In the problems below indicate your answers by drawing boxes around them. You must show your work to get credit for a problem.

1. Let \( f(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x - 1}{x^2} \). Find the intervals over which \( f(x) \) is decreasing.

\[
\begin{align*}
\frac{f'(x)}{x^2} &= -\frac{1}{x^2} + \frac{2}{x^3} \\
&= \frac{1}{x^3} (-x + 2) \\
0 &= f'(x) = \frac{1}{x^3} (-x + 2) \implies x = 2
\end{align*}
\]

\[
\begin{array}{c}
\hline
- \quad 0 \quad + \quad 1 \quad - \quad - \quad - \\
\hline

f'(x)
\end{array}
\]

\( f(x) \) is decreasing over \((-\infty, 0) \cup (2, \infty)\)

2. Find and classify any local extremes of \( f(x) = \frac{1}{x} - \frac{1}{x^2} \).

\( \text{Critical numbers: } x = 2 \quad (x = 0 \text{ not in domain}) \)

\[
\begin{array}{c}
\hline
\hline
- \quad + \quad + \quad - \quad - \quad - \\
\hline

(2, f(2)) = (2, \frac{1}{4}) \text{ is local max}
\end{array}
\]

(First derivative test)
3. On what intervals is \( f(x) = \frac{1}{x} - \frac{1}{x^2} \) concave up? Identify any points of inflection.

\[
f''(x) = \left( -\frac{1}{x^2} + \frac{2}{x^3} \right)' = \frac{2}{x^3} - \frac{2}{x^4}
\]

\[
= \frac{2}{x^4} (x - 3)
\]

\( f''(x) \) not defined at \( x = 0 \)

\( 0 = f''(x) = \frac{2}{x^4} (x - 3) \Rightarrow x = 3 \)

\[
\begin{array}{c|c|c|c|c}
\hline
& - & - & - & + & + & + \\
\hline
& 0 & 3 & & & & \\
\hline
\end{array}
\]

Concave up over \((3, \infty)\)

\((3, f(3)) = (3, \frac{2}{9})\) is an inflection pt.

4. Identify any horizontal or vertical asymptotes for \( f(x) = \frac{1}{x} - \frac{1}{x^2} \) (you needn’t discuss the behavior near the vertical asymptotes).

\[
f(x) = \frac{x - 2}{x^2} \quad \text{\( x = 0 \) is vertical asymptote}
\]

\[
\lim_{x \to \pm \infty} f(x) = \frac{x}{x^2} = \frac{1}{x} \to 0
\]

Thus \( y = 0 \) is a horizontal asymptote \((\text{as} \ x \to \pm \infty)\)
5. Sketch the graph of \( f(x) = \frac{1}{x} - \frac{1}{x^2} \) using (all) the information in the previous problems. State whether there is an absolute maximum and whether there is an absolute minimum for \( f(x) \) and say what they are.

Note: \( 0 = f'(x) = \frac{x-1}{x^2} \Rightarrow x=1 \). Thus \((1,0)\) is x-intercept.

\[ f(2) = \frac{1}{4} \] is absolute max.

No absolute min.
6. A plane flying horizontally at an altitude of 1 mile and speed of 200 m/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 miles away from the station.

\[ \frac{dz}{dt} = ? \]
\[ \frac{dx}{dt} = 200 \text{ mph} \]

\[ 1^2 + x^2 = z^2 \Rightarrow \frac{d}{dt} (1 + x^2) = \frac{d}{dt} (z^2) \]
\[ \Rightarrow 2x \frac{dx}{dt} = 2z \frac{dz}{dt} \]
\[ \Rightarrow 2 \cdot 16 \cdot 200 = 2 \cdot 4 \cdot \frac{dz}{dt} \]
\[ \Rightarrow \frac{dz}{dt} = 50 \sqrt{15} \]

When \( z = 4 \):
\[ 16 = 1 + x^2 \Rightarrow x = \sqrt{15} \]

7. Find \( f(x) \) such that of \( f'(x) = 4x^7 - 3 \) if \( f(1) = -10 \).

\[ f'(x) = 4 \cdot \frac{1}{8} \cdot x^8 - 3 \cdot x + C \]
\[ = \frac{1}{2} x^8 - 3x + C \]
\[-10 = f(1) = \frac{1}{2} - 3 + C \Rightarrow C = -\frac{15}{2} \]
\[ f(x) = \frac{1}{2} x^8 - 3x - \frac{15}{2} \]
8. A rancher wants to fence in a rectangular pen then divide it in half with a fence parallel to one of the two sides of the rectangle. If she has 1800 ft of fence (which must be used for outside of the pen and for the fence dividing the pen), what are the dimensions of such a pen that will enclose the largest area?

\[ A = xy \]

**Constraint:** \[ 1800 = 3x + 2y \] \[ \Rightarrow y = 900 - \frac{3}{2}x \]

\[ A(x) = x \left( 900 - \frac{3}{2}x \right) \]
\[ = 900x - \frac{3}{2}x^2 \]

\[ A'(x) = 900 - 3x \]
\[ \Rightarrow x = 300 \]

When \( x = 300 \), \( A \) achieves absolute max area \( (9600) \)

**Dimensions:** \( x = 300 \) \( y = 900 - \frac{3}{2}(300) = 450 \)