

10.4

State whether the sequence converges as $n \rightarrow \infty$. If it does, find the limit.

9. $\{n^{\alpha/n}\}, \alpha > 0.$

10. $\{\ln(\frac{n+1}{n})\}$

21. $\{\frac{\ln n^2}{n}\}$

22. $\{ \int_{-1+1/n}^{1-1/n} \frac{dx}{\sqrt{1-x^2}} \}$

37. Show that $\lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = 0.$

38. Show that $\lim_{n \rightarrow \infty} (\sqrt{n^2+n} - n) = \frac{1}{2}.$

10.5

Find the limit.

7. $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$

8. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x}$

15. $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$

16. $\lim_{x \rightarrow 0} \frac{x - \ln(x+1)}{1 - \cos 2x}$

37. Find the fallacy:

$$\lim_{x \rightarrow 0} \frac{2+x+\sin x}{x^3+x-\cos x} = \lim_{x \rightarrow 0} \frac{1+\cos x}{3x^2+1+\sin x} = \lim_{x \rightarrow 0} \frac{-\sin x}{6x+\cos x} = \frac{0}{1} = 0.$$

38. Show that, if $a > 0$, then

$$\lim_{n \rightarrow \infty} n(a^{1/n} - 1) = \ln a.$$

10.6

Find the limit.

3. $\lim_{x \rightarrow \infty} \frac{x^3}{1-x^3}$.

4. $\lim_{x \rightarrow \infty} \frac{x^3-1}{2-x}$.

13. $\lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x e^{t^2} dt$.

14. $\lim_{x \rightarrow \infty} \frac{\sqrt{1+x^2}}{x^2}$.

31. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$.

32. $\lim_{x \rightarrow 0} (e^x + 3x)^{1/x}$.