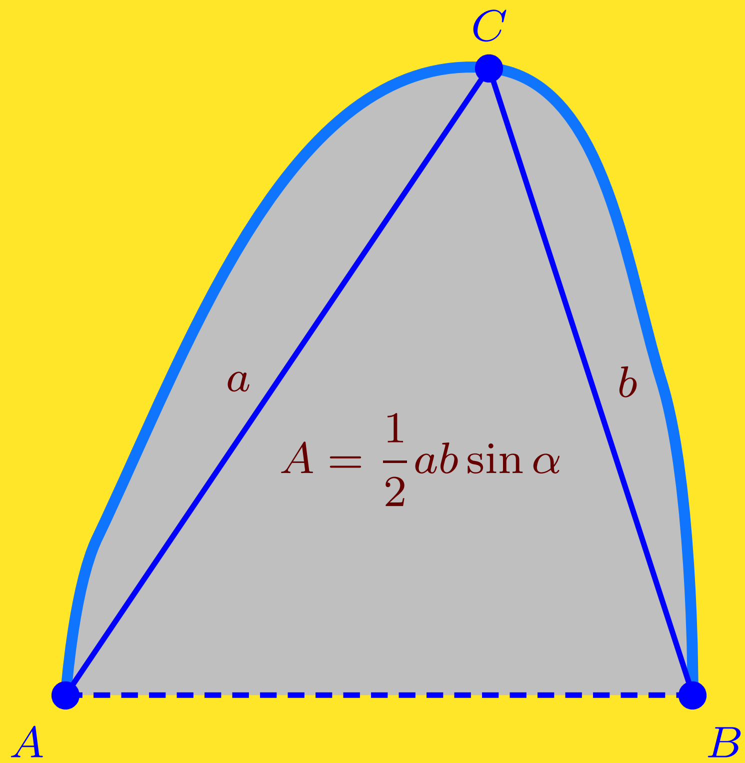
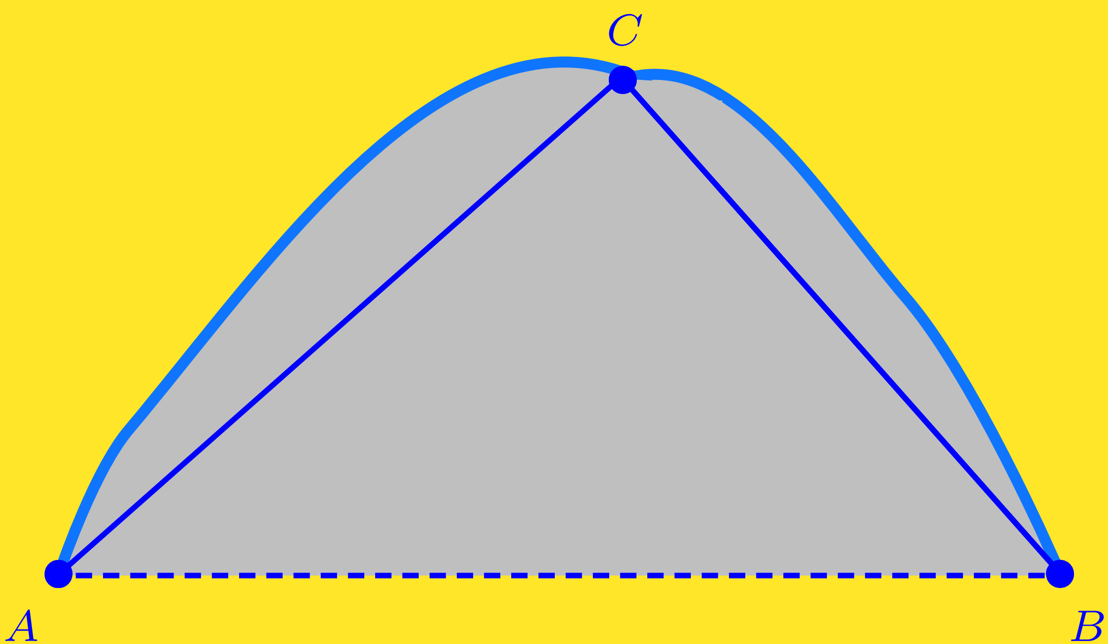

