1.3.2

\[ A_{2000}(5) = 2000\alpha(5) = 2000(1 + 0.04 \cdot 5) = 2000 \cdot 1.2 = 2400. \]

1.3.4 From \( a(0) = 1 \), we conclude that \( \gamma = 1 \).

Next, the given conditions are

\[ 100(\alpha \cdot 4^2 + \beta \cdot 4 + 1) = 152, \]
\[ 200(\alpha \cdot 2^2 + \beta \cdot 2 + 1) = 240. \]

Tidying up the expressions above, we get the following system of two equations with two unknowns \( \alpha \) and \( \beta \):

\[ 4\alpha + \beta = 0.13, \]
\[ 2\alpha + \beta = 0.10. \]

So, \( \alpha = 0.015 \) and \( \beta = 0.07 \) and

\[ a(t) = 0.015t^2 + 0.07t + 1 \text{ for } t \geq 0. \]

Let \( B \) denote the amount in the account at time 8 if 1,600 was deposited at time 6. Then,

\[ B = 1600 \cdot \frac{a(8)}{a(6)} = 1600 \cdot \frac{0.015 \cdot 64 + 0.07 \cdot 8 + 1}{0.015 \cdot 36 + 0.07 \cdot 6 + 1} = 1600 \cdot \frac{2.52}{1.96} \approx 2057.14. \]

1.3.6 According to the information from the problem, the amount of interest accrued from time 0 to time \( n \) is

\[ I_n = 2 + 2^2 + \cdots + 2^{n-1} + 2^n. \]

Using the following formula for the sum of the first \( n \) terms of a geometric sequence:

\[ 1 + Q + Q^2 + \cdots + Q^{n-1} = \frac{1 - Q^n}{1 - Q}. \]

we get

\[ I_n = 2(1 + 2 + \cdots + 2^{n-1}) = 2 \cdot \frac{1 - 2^n}{1 - 2} = 2(2^n - 1) = 2^{n+1} - 2. \]
1.4.6 The accumulation function reads as
\[ a(t) = 1 + 0.05t \quad \text{for } t \geq 0. \]

(a) We need to find the integer \( n \) such that \( i_n = \frac{1}{23} \). In general,
\[ i_n = \frac{a(n) - a(n - 1)}{a(n - 1)} = \frac{0.05}{1 + 0.05(n - 1)}. \]

So, we need to solve for \( n \) in
\[ 1 + 0.05(n - 1) = 23 \cdot 0.05 = 1.15. \]
We get \( n = 4 \).

(b) Using the defining formula for the effective interest rate, we get
\[ i_{[4.6]} = \frac{0.05(6 - 4)}{1 + 0.05 \cdot 4} = \frac{1}{12} \approx 0.0833333. \]

1.5.4 In this problem, the accumulation function is
\[ a(t) = (1 + 0.05)^t \quad t \geq 0. \]
So, the desired amount of interest is
\[ 1000(a(4) - a(3)) = 1000(1.05^4 - 1.05^3) = 1000 \cdot 0.05 \cdot 1.05^3 \approx 57.88. \]

1.5.8 Note that the amount of money original invested is not relevant. It is important to compare the amount of interest earned using the two account options for the time period between time 2 and the end of the investment horizon (time 7 for part (a), and time 10 for part (b)). Let \( K \) denote the principal deposited at time 0.

(a) The amount of interest earned between times 2 and 7 using simple interest is
\[ K(1 + 0.1 \cdot 7) - K(1 + 0.1 \cdot 2) = K \cdot 0.1 \cdot 5 = 0.5K. \]

On the other hand, the amount of interest earned between times 2 and 7 using compound interest is
\[ K(1 + 0.1 \cdot 2)(1 + 0.07)^5 - K(1 + 0.1 \cdot 2) = K \cdot 1.2 \cdot (1.07^5 - 1) \approx 0.48K. \]
So, it is more profitable to leave the money in the simple interest account.

In particular, for \( K = 2,500 \), the amount of interest earned in the simple interest account is 1,250. The total balance in the simple interest account at time 7 is 4,250 (this is the number you can see in the solution in the end of your textbook). Also, the amount of interest earned in the compound interest account is about 1,152.20. The total balance in the compound interest account at time 7 is
\[ 2,500(1 + 0.1 \cdot 2) \cdot (1.07)^5 \approx 4207.66 \]
(this is the other number you can see in the solution in the end of your textbook).
1.6.4 The amount of interest earned on $K$ in one year can be expressed as

\[ Ki_1 = 256. \]

On the other hand, the amount of discount (see box on p. 25 in the textbook) is

\[ Kd_1 = 236. \]

Dividing these two equations, we get

\[ \frac{i_1}{d_1} = \frac{256}{236}. \]

Since \( d_1 \) and \( i_1 \) are equivalent, we also have

\[ d_1 = \frac{i_1}{1 + i_1} \Rightarrow \frac{i_1}{d_1} = 1 + i_1. \]

Combining the last two equations, we get

\[ 1 + i_1 = \frac{256}{236} \Rightarrow i_1 = \frac{20}{236}. \]

Together with the very first equation, this implies

\[ K = \frac{256}{i_1} = \frac{256 \cdot 236}{20} = 3,020.80. \]

1.7.2 This value is

\[ 4850 \cdot v(12) \cdot a(6) = 4850 \cdot (1 - 0.04 \cdot 12) \cdot (1 - 0.04 \cdot 6)^{-1} = 4850 \cdot 0.52 \cdot 0.76^{-1} = 3318.42. \]

1.7.8. Let us denote the present value of project 1 under the annual effective interest rate \( i \) by \( PV(1, i) \). Then,

\[ PV(1, 6\%) = -20,000 + 8,000v_{6\%} + 15,000v_{6\%}^2 \]

with \( v_{6\%} = \frac{1}{1+0.06} = \frac{1}{1.06} \). Then,

\[ PV(1, 6\%) = 897.116. \]

Let us denote the present value of project 2 under the annual effective interest rate \( i \) by \( PV(2, i) \). Then,

\[ PV(2, 6\%) = -10,000 + 3,000v_{6\%} - Xv_{6\%}^2 + 14,000v_{6\%}^3. \]

We are given that \( PV(1, 6\%) = PV(2, 6\%) \). So,

\[ -10,000 + 3,000v_{6\%} - Xv_{6\%}^2 + 14,000v_{6\%}^3 = 897.116. \]

Solving for \( X \) above, we get \( X = 4143.55 \).

Finally, with \( v_{5\%} = \frac{1}{1.05} \),

\[ PV(1, 5\%) - PV(2, 5\%) = -10000 + 5000v_{5\%} + 19143.55v_{5\%}^2 - 14000v_{5\%}^3 \approx 31.94. \]
1.9.2
(a) The accumulation function reads as
\[ a(t) = (1 - 0.035)^{-t} \] for \( t \geq 0 \).

So, she will have to repay
\[ 4000a(6) = 4593.32. \]

(b)
\[ i = \frac{d}{1 - d} = \frac{0.035}{0.965} \approx 0.0363. \]