

M365g

Spring 2017

Bowen

Name \_\_\_\_\_

### List

**Instructions.** Go through this list to test your knowledge.

### Questions

1. For plane curves, how are length, speed, regularity, reparametrization, curvature, torsion, signed curvature, the unit tangent, unit normal and unit bi-normal defined?
2. When can a curve be reparametrized so that it is unit-speed?
3. What are the Frenet-Serret equations?
4. What is Hopf's Umlaufsatz?
5. What is the Inverse Function Theorem? How did we use it?
6. What does it mean to say that the curvature and torsion determine the curve? Are there any restrictions on the curvature and torsion functions?
7. What is a regular surface patch? What are the transition functions?
8. For surface patches, how are regularity, the unit normal, the first fundamental form, conformality, area, the Gauss map, local isometries, the Weingarten map, the second fundamental form, Gauss curvature, mean curvature, principal curvatures defined?
9. Give a matrix representation of the Weingarten map.
10. For curves on surfaces, how are geodesic curvature, normal curvature, covariant derivatives, parallel transport, geodesics, lines of curvature, asymptotic curves defined? Explain methods for computing geodesic curvature and normal curvature.

11. What are ruled surfaces? What can you say about their curvatures, geodesics, etc?
12. Determine the geodesic and normal curvatures of meridians and parallels on surfaces of revolution.
13. What are the Christoffel symbols? You do not have to memorize formulas for them but you should know what they are and how to derive them.
14. What are the geodesic equations? Again, you do not have to memorize them but you should know what they mean and how they are derived.
15. What are the Gauss equations? the Codazzi-Mainardi equations? You don't have to memorize them but you should know what they mean and how they are derived.
16. What are the many forms of the Gauss-Bonnet Theorem? What is holonomy? the Euler characteristic? As an exercise, can you figure out what the Gauss-Bonnet Theorem should be for surfaces with non-empty boundary? What are the multiplicities of stationary points of vector fields?
17. What is Gauss' Theorem Egregium?
18. What is the surface variation theorem? What are minimal surfaces? Which surfaces of revolution are minimal?