Chapter 8
Swaps

Question 8.1.

We first solve for the present value of the cost per two barrels:

$$\frac{\$22}{1.06} + \frac{\$23}{(1.065)^2} = 41.033.$$

We then obtain the swap price per barrel by solving:

$$\begin{aligned} \frac{x}{1.06} + \frac{x}{(1.065)^2} &= 41.033 \\ \Leftrightarrow \quad x &= 22.483, \end{aligned}$$

which was to be shown.

Question 8.2.

a) We first solve for the present value of the cost per three barrels, based on the forward prices:

$$\frac{\$20}{1.06} + \frac{\$21}{(1.065)^2} + \frac{\$22}{(1.07)^3} = 55.3413.$$

We then obtain the swap price per barrel by solving:

$$\begin{aligned} \frac{x}{1.06} + \frac{x}{(1.065)^2} + \frac{x}{(1.07)^3} &= 55.341 \\ \Leftrightarrow \quad x &= 20.9519 \end{aligned}$$

b) We first solve for the present value of the cost per two barrels (year 2 and year 3):

$$\frac{\$21}{(1.065)^2} + \frac{\$22}{(1.07)^3} = 36.473.$$

We then obtain the swap price per barrel by solving:

$$\begin{aligned} \frac{x}{(1.065)^2} + \frac{x}{(1.07)^3} &= 36.473 \\ \Leftrightarrow \quad x &= 21.481 \end{aligned}$$

Question 8.3.

Since the dealer is paying fixed and receiving floating, she generates the cash-flows depicted in column 2. Suppose that the dealer enters into three short forward positions, one contract for each year of the active swap. Her payoffs are depicted in columns 3, and the aggregate net cash flow position is in column 4.

Year	Net Swap Payment	Short Forwards	Net Position
1	$S_1 - \$20.9519$	$\$20 - S_1$	-0.9519
2	$S_1 - \$20.9519$	$\$21 - S_1$	$+0.0481$
3	$S_1 - \$20.9519$	$\$22 - S_1$	$+1.0481$

We need to discount the net positions to year zero. We have:

$$PV\ (netCF) = \frac{-0.9519}{1.06} + \frac{0.0481}{(1.065)^2} + \frac{1.0481}{(1.07)^3} = 0.$$

Indeed, the present value of the net cash flow is zero.

Question 8.4.

The fair swap rate was determined to be \$20.952. Therefore, compared to the forward curve price of \$20 in one year, we are overpaying \$0.952. In year two, this overpayment has increased to $\$0.952 \times 1.070024 = 1.01866$, where we used the appropriate forward rate to calculate the interest payment. In year two, we underpay by \$0.048, so that our total accumulative underpayment is \$0.97066. In year three, this overpayment has increased again to $\$0.97066 \times 1.08007 = 1.048$. However, in year three, we receive a fixed payment of 20.952, which underpays relative to the forward curve price of \$22 by $\$22 - \$20.952 = 1.048$. Therefore, our cumulative balance is indeed zero, which was to be shown.

Question 8.5.

Since the dealer is paying fixed and receiving floating, she generates the cash-flows depicted in column 2. Suppose that the dealer enters into three short forward positions, one contract for each year of the active swap. Her payoffs are depicted in columns 3, and the aggregate net position is summarized in column 4.

Year	Net Swap Payment	Short Forwards	Net Position
1	$S_1 - \$20.952$	$\$20 - S_1$	-0.952
2	$S_1 - \$20.952$	$\$21 - S_1$	$+0.048$
3	$S_1 - \$20.952$	$\$22 - S_1$	$+1.048$

We need to discount the net positions to year zero, taking into account the uniform shift of the term structure. We have:

$$PV\,(netCF) = \frac{-0.9519}{1.065} + \frac{0.0481}{(1.07)^2} + \frac{1.0481}{(1.075)^3} = -0.0081.$$

The present value of the net cash flow is negative: The dealer never recovers from the increased interest rate he faces on the overpayment of the first swap payment.

$$PV\,(netCF) = \frac{-0.9519}{1.055} + \frac{0.0481}{(1.06)^2} + \frac{1.0481}{(1.065)^3} = +0.0083$$

The present value of the net cash flow is positive. The dealer makes money, because he gets a favorable interest rate on the loan he needs to take to finance the first overpayment.

