In the problems below indicate your answers by drawing boxes around them. You must show your work to get credit for a problem.

1. Let \( f(x) = \frac{x^2 + 4}{x^2 - 1} \). Find the intervals over which \( f(x) \) is increasing.

\[
\begin{align*}
\frac{d}{dx} f(x) &= \frac{(2x)(x^2-1) - (x^2+4)(2x)}{(x^2-1)^2} \\
&= \frac{-10x}{(x^2-1)^2}
\end{align*}
\]

Increasing \((-\infty, -1) \cup (-1, 0)\)
2. Find and classify any local extremes of \( f(x) = \frac{x^2 + 4}{x^2 - 1} \).

Critical numbers: \( x = 0 \) only (see #1)

First derivative test \( \Rightarrow \) \((0, -4)\) local max

\[
\begin{array}{c|c|c|c|c|c}
\pm & + & 1 & - & + \\
-1 & 0 & 1 & f
\end{array}
\]

3. Find any horizontal or vertical asymptotes for \( f(x) = \frac{x^2 + 4}{x^2 - 1} \).

As \( x \to \pm \infty \), \( f(x) = \frac{x^2 + 4}{x^2 - 1} \approx \frac{x^2}{x^2} = 1 \)

Thus \( y = 1 \) is horizontal asymptote as \( x \to \pm \infty \).

Thus \( x = 1, x = -1 \) are vertical asymptotes.
4. Sketch the graph of \( f(x) = \frac{x^2 + 4}{x^2 - 1} \) using the information in the previous problems. Include the asymptotes, but don’t worry about concavity or inflection points. State whether there is an absolute maximum and whether there is an absolute minimum for \( f(x) \) and say what they are.
5. Over what intervals is \( g'' \) negative, if the graph of \( y = g(x) \) is as pictured to the right?

\[
g'' < 0 \Leftrightarrow \text{concave down}
\]

Concave down:
\((-\infty, -1) \cup (2, \infty)\)

6. At which points on the curve of \( y = 1 + 40x^3 - 3x^5 \) does the tangent line have the largest slope? (Hint: What is the formula, \( m(x) \), for the slope at each point of the curve? How would you find its largest value?)

\[
m(x) = y' = 120x^2 - 15x^4
\]

\[
m'(x) = 240x - 60x^3 = 60x(4-x^2)
\]

Critical numbers: \( x = -2, 0, 2 \)

\[
\begin{array}{cccccc}
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& & & & & \\
& \downarrow & - & + & + & - \\
-2 & - & 0 & 2
\end{array}
\]

\[
m'(x)
\]

\( x = -2, x = 2 \) local max

\[
M(-2) = 480 - 240 = 240 = m(2)
\]

Maximal slope is attained when \( x = -2, 2 \)

\( m(x) \) increases to \( m(-2) \), then decreases, then increases to \( m(2) \), then decreases.

That is, maximal slope at pts \((-2, -223), (2, 225)\)
7. A particle moves along the ellipse given by \( x^2 + 5y^2 = 29 \). If the particle is moving at a velocity of 4 m/sec in the x-coordinate (i.e. horizontally) when it crosses the point \( (3, 2) \), at what velocity is it moving in the y-coordinate (i.e. vertically)

\[
\frac{dx}{dt} = 4 \quad \text{at} \quad (3, 2)
\]

\[
\frac{dy}{dt} = ?
\]

\[
\frac{d}{dt} \left( x^2 + 5y^2 = 29 \right)
\]

\[
\Rightarrow \quad 2x \frac{dx}{dt} + 10y \frac{dy}{dt} = 0
\]

\[
\Rightarrow \quad 2 \cdot 3 \cdot 4 + 10 \cdot 2 \cdot \frac{dy}{dt} = 0
\]

\[
\Rightarrow \quad \frac{dy}{dt} = -\frac{2y}{20} = \boxed{-1.2 \text{ m/sec}}
\]

Note \( \frac{dy}{dt} < 0 \) means particle is moving down at \( (3, 2) \)
8. Find the anti-derivative, $F(x)$, of $f(x) = \sec^2(3x)$ if $F(0) = 2$.

\[
F(x) = \frac{1}{3} \tan(3x) + C
\]

\[
\text{(Check: } F'(x) = \frac{1}{3} \cdot \sec^2(3x) \cdot 3)\]

\[
2 = F(0) = \frac{1}{3} \tan(0) + C
\]

\[
= C
\]

\[
\boxed{F(x) = \frac{1}{3} \tan(3x) + 2}
\]