Continuously paying annuities

1. Compound interest: Increasing payments

2. General Accumulation Function
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The Set-up: Unit increase in payments

Assume that we have compound interest with the effective interest rate per interest period equal to $i$.

Consider the following continuous annuity:

- the annuity lasts for $n$ interest periods;
- the payments take place continuously, at a rate of $t$ per interest period at time $t$.
- $(\bar{a})_{\overline{n}}$ stands for the present value of the above annuity, i.e.,
  \[
  (\bar{a})_{\overline{n}} = \lim_{m \to \infty} (\bar{a})_{\overline{m}} = \frac{\bar{a}_{\overline{n}} - n v^n}{\delta}
  \]
- It is easier to see what happens by noting that
  \[
  (\bar{a})_{\overline{n}} = \int_{0}^{n} t \cdot v^t dt
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- $(\bar{s})_{\overline{n}}$ stands for the accumulated value of the above annuity, i.e.,
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$$
(\bar{a}_m\bar{a})_n\ i = \lim_{m \to \infty} (I^{(m)}a)^{(m)}_n = \frac{\bar{a}_m\ i - n\delta^n}{\delta}
$$

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Unit payment stream

• Let \( v(t) \) denote the general discount function

• Let us first consider the basic continuous annuity, i.e., the annuity that pays at the unit rate at all times.

• Then, the present value of such an annuity with length \( n \) equals

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\int_0^n v(t) \, dt
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• We still denote the above present value by \( \bar{a}_{\overline{n}} \)

• In the special case of compound interest, the above formula collapses to the one already familiar to us from the compound interest set-up. You can verify this through simple integration . . .
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Any payment stream

- Let $f(t)$ be a continuous function which represents the rate of payments of a continuous annuity on the time interval $[0, n]$
- Then, the present value of this annuity can be obtained as

$$\int_{0}^{n} f(t) \cdot v(t) \, dt$$

- Assignment: Problems 4.6.1, 2, 3, 5, 7
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