In the problems below indicate your answers by drawing boxes around them. You must show your work to get credit for a problem.

1. Compute $f'(0)$ when $f(x) = \frac{\sqrt{3} \sec(x) - x}{\tan(x) - \cos(x)}$.

   
   \[
   f'(x) = \frac{\sqrt{3} \sec(x) \tan(x) - 1}{\tan(x) - \cos(x)} \left( \frac{\tan(x) - \cos(x)}{(\tan(x) - \cos(x))^2} - \frac{\sqrt{3} \sec(x) - x}{(\tan(x) - \cos(x))^2} \right) 
   
   \]

   
   \[
   f'(0) = \frac{(0-1)(0-1) - (\sqrt{3} \cdot 1)(1+0)}{(-1)^2} 
   
   = \left[ 1 - \sqrt{3} \right] 
   
   \]

2. Find any vertical asymptotes of $\frac{-x^2 + 6x + 9}{x + 5}$. Sketch the graph of this function near these asymptotes. (You don’t need to sketch the whole graph, just the part near the asymptotes).

   \[ X = -5 \text{ is vertical asymptote} \]

   \[
   -x^2 + 6x + 9 \rightarrow -25 + -30 + 9 = -46 \\
   \text{as } x \rightarrow -5 
   
   \text{Thus } \frac{-x^2 + 6x + 9}{x + 5} \rightarrow -\infty \text{ as } x \rightarrow -5^+ 
   
   \frac{-x^2 + 6x + 9}{x + 5} \rightarrow \infty \text{ as } x \rightarrow -5^- 
   
   \]
3. Using the definition of derivative (i.e. as a limit of a difference quotient) compute \( f'(5) \) when \( f(x) = \frac{1}{2x + 1} \).

\[
\begin{align*}
    f'(5) &= \lim_{h \to 0} \frac{f(5+h) - f(5)}{h} \\
    &= \lim_{h \to 0} \frac{\frac{1}{2(5+h) + 1} - \frac{1}{2(5) + 1}}{h} \\
    &= \lim_{h \to 0} \frac{1 - \frac{2(5+h) + 1}{2(5+h) + 1}}{11h} \\
    &= \lim_{h \to 0} \frac{-2h}{(11h)(11) - 11} \\
    &= \lim_{h \to 0} \frac{-2}{11(h+1) - 11} = \boxed{\frac{-2}{121}}
\end{align*}
\]

4. The graphs of \( y = f(x) \) and \( y = g(x) \) are given at the right. Find \( \lim_{x \to 1} f(x)g(x) \) and \( \lim_{x \to -3} f(x)g(x) \).

\[
\begin{align*}
    \lim_{x \to 1} f(x)g(x) &= \lim_{x \to 1} f(x) \lim_{x \to 1} g(x) \\
    &= 4 \cdot -2 = \boxed{-8}
\end{align*}
\]

As \( x \to 3^+ \), \( f(x) \to 2 \), \( g(x) \to 0 \)

Thus \( f(x)g(x) \to 0 \)

As \( x \to 3^- \), \( f(x) \to -3 \), \( g(x) \to 0 \)

Thus \( f(x)g(x) \to 0 \)

\[
\begin{align*}
    \lim_{x \to 3^-} f(x)g(x) &= \lim_{x \to 3^-} f(x)g(x) \\
    &= \lim_{x \to 3^-} f(x)g(x) = 0
\end{align*}
\]
5. Find the slope of the tangent line to \( y = \frac{1}{2x^2 - \frac{1}{x}} \) at the point (1,1).

\[
y' = \frac{1' \left( 2x^2 - \frac{1}{x} \right) - 1 \cdot \left( 2x^2 - \frac{1}{x} \right)'}{(2x^2 - \frac{1}{x})^2}
\]

\[
= \frac{0 - \left( 4x + x^{-2} \right)}{(2x^2 - \frac{1}{x})^2}
\]

\[
y'(1) = \frac{-(4 + 1)}{(2 - 1)^2} = -5
\]

Slope at (1,1) is -5

6. Suppose the function \( C(x) \) is the cost for producing \( x \) units of a certain toy. Let \( C(x) = (1000 - x - x^2)(x^2 + 4x + 50) \). How much is the cost changing per unit change in the number of toys produced when \( x = 10 \) toys are being made? This quantity is called marginal cost.

