More Exotic Options

1. Barrier Options
2. Compound Options
3. Gap Options
More Exotic Options

1 Barrier Options

2 Compound Options

3 Gap Options
Definition; Some types

- The payoff of a Barrier option is path dependent
- More precisely, the payoff depends on whether over the option life the underlying price reaches the barrier; note that this is a simplistic view of things - the stock prices are observed at discrete times and the wording above implies continuous observation of stock prices
- Knock-out options go out of existence if the asset price reaches the barrier; the variants are:
  - down-and-out: has to fall to reach the barrier
  - up-and-out: has to rise to reach the barrier
- Knock-in options come into existence if the asset price reaches the barrier; the variants are:
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Rebate options

- **Rebate options** make a fixed payment if the asset price reaches the barrier; we have
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- More complex barrier options require the asset price to not only cross a barrier, but spend a certain amount of time across the barrier in order to knock in or knock out
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Up-and-Out Call

- Let us spend some time studying a particular barrier option - the analysis of the other options would be analogous
- Assume that the underlying risky asset $S$ is a geometric Brownian motion

$$dS(t) = rS(t) \, dt + \sigma S(t) \, d\tilde{W}(t),$$

where $\tilde{W}$ is a standard Brownian motion under the risk-neutral measure $\tilde{P}$
- Consider a European call, expiring at time $T$ with strike price $K$ and up-and-out barrier $B$ (of course, $K < B$)
- As we have seen before, the closed form for the solution of the above SDE for the asset price $S$ is

$$S(t) = S(0)e^{\sigma \tilde{W}(t)+(r-\frac{1}{2}\sigma^2)t} = S(0)e^{\sigma \hat{W}(t)}$$

where we introduced the stochastic process

$$\hat{W}(t) = \frac{1}{\sigma}(r - \frac{1}{2}\sigma^2)t + \tilde{W}(t)$$
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Up-and-Out Call: On the maximum of $S$

- Define

$$\hat{M}_T = \max_{0 \leq t \leq T} \hat{W}(t)$$

- Then, we have that

$$\max_{0 \leq t \leq T} S(t) = e^{\sigma \hat{M}(T)}$$

- The option kicks out if and only if

$$S(0)e^{\sigma \hat{M}(T)} > B$$

- If the above happens - the option is rendered worthless

- If

$$S(0)e^{\sigma \hat{M}(T)} \leq B$$

then the payoff is

$$(S(T) - K)^+ = (S(0)e^{\sigma \hat{W}(T)} - K)^+$$
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Up-and-Out Call: The payoff

- Altogether, when we combine the above two cases, the payoff of the option is

\[ V(T) = (S(0)e^{\sigma \hat{W}(t)} - K)^+ \mathbb{I}_{\{S(0)e^{\sigma \hat{M}(t)} \leq B\}} \]

\[ = (S(0)e^{\sigma \hat{W}(t)} - K) \mathbb{I}_{\{S(0)e^{\sigma \hat{W}(t)} \geq K, S(0)e^{\sigma \hat{M}(t)} \leq B\}} \]

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where

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The Valuation

- In general, barrier options are “cheaper” than the otherwise identical “ordinary” options
- Under some minor assumptions, it is possible to find the price of the barrier options in the Black-Scholes setting
- The formula for the price is quite long - but it contains only $N$ as a special function
- In fact, the up-and-out option’s price satisfies the Black-Scholes-Merton partial differential equation (as the price for the ordinary European call) - it is the boundary conditions that are different (they have to account for the barrier) and they complicate matters somewhat
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The Set-up

• A compound option is an option to buy an option

• Let us draw a timeline

• Consider a call on a call option, i.e., an option to buy a call option with maturity \( T \) and strike price \( K \) at some exercise time \( T_1 < T \), for some strike price \( K_1 \)

• This call on a call should be exercise at time \( T_1 \) only if the strike price \( K_1 \) is lower than the price of the underlying call option at time \( T_1 \)

• So, the payoff of this option at time \( T_1 \) is

\[
(C(T_1) - K_1)^+ = (C(S(T_1), K, T - T_1) - K_1)^+
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where \( C(T_1) = C(S(T_1), K, T - T_1) \) is the current price of the underlying call option
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The Pricing

- Assume the Black-Scholes-Merton setting and denote the price of the underlying call option at time $T_1$ by $C(T_1)$ - we know the formula for this value.

- Then the time 0 price of the call on a call option can be represented as

$$ \tilde{E}_0[e^{-rT_1}(C(T_1) - K_1)^+] $$

$$ = \tilde{E}_0[e^{-rT_1}(\tilde{E}_{T_1}[e^{-r(T-T_1)}(S(T) - K)^+] - K_1)^+] $$

$$ = \tilde{E}_0[e^{-rT}(S(T) - K)^+ - e^{-rT_1}K_1] \mathbb{I}_{\{C(T_1) \geq K_1\}} $$

where $\tilde{E}$ denotes the expectation with respect to the risk-neutral probability $\tilde{P}$.
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where $\tilde{E}$ denotes the expectation with respect to the risk-neutral probability $\tilde{P}.$
Parity

- Let $CallOnCall$ denote the price of the compound call on an underlying call option with maturity $T_1$
- Let $PutOnCall$ denote the price of the compound put on an underlying call option (the exact analogue of the above call-on-call)
- Let $Call$ denote the price of the underlying call option
- Then the parity for compound options reads as

$$CallOnCall - PutOnCall = Call - x^{-rT_1}$$
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A Gap Call

- A gap call option pays $SK_1$ when $S > K_2$
- There is a gap in the payoff diagram - hence the name of the option
- The Black-Scholes price of a gap call option is

$$C(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)$$

where

$$d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{S}{K_2} \right) + (r - \delta + \frac{1}{2} \sigma^2) T \right]$$

and $d_2 = d_1 - \sigma \sqrt{T}$
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- A gap call option pays $SK_1$ when $S > K_2$
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