Annuities with Payments More Frequent than Each Interest Period and Payments in Arithmetic Progression

1. A Constant Increase Each Payment Period
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**The Set-up**

Assume that we have compound interest with the effective interest rate per interest period equal to $i$.

Consider the following annuity-immediate:

- the annuity lasts for $n$ interest periods;
- the payments take place at the end of an $m^{th}$ of an interest period;
- the $j^{th}$ payment is equal to $j/m^2$, for $j = 1, 2, \ldots, nm$;
- Note that the payments increase by a constant amount $1/m^2$ for each payment period;
- Note that the total increase in payments in every interest period equals

$$m \cdot \frac{1}{m^2} = \frac{1}{m}$$

- $(I^{(m)} a)^{(m)}_{nm}$ stands for the present value of the above annuity, i.e.,

$$(I^{(m)} a)^{(m)}_{nm} i = \frac{1}{m^2} (I a)_{nm} J$$

where $J$ denotes the effective interest rate per payment period:

$$J = (1 + i)^{1/m} - 1$$
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- Note that the payments increase by a constant amount $1/m^2$ for each payment period
- Note that the total increase in payments in every interest period equals

$$m \cdot \frac{1}{m^2} = \frac{1}{m}$$

- $(I^{(m)}a)^{(m)}_{n|}$ stands for the present value of the above annuity, i.e.,

$$(I^{(m)}a)^{(m)}_{n|} = \frac{1}{m^2} (Ia)^{nm}_{nm}$$

where $J$ denotes the effective interest rate per payment period:

$$J = (1 + i)^{1/m} - 1$$
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- Note that the payments increase by a constant amount $1/m^2$ for each payment period.
- Note that the total increase in payments in every interest period equals

\[ m \cdot \frac{1}{m^2} = \frac{1}{m} \]

- $(I^{(m)} a)_{m}^{(m)} \ldots$ stands for the present value of the above annuity, i.e.,

\[ (I^{(m)} a)_{m}^{(m)} \cdot i = \frac{1}{m^2} (la)_{nm} \cdot J \]

where $J$ denotes the effective interest rate per payment period:

\[ J = (1 + i)^{1/m} - 1 \]
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Consider the following annuity-immediate:

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where $J$ denotes the effective interest rate per payment period:

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Assume that we have compound interest with the effective interest rate per interest period equal to \( i \).

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- the annuity lasts for \( n \) interest periods;
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- the \( j^{th} \) payment is equal to \( j/m^2 \), for \( j = 1, 2, \ldots, nm \);
- Note that the payments increase by a constant amount \( 1/m^2 \) for each payment period;
- Note that the total increase in payments in every interest period equals

\[
m \cdot \frac{1}{m^2} = \frac{1}{m}
\]

- \((l^{(m)} a)^{(m)}_{\overline{n|}}\) stands for the present value of the above annuity, i.e.,

\[
(l^{(m)} a)^{(m)}_{\overline{n|}} i = \frac{1}{m^2} (l a)_{\overline{nm|}} J
\]

where \( J \) denotes the effective interest rate per payment period:

\[
J = (1 + i)^{1/m} - 1
\]
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Assume that we have compound interest with the effective interest rate per interest period equal to $i$.

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- Note that the total increase in payments in every interest period equals

$$m \cdot \frac{1}{m^2} = \frac{1}{m}$$

- $(I^{(m)}a)^{(m)}_{n|m}$ . . . stands for the present value of the above annuity, i.e.,

$$(I^{(m)}a)^{(m)}_{n|m} i = \frac{1}{m^2} (I a)_{nm | m} J$$

where $J$ denotes the effective interest rate per payment period:

$$J = (1 + i)^{1/m} - 1$$
Value at issuance and accumulated value: Formulae

- So, the **present value** of the annuity-immediate described above is

\[
(I^{(m)} a^{(m)})^{(m)}_i = \frac{1}{m^2} (I a)^{nm\|} J = \frac{1}{m} \bar{a}_{nm}^m J - n v^n = \ddot{a}_{m}^{(m)} i - n v^n
\]

- Similarly, the **accumulated value** of the annuity-immediate described above is

\[
(I^{(m)} s^{(m)})^{(m)}_i = \ddot{s}_{m}^{(m)} i - n
\]
Value at issuance and accumulated value: 

Formulae

- So, the **present value** of the annuity-immediate described above is

\[
(I^{(m)} a^{(m)})_{\overline{nm}} i = \frac{1}{m^2} (Ia)_{\overline{nm}} j = \frac{1}{m} \frac{\ddot{a}_{\overline{nm}} j - n v^n}{mJ} = \frac{\ddot{a}^{(m)} (m) i - n v^n}{i^{(m)}}
\]

- Similarly, the **accumulated value** of the annuity-immediate described above is

\[
(I^{(m)} s^{(m)})_{\overline{nm}} i = \frac{\ddot{s}^{(m)} (m) i - n}{i^{(m)}}
\]