\[
C'(x) = (1000 - x - x^2)' \cdot (x^2 + 4x + 50) + (1000 - x - x^2) \cdot (x^2 + 4x + 50)'
\]

\[
= (-1 - 2x) \cdot (x^2 + 4x + 50) + (1000 - x - x^2) \cdot (2x + 4)
\]

\[
C'(10) = (-21) \cdot (190) + (890) \cdot 24
\]

\[
= -3990 + 21,360
\]

\[
= 17,370
\]
7. Let \( f(x) = \cos(x) \sin(x)(x^4 - \sqrt{x}) \). Compute \( f'(x) \).

\[
\begin{align*}
  f'(x) &= -\sin(x) \left( \sin(x) \left( x^4 - \sqrt{x} \right) \right) + \cos(x) \left( \sin(x) \left( x^4 - \sqrt{x} \right) \right)'
  \\
  &= -\sin(x) \left( \sin(x) \left( x^4 - \sqrt{x} \right) \right) + \cos(x) \left[ \cos(x) \left( x^4 - \sqrt{x} \right) + \sin(x) \left( 4x^3 - \frac{1}{2} x^{-\frac{1}{2}} \right) \right]
  \\
  &= \left( \cos^2(x) - \sin^2(x) \right) \left( x^4 - \sqrt{x} \right) + \cos(x) \sin(x) \left( 4x^3 - \frac{1}{2} x^{-\frac{1}{2}} \right)
  \\
  \left( \sqrt{x} \right)' \cdot \left( \sqrt{x} \right)' &= \frac{1}{2} x^{\frac{1}{2}-1}
  \\
  &= \frac{1}{2} x^{-\frac{1}{2}}
\end{align*}
\]

8. A particle is moving along the x-axis. Its position at time \( t \), \( X(t) \), is given by \( X(t) = -4t^3 + 6t^2 + 8 \). This particle starts out at position \( X = 8 \) (when \( t = 0 \)). The particle moves along the axis until it reaches its maximum position, then turns around and returns. What is this maximum position?

\( 0 = X'(t) = -12t^2 + 12t = -12t(1-t) \Rightarrow t = 0, t = 1 \)

At \( t = 1 \) it reaches its maximum position

\( X(1) = 10 \) is its maximum position.

What is the particle’s velocity when it returns to position \( X = 8 \)?

\( 8 = X(t) = -4t^3 + 6t^2 + 8 \Rightarrow 0 = -4t^3 + 6t^2 \\
= t^2(-4t + 6) \Rightarrow t = 0, t = \frac{3}{2} \)

\( X'(\frac{3}{2}) = -12(\frac{3}{2})^2 + 12(\frac{3}{2}) = -9 \)
9. Let \( s(t) \) be the position of a car at time \( t \) moving along a straight road. The graph of \( s(t) \) is pictured on the right. Use the graph to answer the following questions.

(i). What is the average velocity of the car between the times \( t = 0 \) and \( t = 10 \)?

\[
\text{avg. velocity} = \frac{s(10) - s(0)}{10 - 0} = \frac{15 - 5}{10} = 1
\]

(ii). Is the car going faster at A or B?

faster at B because the slope of tangent \((= s'(x))\) is greater

10. The graph of \( y = f(x) \) is pictured on the right. Where does the graph of \( y = f'(x) \) intersect the x-axis (i.e., what are the x-intercepts of \( f' \))? Where does the graph of \( y = f'(x) \) intersect the y-axis?

\( x \)-intercepts of \( f' \) are where \( f' = 0 \):

\[ x = 2, \ x = 4 \]

\( \therefore \) \( (2, 0), (4, 0) \)

\( y \)-intercept: \( (0, f'(0)) \)

\[ f'(0) = \text{slope at } (0, 2) \text{ on } f(x) = 1 \]

y-intercept \((0, 1)\)
11.6. Compute the slope of the tangent line to \( y = \sqrt[3]{\tan(3x) + \sec(\pi x)} \) at \((0, 1)\).

\[
y' = \left( (\tan(3x) + \sec(\pi x))^{\frac{1}{3}} \right)'
\]
\[
= \frac{1}{3} (\tan(3x) + \sec(\pi x))^{-\frac{2}{3}} \\
\cdot (\sec^2(3x) \cdot 3 + \sec(\pi x) \cdot \tan(\pi x) \cdot \pi)
\]

At \( x = 0 \),
\[
y' = \frac{1}{3} (0 + 1)^{-\frac{2}{3}} \left( 1 \cdot 3 + 1 \cdot 0 \cdot \pi \right)
\]
\[
= \frac{1}{3} \cdot 3 = \boxed{1}
\]