C
a
b
$A = \frac{1}{2}ab\sin\alpha$
A
B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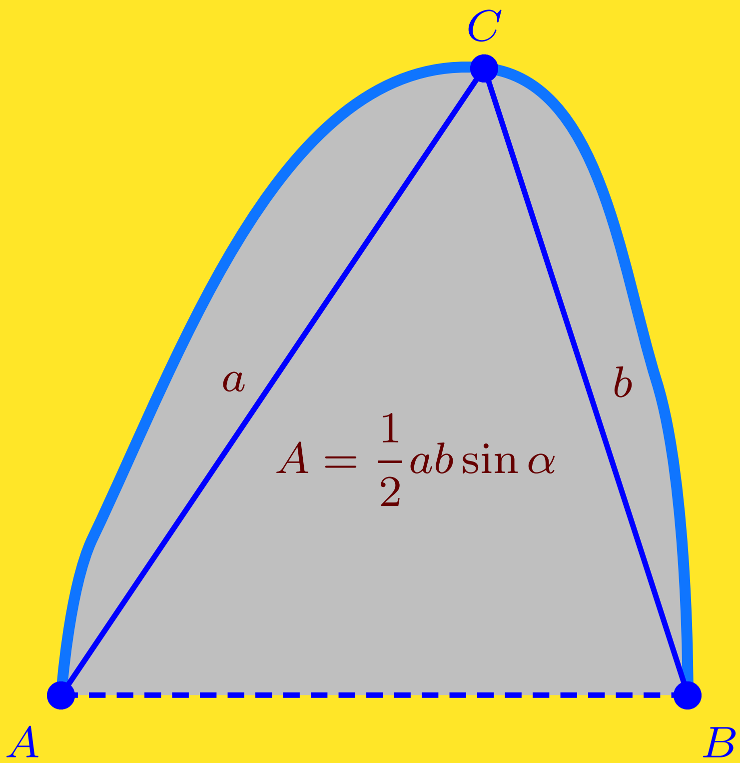
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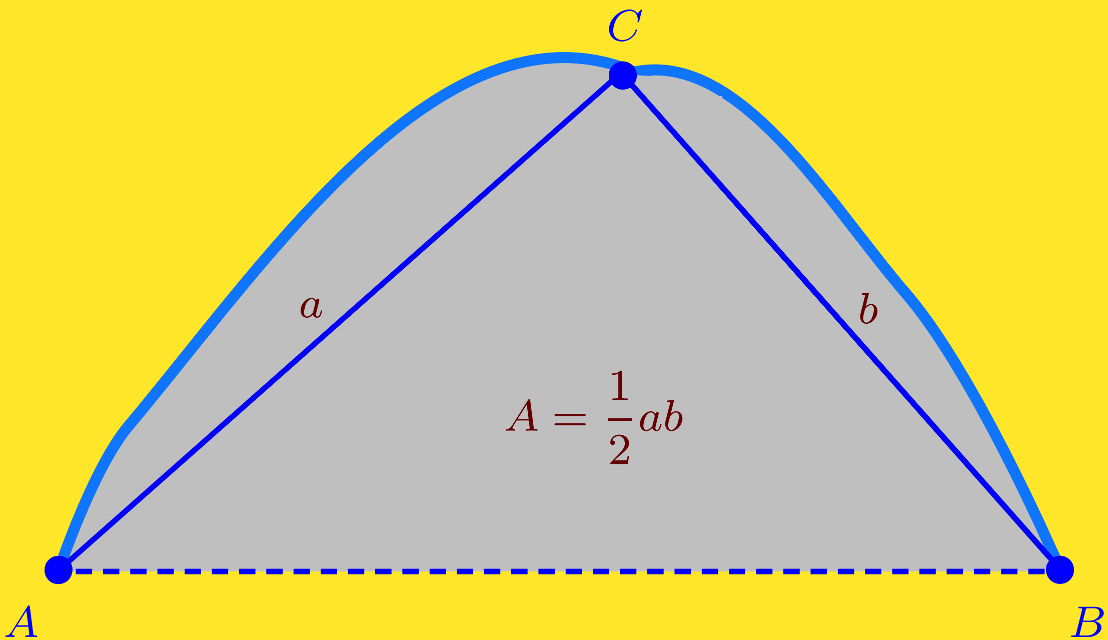
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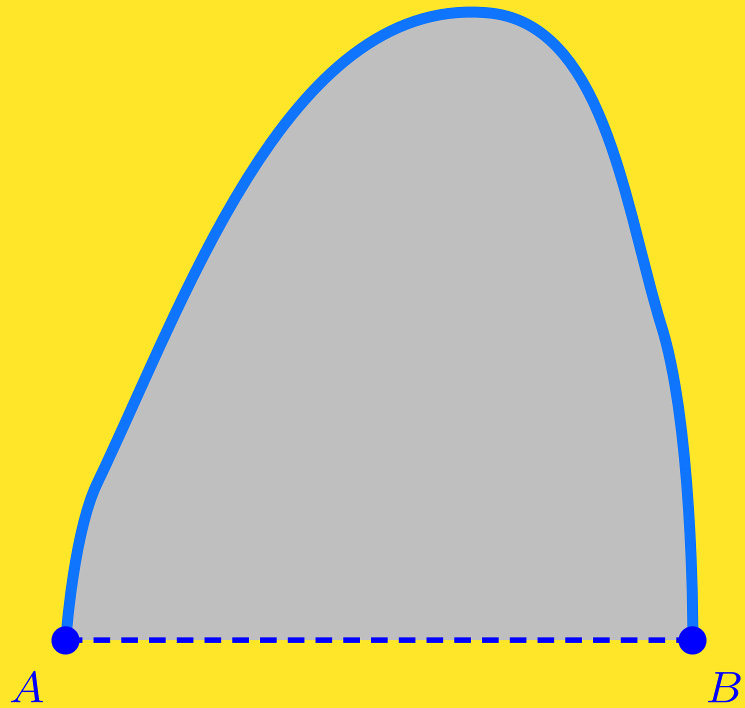
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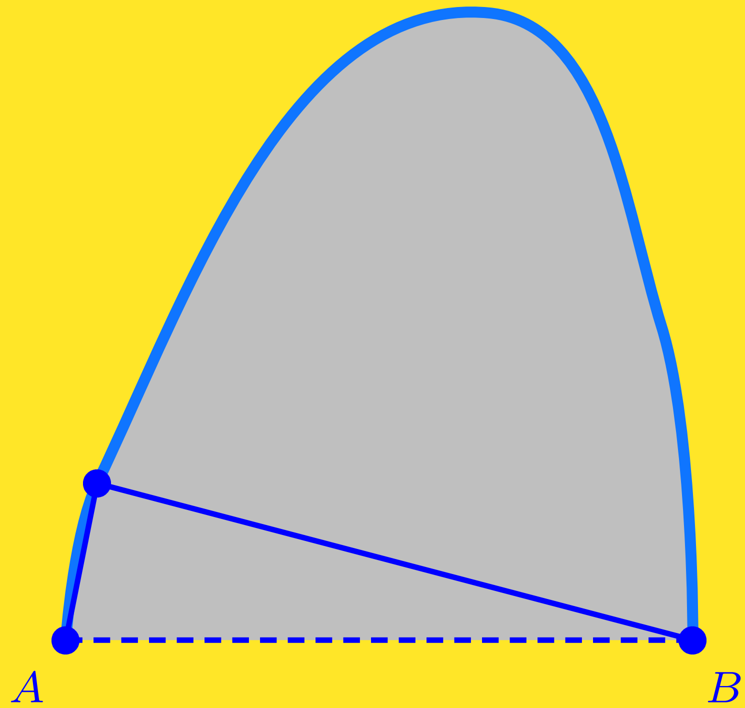
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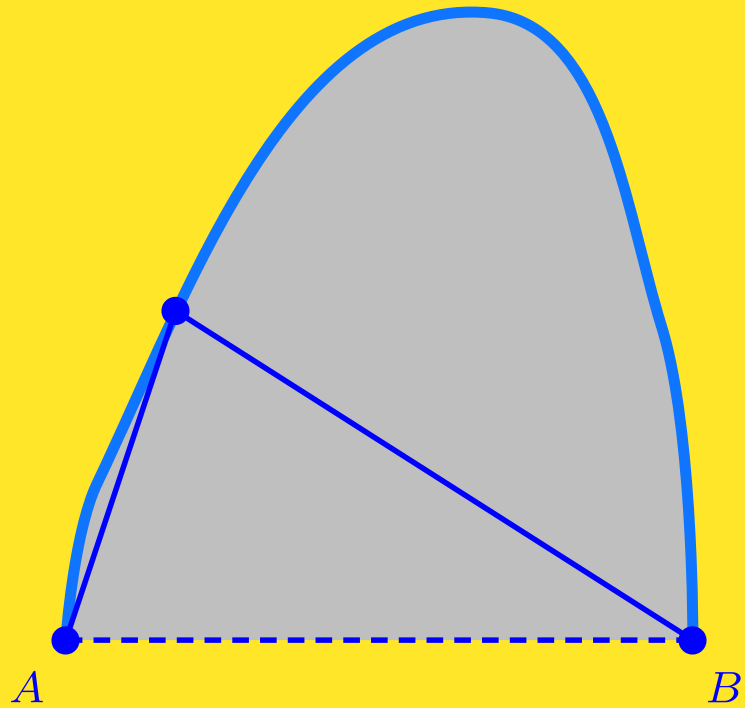
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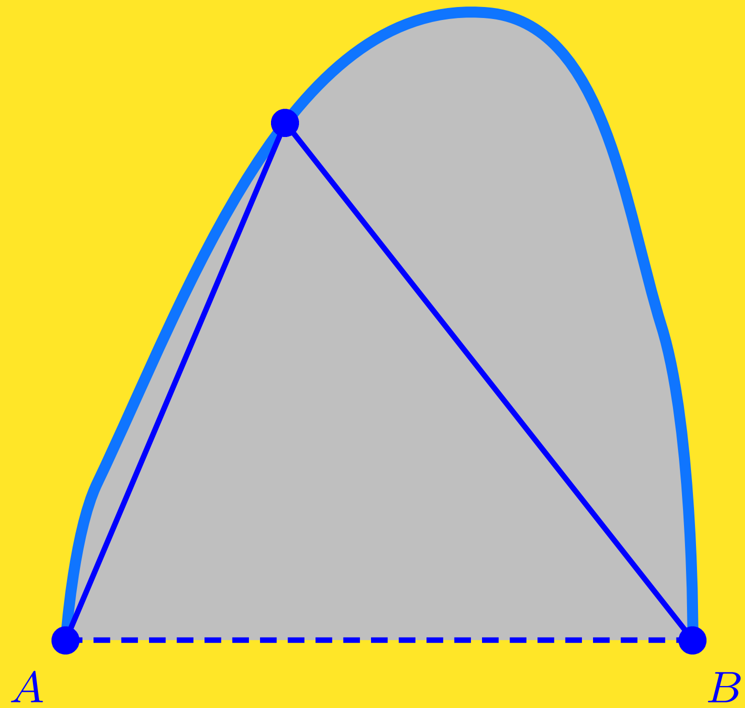
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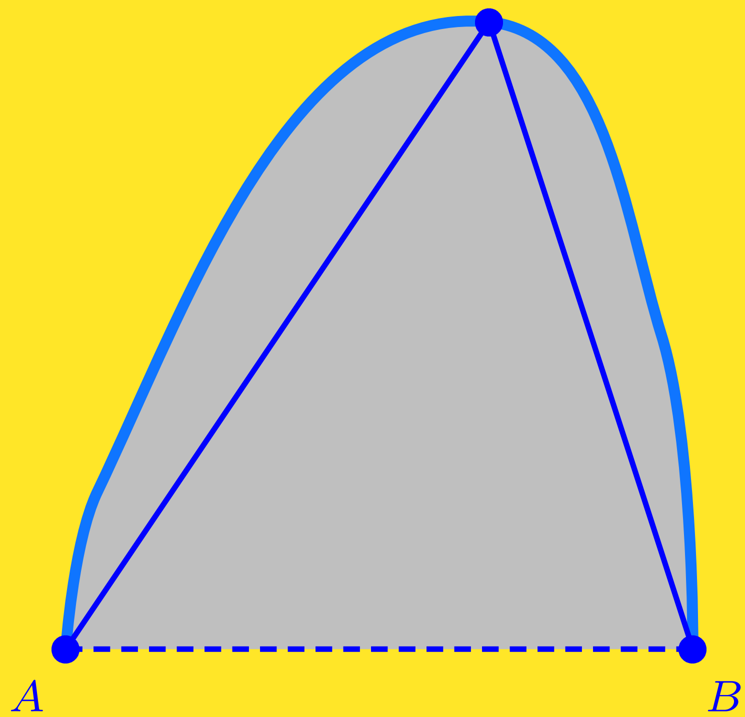
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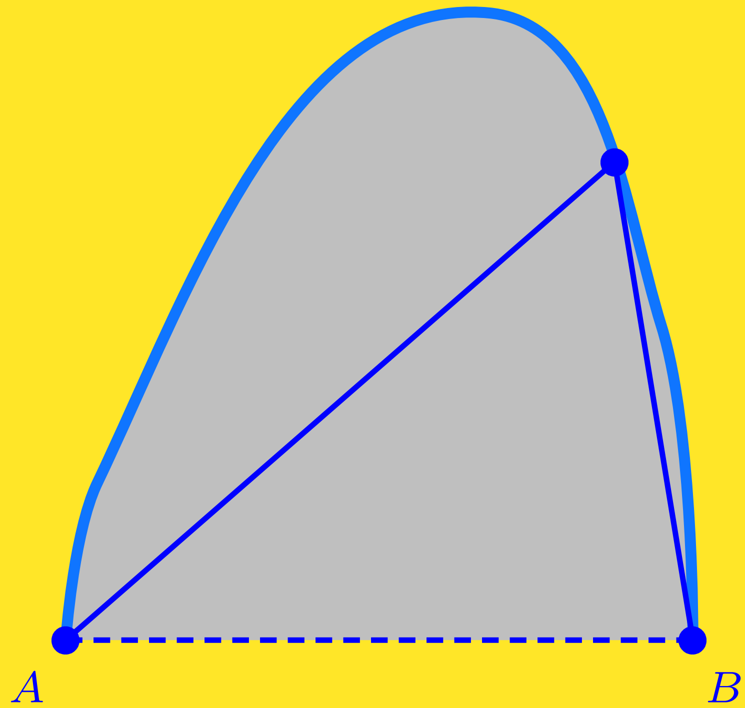

