**RESEARCH STATEMENT**

Katja Mallmann

My general area of study is Brauer groups. The Brauer group of a field $K$ is the set of all equivalence classes of central simple algebras over $K$. Over rings, central simple algebras correspond to Azumaya algebras. In the realm of Azumaya algebras we are able to apply specialization methods, for example, which makes their study valuable (cf. [S99]). The main focus of my research is the study of Azumaya algebras with involutions, invariants of involutions and algebraic structures defined by them. Ultimately, I seek to give a specific description of the discriminant algebra.

An involution is an anti-automorphism of order two of an algebra. The center of an algebra $A$ is preserved under any involution $\sigma$ and the restriction of $\sigma$ to the center of the algebra is an automorphism which is either the identity or of order two. Involutions which leave the center elementwise fixed are called involutions of the first kind while involutions whose restriction to the center is an automorphism of order two are called of the second kind (or unitary).

In the case of involutions of the first kind, their structures, like the Clifford algebra for orthogonal involutions, are associated to classical quadratic forms and have been well-studied. The study of the discriminant algebra, which we build from an algebra with unitary involution, however, is related both to hermitian forms and to the corestriction map in cohomology. The latter is true due to a result on the existence of involutions of the second kind which says that an algebra carries a unitary involution if and only if its corestriction is trivial (cf. [BI98] or [S77]).

An explicit description of the discriminant algebra is only known in very few cases like the quaternion algebras and algebras of degree four. My goal is to describe the discriminant algebra as an algebra, as a crossed product and as a cohomology class. In order to begin my research I first learned about quadratic forms, Azumaya algebras, crossed products, involutions and group cohomology.

While in the Book of Involutions, [BI98], the authors are mainly concerned with involutions and invariants of involutions on central simple algebras, the question naturally arises as to whether some of these concepts can be generalized to the Azumaya case. In [S05], Saltman defines Clifford algebras for Azumaya algebras with involutions. He introduces the so-called $G-H$-cocyles and the associated Azumaya crossed products in order to construct a "generic" Azumaya algebra with orthogonal involution and give a cohomological description of the Clifford algebra. I use, modify and extend some of his techniques in order to study the structure of the discriminant algebra.

My specific work has been in generalizing the definition of the discriminant algebra to the Azumaya case. It is important to note that in order to do so, I had to modify some of the assumptions, like asking that the original algebra has a separable splitting field $M$.

Starting with an Azumaya $R$-algebra $(A, \tau)$ of even degree $n = 2m$ with a unitary involution $\tau$ and center $S$, the definition is done in a couple of steps. It is prepared for by the construction of the so-called $k$-th $\lambda$-powers of an algebra. For each $k$, this is an Azumaya algebra, denoted by $\lambda^k A$, which is Brauer-equivalent to $A^{\otimes k}$ of degree $\binom{n}{k}$. In the case where $k = m$ is half the degree of the original algebra, the $\lambda$-power carries both a canonical involution of the first kind and an involution of the second kind defined by $\tau$. The discriminant algebra $D(A, \tau)$ is defined as the $R$-subalgebra of invariants under the composition of those two involutions. It is an Azumaya algebra over $R$ of degree $\binom{n}{m}$ and carries an involution of the first kind, the restriction of either of the involutions on $\lambda^m A$.

In the course of my research I found that the exterior power algebra $E_m(M)$ as defined by Saltman in [S83] is naturally the splitting field of the $\lambda$-power algebra. I also accomplished to describe the involutions on $E_m(M)$ concretely and thus give one explicit description of the structure of a splitting subalgebra for the discriminant algebra. In order to give a fully coherent picture I am performing further cohomology and lattice computations to find the corresponding cocycle that will enable us to exhibit the cohomology class and crossed product structure.

As for future research, my plans include a more thorough study of Brauer groups and crossed products. It also look forward to further learning about quadratic and hermitian forms. While the Clifford algebra, for example, has already found some important applications in physics, engineering and related fields, I consider it of great interest to explore more interdisciplinary questions between mathematics and the natural sciences.

## References


[BI98] Max-Albert Knus, Alexander Merkurjev, Markus Rost, Jean-Pierre Tignol, *The Book of Involutions*, American Mathematical Society, Providence, RI 1998

[S77] David J. Saltman, *Azumaya Algebras with Involution*, Journal of Algebra, 52 (1978), 526-539

[S83] David J. Saltman, *Exterior Powers of Fields and Subfields*, Nagoya Mathematical Journal, Vol. 89 (1983), 119-127

[S99] David J. Saltman, *Lectures on Division Algebras*, American Mathematical Society, Providence, RI 1999 (CBMS Lecture Note Series #84)

[S05] David J. Saltman, *Triality, Cocycles, Crossed Products, Involutions, Clifford Algebras and Invariants*, preprint