10.2

Find an explicit formula for a_n:

5. $2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}, \frac{26}{5}, ...$

6. $-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, -\frac{5}{36}, ...$

Determine the boundedness and monotonicity of each sequence:

29. $\{\frac{2^n-1}{2^n}\}$

30. $\{\frac{1}{2n} - \frac{1}{2n+3}\}$

A sequence $\{a_n\}$ is said to be defined recursively if, for some $k \geq 1$, the terms $a_1, a_2, ..., a_k$ are given and a_n is specified in terms of $a_1, a_2, ..., a_{n-1}$ for each $n \geq k$. The formula specifying a_n for $n \geq k$ in terms of some (or all) of its predecessors is called a recurrence relation. Write down the first six terms of each sequence and then give the general formula for a_n.

51. $a_1 = 1; a_{n+1} = a_n + 2n + 1$

52. $a_1 = 1; a_{n+1} = 2a_n + 1$

61. Let r be a real number, $r \neq 0$. Define a sequence $\{S_n\}$ by:

$S_1 = 1$

$S_2 = 1 + r$

$S_3 = 1 + r + r^2$

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$S_n = 1 + r + r^2 + ... + r^{n-1}$

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(a) Suppose $r = 1$. What is S_n for $n = 1, 2, 3, ...$?

(b) Suppose $r \neq 1$. Find a formula for S_n that does not involve adding up the powers of r. HINT: Calculate $S_n - rS_n$.

62. Let $a_n = \frac{1}{n(n+1)}, n = 1, 2, 3, ...$, and let $\{S_n\}$ be the sequence defined by

$S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

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$S_n = a_1 + a_2 + a_3 + ... + a_n$

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Find a formula for $S_n, n = 1, 2, 3, ...,$ that does not involve adding up the terms $a_1, a_2, a_3, ...$ HINT: Use partial fractions to write $1/[k(k+1)]$ as the sum of two fractions.

10.3

State whether the sequence converges and, if it does, find the limit.

5. $\{\frac{n-1}{n}\}$

6. $\{\frac{n+(-1)^n}{n}\}$

31. $\{(1+\frac{1}{n})^{2n}\}$

32. $\{(1+\frac{1}{n})^{n/2}\}$

These sequences are defined recursively. Determine in each case whether the sequence converges and, if so, find the limit. Start each sequence with $a_1 = 1$.

53. $a_{n+1} = \frac{1}{n+1}a_n$

54. $a_{n+1} = \frac{n}{n+1}a_n$