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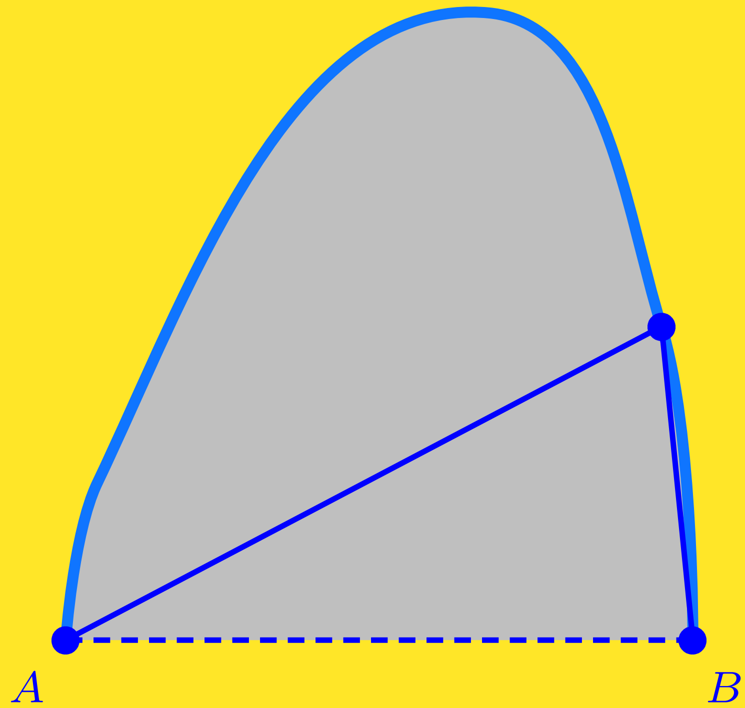
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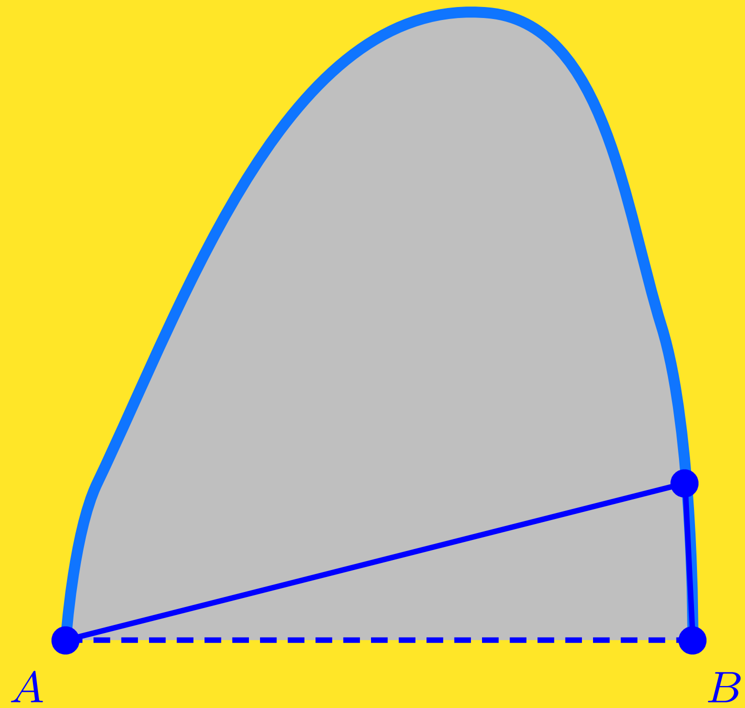
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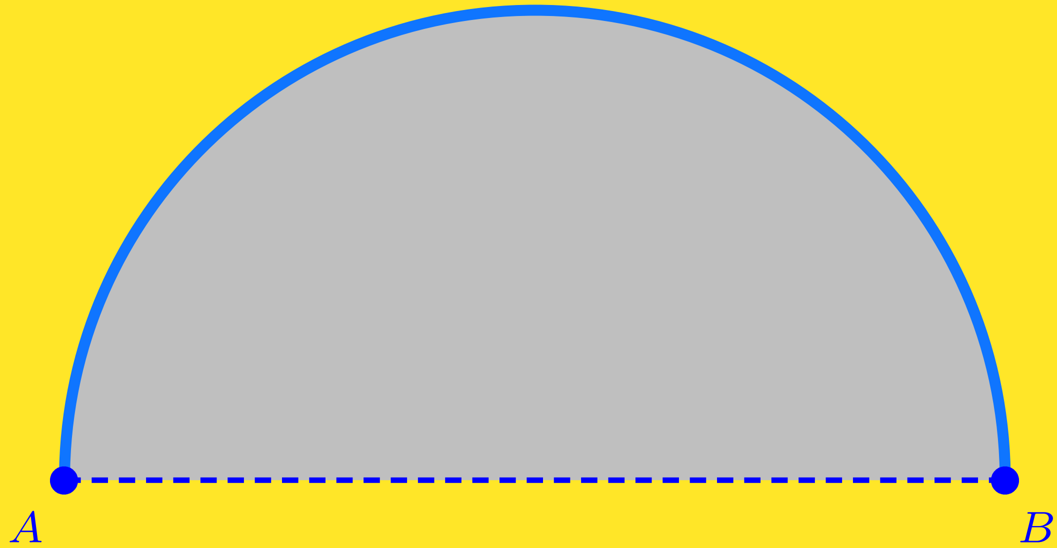
A
B