The dealer could have tried to hedge his exposure with a forward rate agreement or any other derivative protecting against interest rate risk.

Question 8.6.

In order to answer this question, we use equation (8.13.) of the main text. We assumed that the interest rates and the corresponding zero-coupon bonds were:

Quarter	Interest rate	Zero-coupon price
1	0.0150	0.9852
2	0.0302	0.9707
3	0.0457	0.9563
4	0.0614	0.9422
5	0.0773	0.9283
6	0.0934	0.9145
7	0.1098	0.9010
8	0.1265	0.8877

Using formula 8.13., we obtain the following per barrel swap prices:

4-quarter swap price: $20.8533

8-quarter swap price: $20.4284

The total costs of prepaid 4- and 8-quarter swaps are the present values of the payment obligations. They are:

4-quarter prepaid swap price: $80.3768

8-quarter prepaid swap price: $152.9256

Question 8.7.

Using formula 8.13., and plugging in the given zero-coupon prices and the given forward prices, we obtain the following per barrel swap prices:

Quarter	Zero-bond	Swap price
1	0.9852	21.0000
2	0.9701	21.0496
3	0.9546	20.9677
4	0.9388	20.8536
5	0.9231	20.7272
6	0.9075	20.6110
7	0.8919	20.5145
8	0.8763	20.4304

Question 8.8.

We use formula (8.4), and replace the forward interest rate with the forward oil prices. In particular, we calculate:

$$X = \frac{\sum_{i=3}^{6} P_0(0, t_i) F_{0,t_i}}{\sum_{i=3}^{6} P_0(0, t_i)} = \$20.3807$$

Therefore, the swap price of a 4-quarter oil swap with the first settlement occurring in the third quarter is $20.3807.

Question 8.9.

Using the 8-quarter swap price of $20.43, we can calculate the net position by subtracting the swap price from the forward prices. The 1-quarter implied forward rate is calculated from the zero-coupon bond prices. The column implicit loan balance adds the net position of each quarter and the implicit loan balance plus interest of the previous quarter. Please note that the forward curve is inverted—we are initially loaning money because the swap price is lower than the forward price.

Quarter	Net position	Implied forward rate	Implicit loan balance
1	0.5696	1.0150	0.5696
2	0.6696	1.0156	1.2480
3	0.3696	1.0162	1.6379
4	0.0696	1.0168	1.7350
5	−0.2304	1.0170	1.5341
6	−0.4304	1.0172	1.1300
7	−0.5304	1.0175	0.6194
8	−0.6304	1.0178	0.0000

Question 8.10.

We use equation (8.6) of the main text to answer this question:

$$X = \frac{\sum_{i=1}^{8} Q_{t_i} P_0(0, t_i) F_{0,t_i}}{\sum_{i=1}^{8} Q_{t_i} P_0(0, t_i)}, \text{ where} Q = [1, 2, 1, 2, 1, 2, 1, 2]$$

After plugging in the relevant variables given in the exercise, we obtain a value of $20.4099 for the swap price.

Question 8.11.

We are now asked to invert our equations. The swap prices are given, and we want to back out the forward prices. We do so recursively. For a one-quarter swap, the swap price and the forward price are identical. Given the one-quarter forward price, we can find the second quarter forward price, etc. Doing so yields the following forward prices:

Quarter	Forward price
1	2.25
2	2.60
3	2.20
4	1.90
5	2.20
6	2.50
7	2.15
8	1.80

Question 8.12.

With a swap price of $2.2044, and the forward prices of question 8.11., we can calculate the implied loan amount. We can calculate the net position by subtracting the swap price from the forward prices. The 1-quarter implied forward rate is calculated from the zero-coupon bond prices. The column implicit loan balance adds the net position each quarter and the implicit loan balance plus interest of the previous quarter. Please note that the shape of the forward curve—we are initially loaning money, because the swap price is lower than the forward price.

Quarter	Forward price	Net balance	Forward interest rate	Implicit loan balance
1	2.25	0.0456	1.0150	0.0456
2	2.60	0.3955	1.0156	0.4418
3	2.20	−0.0042	1.0162	0.4447
4	1.90	−0.3046	1.0168	0.1476
5	2.20	−0.0043	1.0170	0.1458
6	2.50	0.2954	1.0172	0.4437
7	2.15	−0.0543	1.0175	0.3971
8	1.80	−0.4042	1.0178	0.0000

Question 8.13.

