Premium-Discount Formula and Other Bond Pricing Formulas

1. Premium-Discount Formula

2. Other Pricing Formulas for Bonds

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The formula and selling at a premium

Assignment: All the examples in section 6.2!

- The premium-discount pricing formula for bonds reads as

$$P = C(g - j)a_{\overline{n}|j} + C$$

where C is the redemption amount, g is the modified coupon rate, j is the effective yied rate per coupon period, and n is the number of coupons.

- If $P > C$, we say that the bond sells at a premium
- The value $P - C$ is called the premium or amount of premium for the bond, i.e.,

$$P - C = C(g - j)a_{\overline{n}|j}$$

- So, the bond sells at a premium iff $g > j$

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Selling at a discount

- If $P < C$, we say that the bond sells at a discount
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An Example

- Find the price of a \$1,000 par value 10-year bond with coupons at 8.4% convertible semiannually, which will be redeemed at \$1,050. The bond is bought to yield 10% convertible semiannually.

$\Rightarrow$ In this example, the parameters are:

$$F = 1000$$

$$C = 1050$$

$$r = \frac{0.084}{2} = 0.042$$

$$g = \frac{1000}{1050} \cdot 0.042 = 0.04$$

$$j = \frac{0.1}{2} = 0.05$$

$$n = 20$$

$$K = 1050 \cdot 1.05^{-20} = 395.7340$$

$$G = \frac{0.042}{0.05} \cdot 1000 = 840$$

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An Example (cont'd)

Using the **basic pricing formula**, we get

$$\begin{aligned} P &= Fra_{\overline{n}|} + K \\ &= 1000 \cdot 0.042 \cdot a_{\overline{20}|\,0.05} + 395.7340 \\ &= 42 \cdot 12.4622 + 395.7340 \\ &= 919.15 \end{aligned}$$

Using the **premium-discount formula**, we get

$$\begin{aligned} P &= C + (Fr - Cj)a_{\overline{n}|} \\ &= 1050 + (42 - 52.50)a_{\overline{20}|\,0.05} \\ &= 1050 + (-9.50) \cdot 12.4622 \\ &= 919.15 \end{aligned}$$

Of course, the two prices are the same

- *Assignment:* Examples 6.3.5 and 6.3.6

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The Base Amount Formula

- If we substitute the expression for the value of the annuity in the basic formula, we get

$$P = G - Gv_j^n + Cv_j^n = (C - G)v_j^n + G$$

  where G denotes the base amount, v_j is the discount factor per coupon period and n is the number of coupons
- The above formula is referred to as the base amount formula

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An Example (cont'd): Base amount formula

⇒ Reconsidering the earlier example, we can reevaluate the price of the bond using the base amount formula as

$$\begin{aligned} P &= G + (C - G)v_j^n \\ &= 840 + (1050 - 840)\left(\frac{1}{1.05}\right)^{20} \\ &= 840 + 210 \cdot 0.37689 \\ &= 919.15 \end{aligned}$$

Makeham's Formula

- If we do not know the number of coupons n, but we know the present value K of the redemption amount, we use Makeham's formula:

$$P = K + \frac{g}{j} \cdot (C - K)$$

where g stands for the coupon rate, j is the effective yield rate per coupon period, C is the redemption amount and K is the present value of the redemption amount

An Example (cont'd): Makeham's formula

$\Rightarrow$ If we look at our example again, using Makeham's formula, we obtain:

$$\begin{aligned} P &= K + \frac{g}{j}(C - K) \\ &= 395.7340 + \frac{0.04}{0.05}(1050 - 395.7340) \\ &= 919.15 \end{aligned}$$