THEOREM:
Among all planar shapes with the same perimeter the circle has the largest area.

LEMMA 1:
The figure that solves the Theorem must be convex.

LEMMA 2:
Choose points S and T on the curve that the points divide the perimeter into two equal parts. Then the segment ST must divide the area of the shape into two equal parts.

LEMMA 3:
Consider all arcs with a given length and endpoints on a line. The curve that encloses the maximum area between it and the line is a semicircle.

DEFINITION:

A circle is the set of points in a plane that are equidistant from a given point.

Jakob Steiner (1796 - 1863)

Lejeune Dirichlet (1805 - 1859)

1. $C = 2\pi r$
2. $A = \pi r^2$
3. **Isoperimetric Problem**

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Is it possible to construct a square equal in area to a circle using only a straight-edge and compass?

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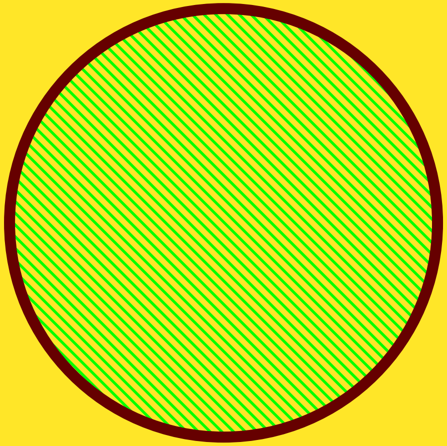

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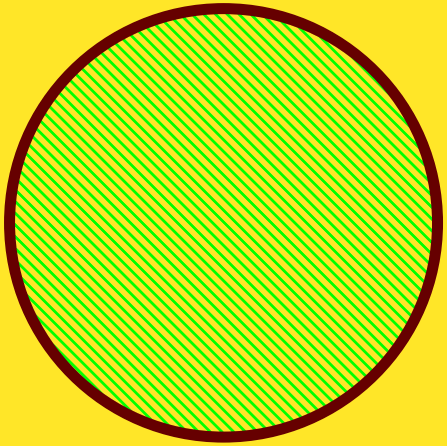

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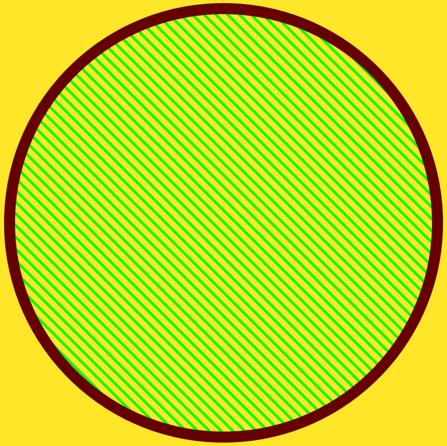

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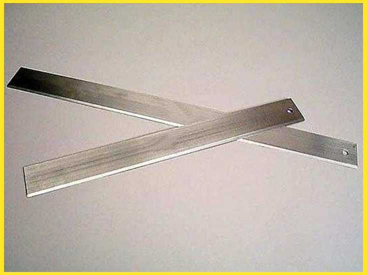

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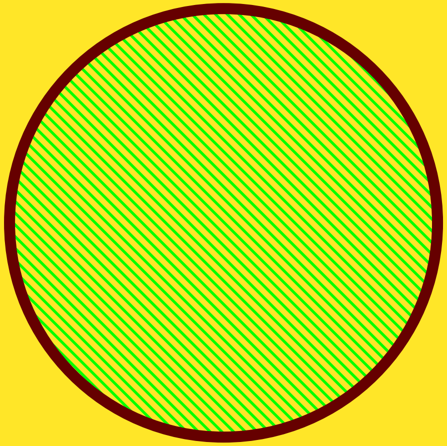

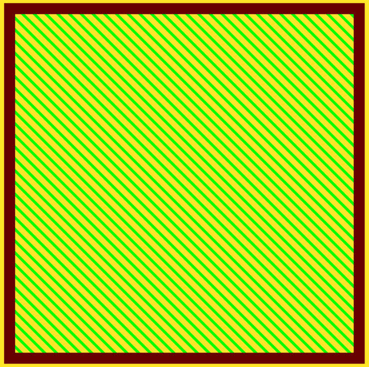

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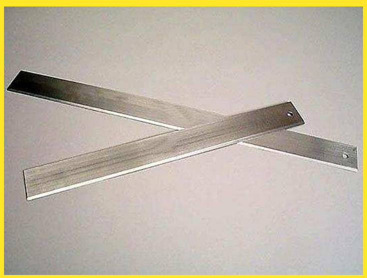

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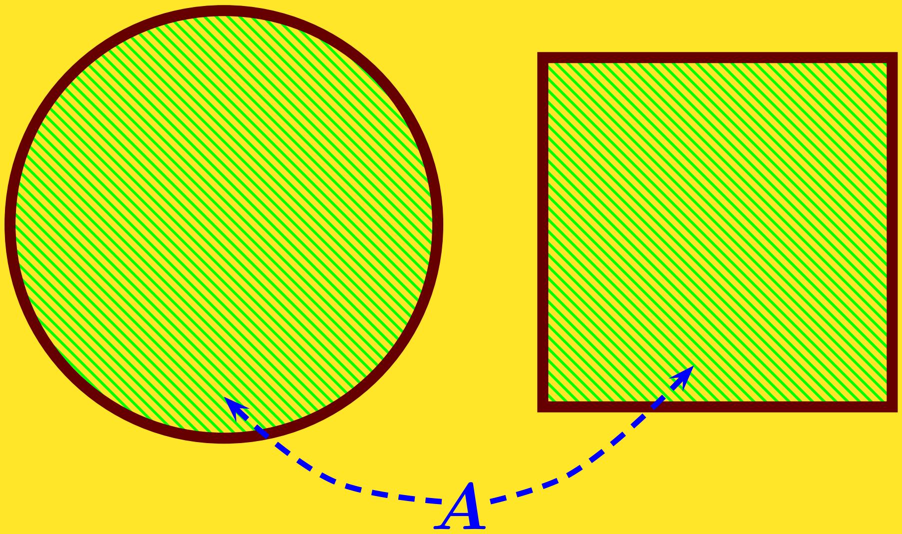