From the given zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Forward interest rate
1	1.0150
2	1.0156
3	1.0162
4	1.0168
5	1.0170
6	1.0172
7	1.0175
8	1.0178

Now, we can calculate the deferred swap price according to the formula:

$$X = \frac{\sum_{i=2}^{6} P_0\left(0, t_i\right) r_0\left(t_{i-1}, t_i\right)}{\sum_{i=2}^{6} P_0\left(0, t_i\right)} = 1.66\%$$

Question 8.14.

From the given zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Forward interest rate
1	1.0150
2	1.0156
3	1.0162
4	1.0168
5	1.0170
6	1.0172
7	1.0175
8	1.0178

Now, we can calculate the swap prices for 4 and 8 quarters according to the formula:

$$X = \frac{\sum_{i=1}^{n} P_0(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=1}^{n} P_0(0, t_i)}, \text{ where } n = 4 \text{ or } 8$$

This yields the following prices:

4-quarter fixed swap price:	1.59015%
8-quarter fixed swap price:	1.66096%

Question 8.15.

We can calculate the value of an 8-quarter dollar annuity that is equivalent to an 8-quarter Euro annuity by using equation 8.8. of the main text. We have:

$$X = \frac{\sum_{i=1}^{8} P_0(0, t_i) R^* F_{0,t_i}}{\sum_{i=1}^{8} P_0(0, t_i),} \text{ where } R^* \text{ is the Euro annuity of 1 Euro.}$$

Plugging in the forward price for one unit of Euros delivered at time t_i, which are given in the price table, yields a dollar annuity value of \$0.9277.

Question 8.16.

The dollar zero-coupon bond prices for the three years are:

$$P_{0,1} = \frac{1}{1.06} = 0.9434$$

$$P_{0,2} = \frac{1}{(1.06)^2} = 0.8900$$

$$P_{0,3} = \frac{1}{(1.06)^3} = 0.8396$$

R^* is 0.035, the Euro-bond coupon rate. The current exchange rate is 0.9\$/E. Plugging all the above variables into formula (8.9) indeed yields 0.06, the dollar coupon rate:

$$R = \frac{\sum_{i=1}^{3} P_0(0, t_i) R^* F_{0,t_i}/x_0 + P_{0,3}\left(F_{0,t_n}/x_0 - 1\right)}{\sum_{i=1}^{3} P_0(0, t_i)} = \frac{0.098055 + 0.062317}{2.673} = 0.060$$

Question 8.17.

We can use the standard swap price formula for this exercise, but we must pay attention to taking the right zero-coupon bonds, and the right Euro-denominated forward interest rates. From the given Euro zero-coupon bond prices, we can calculate the one-quarter forward interest rates. They are:

Quarter	Euro denominated implied forward interest rate
1	0.0088
2	0.0090
3	0.0092
4	0.0095
5	0.0096
6	0.0097
7	0.0098
8	0.0100

Now, we can calculate the swap prices for 4 and 8 quarters according to the formula:

$$X = \frac{\sum_{i=1}^{n} P_0\left(0, t_i\right) r_0\left(t_{i-1}, t_i\right)}{\sum_{i=1}^{n} P_0\left(0, t_i\right),} \text{ where } n = 4 \text{ or } 8$$

This yields the following prices:

4-quarter fixed swap price:	0.91267%
8-quarter fixed swap price:	0.94572%

Question 8.18.

We can use equation (8.9), but there is a complication: We do not have the current spot exchange rate. However, it is possible to back it out by using the methodology of the previous chapters: We know that the following relation must hold:

$$F_{0,1} = X_0 e^{(r-r^*)}, \text{ where the interest rates are already on a quarterly level.}$$

We can back out the interest rates from the given zero-coupon prices. Doing so yields a current exchange rate of 0.90 \$/Euro. $R*$ is the 8-quarter fixed swap price payment of 0.0094572.

By plugging in all the relevant variables into equation 8.9, we can indeed see that this yields a swap rate of 1.66%, which is the same rate that we calculated in exercise 8.14.