With the sketched modifications we are able to perform the intended "blow-up" procedure.

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Symmetry results for Plateau's surfaces

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(joint work with Jacob Bernstein)

A **minimal Plateau's surface** Σ is defined here as a closed subset of $\mathbb{R}^3$ such that, for each point $p \in \Sigma$, one can find $r > 0$, $\alpha \in (0, 1)$, and a $C^{1,\alpha}$-diffeomorphism $f_p : B_r(p) \to B_r(p)$, with $Df_p(p)$ a linear isometry, such that $f_p(\Sigma \cap B_r(p)) = K \cap B_r(p)$, where K is either a plane P, a half-plane H, a union Y of three half-planes meeting along a common line at 120-degrees, or a regular tetrahedral cone T. Moreover, the interior set Σ of the points p with $K = P$ is assumed to have vanishing mean curvature – given the $C^{1,\alpha}$-regularity, at first in distributional sense, and thus, by elliptic regularity, in the classical, smooth sense.

The above definition captures, in elementary mathematical terms, the content of the experimental laws of Plateau for soap films at equilibrium. It should be noted that Plateau also experimented with soap bubbles, where the mean curvature of the interior set may take different constant values on different connected components. Also, the above definition does not include the possibility of "singular boundary points", which are indeed physically possible, although (apparently) not exhaustively described in the physical and mathematical literature.

As shown in the works of Almgren [2] and Taylor [8], minimal Plateau's surfaces arise as *Almgren minimal sets*, i.e., as closed sets locally minimizing the two-dimensional Hausdorff measure $\mathcal{H}^2$ in $\mathbb{R}^3$ with respect to local Lipschitz deformations. Variational characterization of Almgren minimal sets as global minimizers in suitable variational problems have been first proposed, and then obtained, by several authors in recent years. Limiting ourselves to the first results concerning area minimization in codimension one we mention here [4, 5, 6] as entry points in a vaster literature.

Classical minimal surfaces are often motivated in terms of their application to the description of soap films. From this viewpoint, given the ubiquity of Y-type and T-type singularity, we consider the fascinating idea of reviewing classical results for smooth minimal surfaces in the physically more relevant context of minimal Plateau's surfaces.

In this direction, we consider as a case study a rigidity theorem of Schoen [7] for catenoids: given two co-axial circles in $\mathbb{R}^3$, the only minimal surfaces bounded by those circles are either catenoids or disks. The expected result in the context of minimal Plateau's surfaces should include an additional rigidity case, which will be present depending on the metric data of the problem (radii of the circles and their distance), and consists of singular catenoids, i.e. union of two catenoidal necks and a disk meeting along a common boundary circle of Y-points.

In [3] we obtain the expected extension of Schoen's rigidity theorem to minimal Plateau's surfaces. For reasons whose nature is likely just technical, this is done under the assumptions that the two circles have the same radii, and under a global to local topological assumption called "cellular structure" (for each $p \in \Sigma$ there exists $r_p > 0$ such that $\mathbb{R}^3 \setminus \Sigma$ and $B_r(p) \setminus \Sigma$ have the same finite number of connected components if $r < r_p$).

The result is obtained by an original application of the classical moving planes method introduced by Alexandrov in [1]. Interestingly, Alexandrov's method has been concurrently applied in the non–smooth setting of varifold solutions to the mean curvature flow in [9], and, independently from our work, to the study of Schoen's rigidity theorem in the varifold setting, but assuming *a priori* the absence of Y-singularities, in [10]. A novel contribution we can offer is the insight that the application of the moving planes can be compatible with the actual presence of singularities, while still working as a tool to obtain additional regularity (we exclude T-type singularities in a situation where they could indeed be possible).

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