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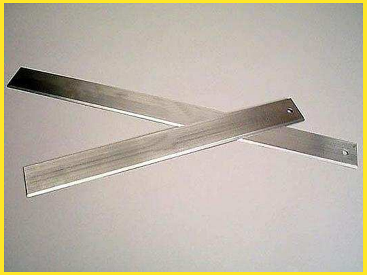

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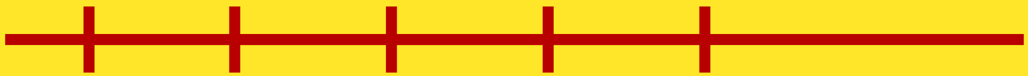

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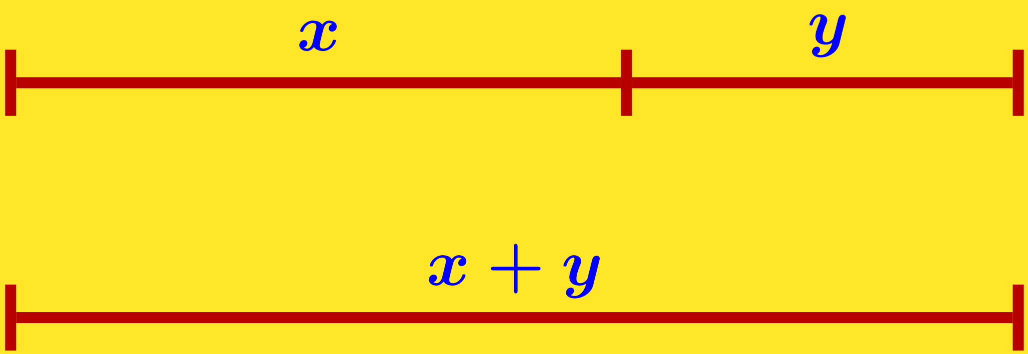

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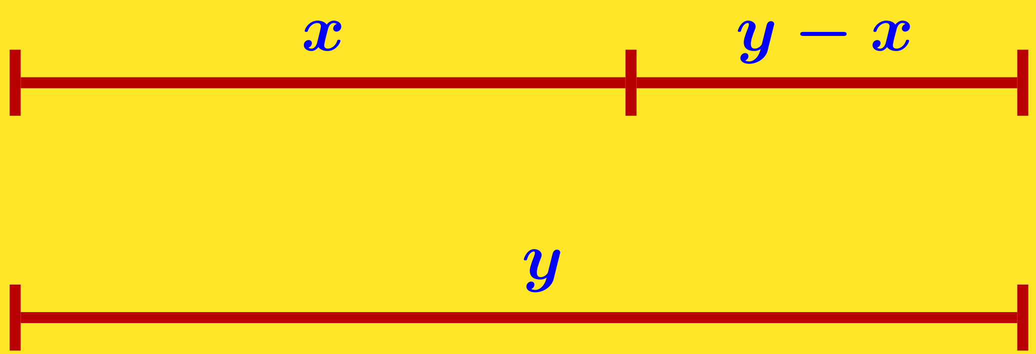

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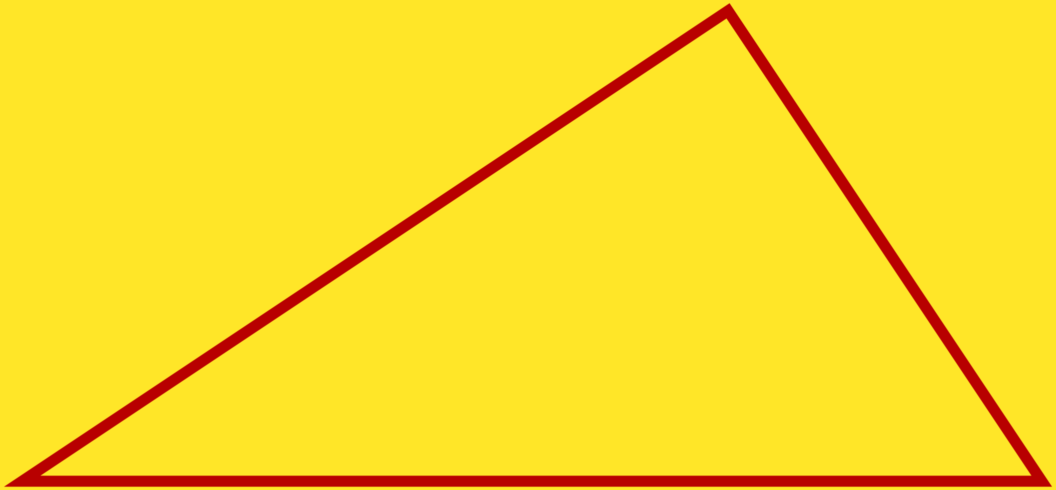

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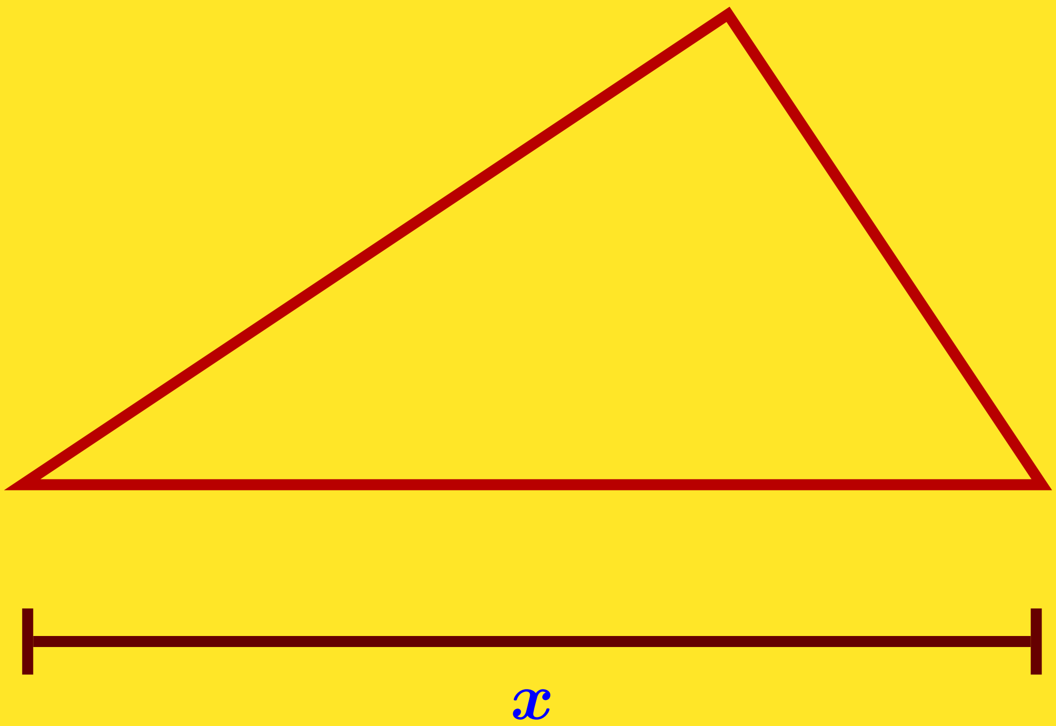

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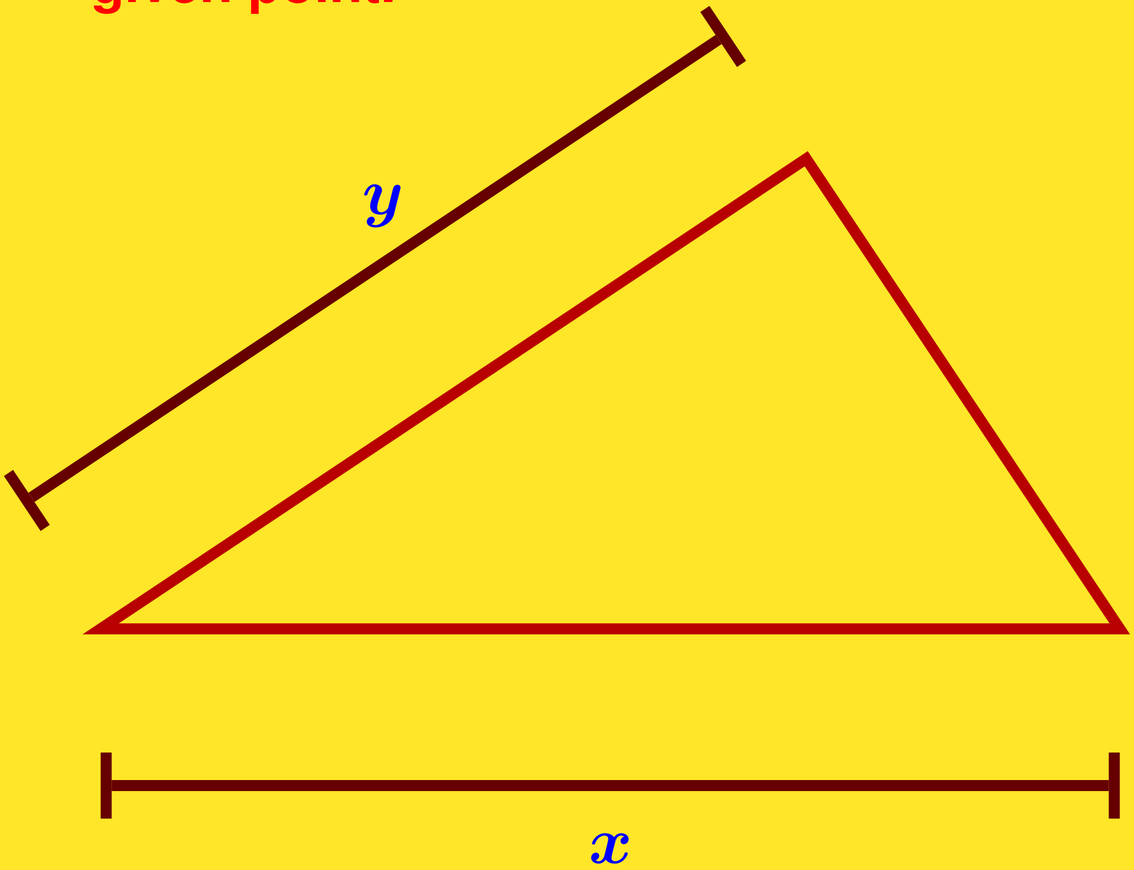

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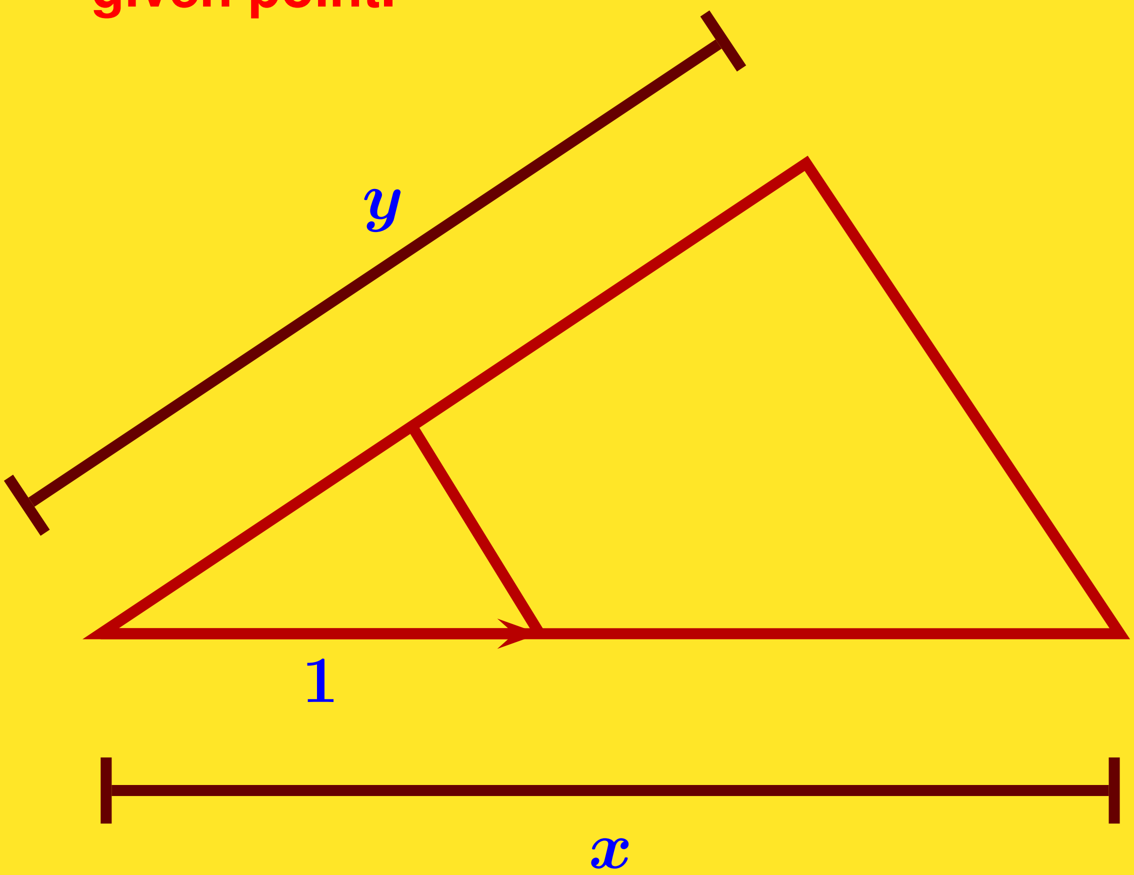

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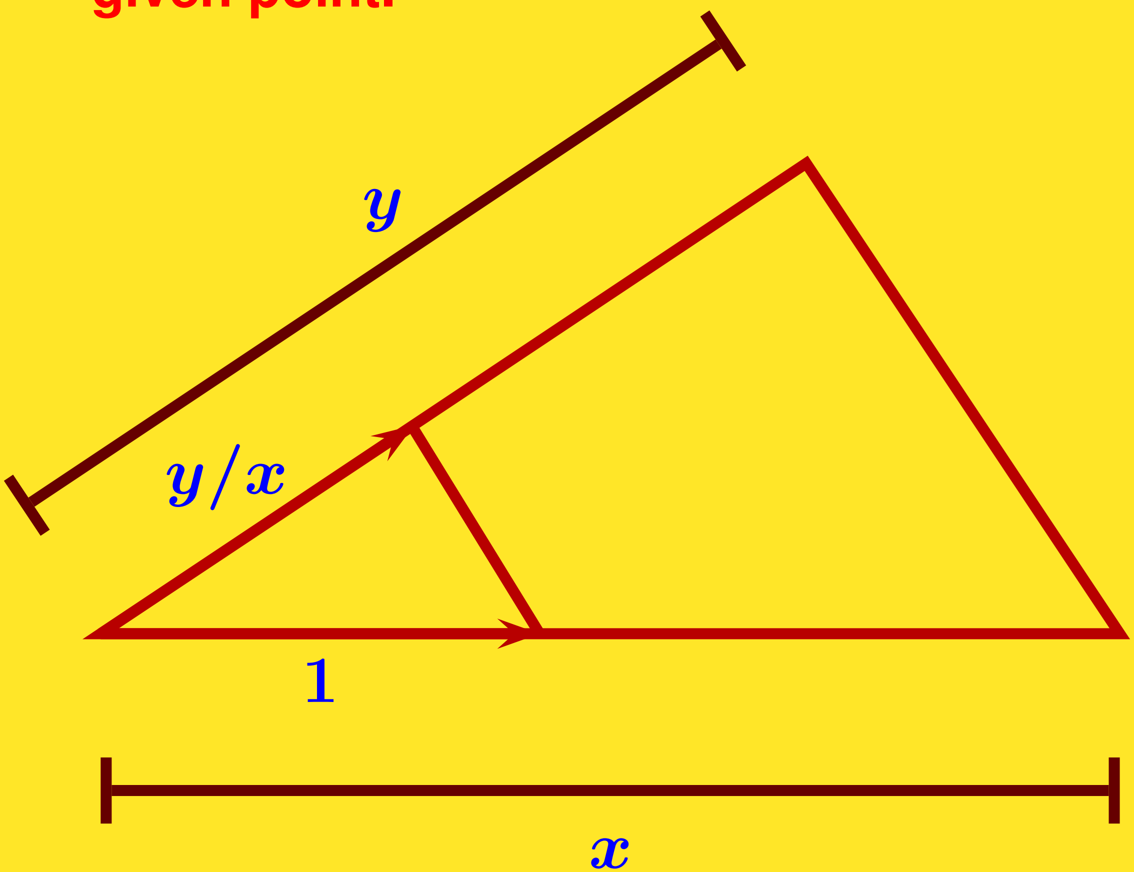

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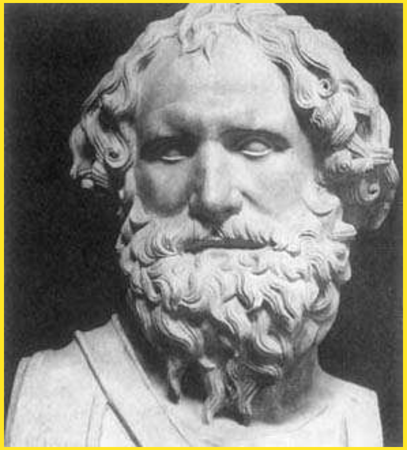

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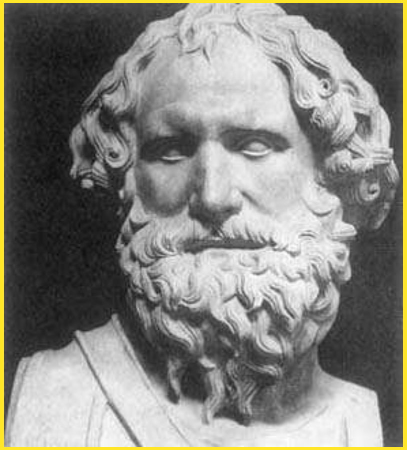

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$$\pi \approx \frac{22}{7}$$

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$$\pi \approx \frac{355}{113}$$

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THEOREM (Lambert, 1761):

The number π is not rational.

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PROOF:

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PROOF: Assume to the contrary that

$$\sqrt{2} = \frac{p}{q}$$

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where p and q have no common divisor.

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$$\sqrt{2} = \frac{1414213562}{1000000000}$$

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$$\sqrt{2} = \frac{2 \cdot 707106781}{2 \cdot 500000000}$$

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(1707 - 1783)

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$$0 < R_n < \frac{3}{(n+1)!}$$

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$$\frac{a}{b} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + R_n$$

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$$n!b\frac{a}{b} = n!b\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + R_n\right)$$

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This contradicts (1) if n is sufficiently large. ■

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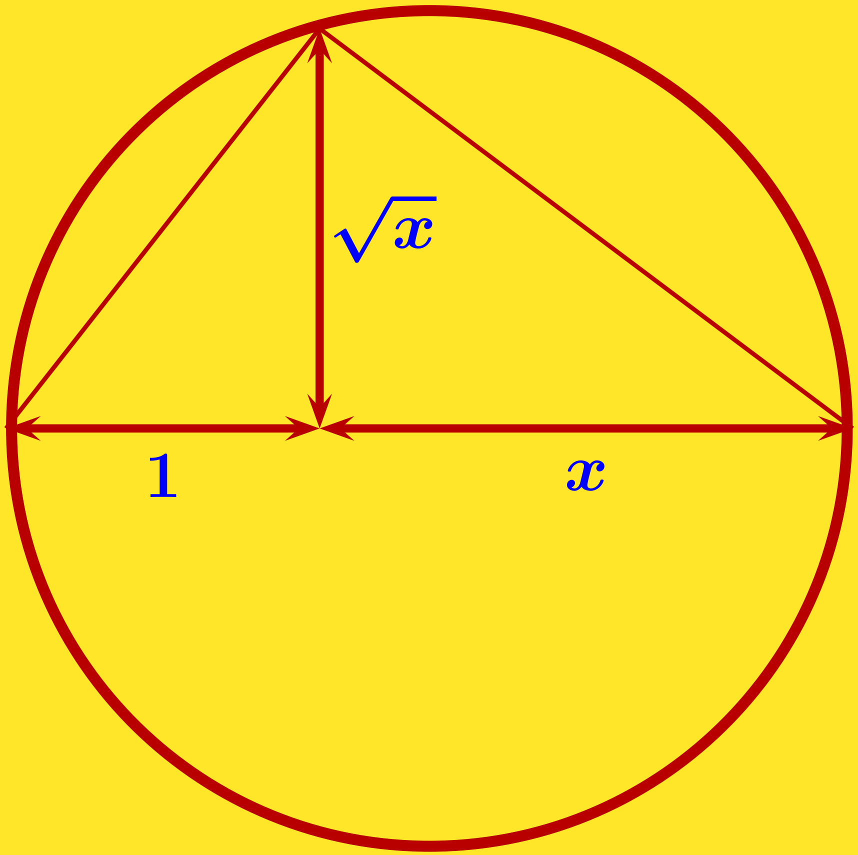
1. You can't confuse e with a food product.

DEFINITION:

A circle is the set of points in a plane that are equidistant from a given point.



1. $C = 2\pi r$
2. $A = \pi r^2$
3. **Isoperimetric Problem**
4. **Squaring the Circle**

Is it possible to construct a square equal in area to a circle using only a straight-edge and compass?

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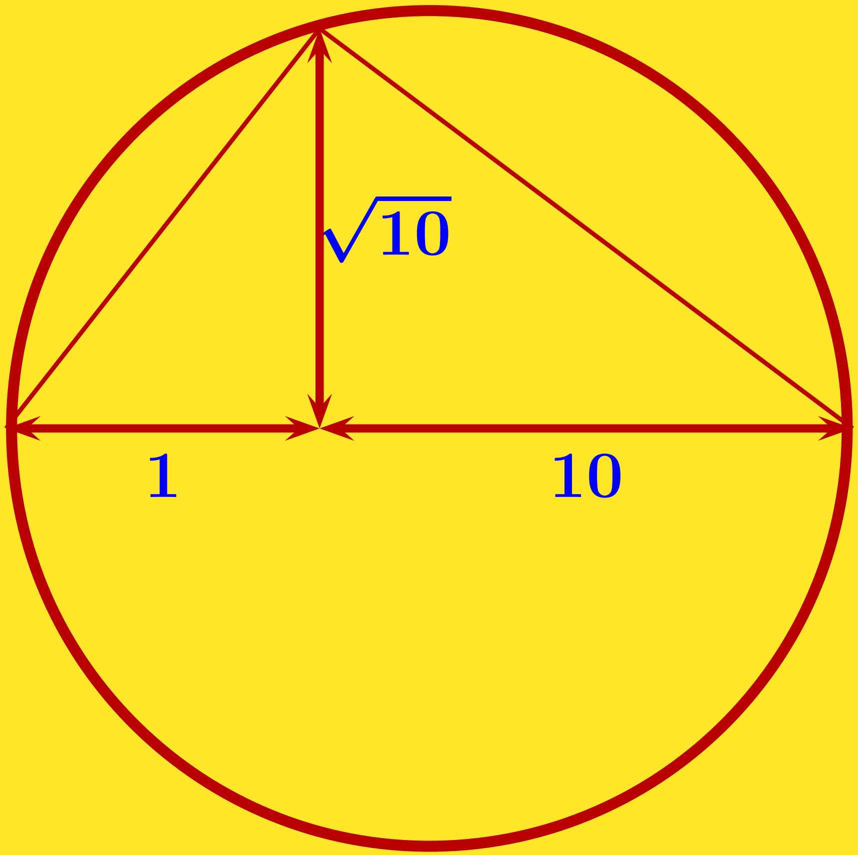

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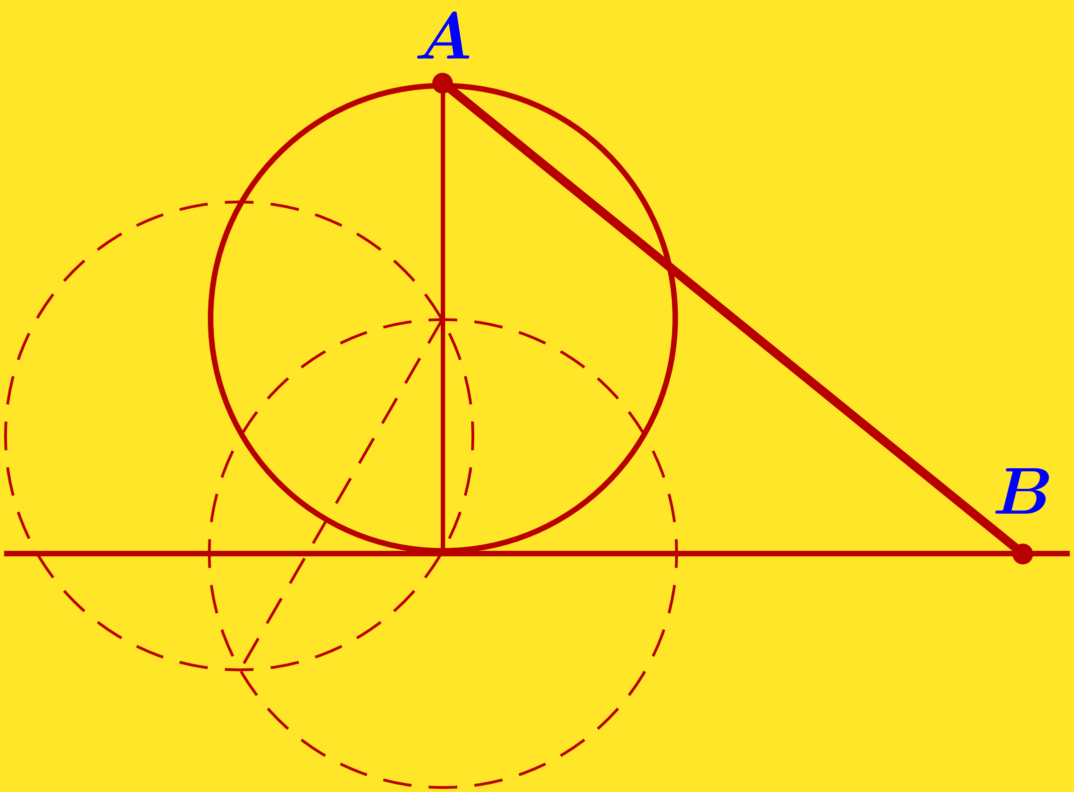

$$\pi \approx \sqrt{40/3 - 2\sqrt{3}} = 3.1415\ldots$$

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Approximation	Digits
$\pi \approx \sqrt{10}$	1
$\pi \approx \sqrt{\frac{40}{3} - 2\sqrt{3}}$	4
$\pi \approx \sqrt{\sqrt{\frac{767}{\sqrt{62}}}}$	5
$\pi \approx \sqrt{\sqrt{\frac{2143}{22}}}$	6
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Approximation	Polynomial
$\pi \approx \frac{22}{7}$	$7x - 22 = 0$
$\pi \approx \frac{355}{113}$	$113x - 355 = 0$
$\pi \approx \sqrt{10}$	$x^2 - 10 = 0$
$\pi \approx \sqrt{\frac{40}{3} - 2\sqrt{3}}$	$9x^4 - 240x^2 + 1492 = 0$
$\pi \approx \sqrt{\sqrt{\frac{767}{\sqrt{62}}}}$	$62x^8 - 588289 = 0$
$\pi \approx \sqrt{\sqrt{\frac{2143}{22}}}$	$22x^4 - 2143 = 0$

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THEOREM (Lindemann, 1882):
The number π is transcendental.

Transcendental Number	Reference
$\sum_{n=1}^{\infty} 10^{-n!}$	Liouville (1850)

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$\pi^{e},\ e^{e},\ \pi^{\pi}$	???????

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$\pi^e,\ e^e,\ \pi^\pi$	???????
$e+\pi,\ e\pi$	???????

Approximation	Digits
$\pi \approx \frac{31}{10}$	1
$\pi \approx \frac{314}{100}$	2
$\pi \approx \frac{3141}{1000}$	3
$\pi \approx \frac{31415}{10000}$	4
$\pi \approx \frac{314159}{100000}$	5
$\pi \approx \frac{3141592}{1000000}$	6

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Approximation	Digits
$\pi \approx \frac{22}{7}$	2
$\pi \approx \frac{333}{106}$	4
$\pi \approx \frac{355}{113}$	6
$\pi \approx \frac{103993}{33102}$	9
$\pi \approx \frac{833719}{265381}$	11
$\pi \approx \frac{4272943}{1360120}$	12

Lejeune Dirichlet

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<u>THEOREM</u> (Dirichlet, 1842): For any real irrational number ξ there exist infinitely many rational numbers p/q such that

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