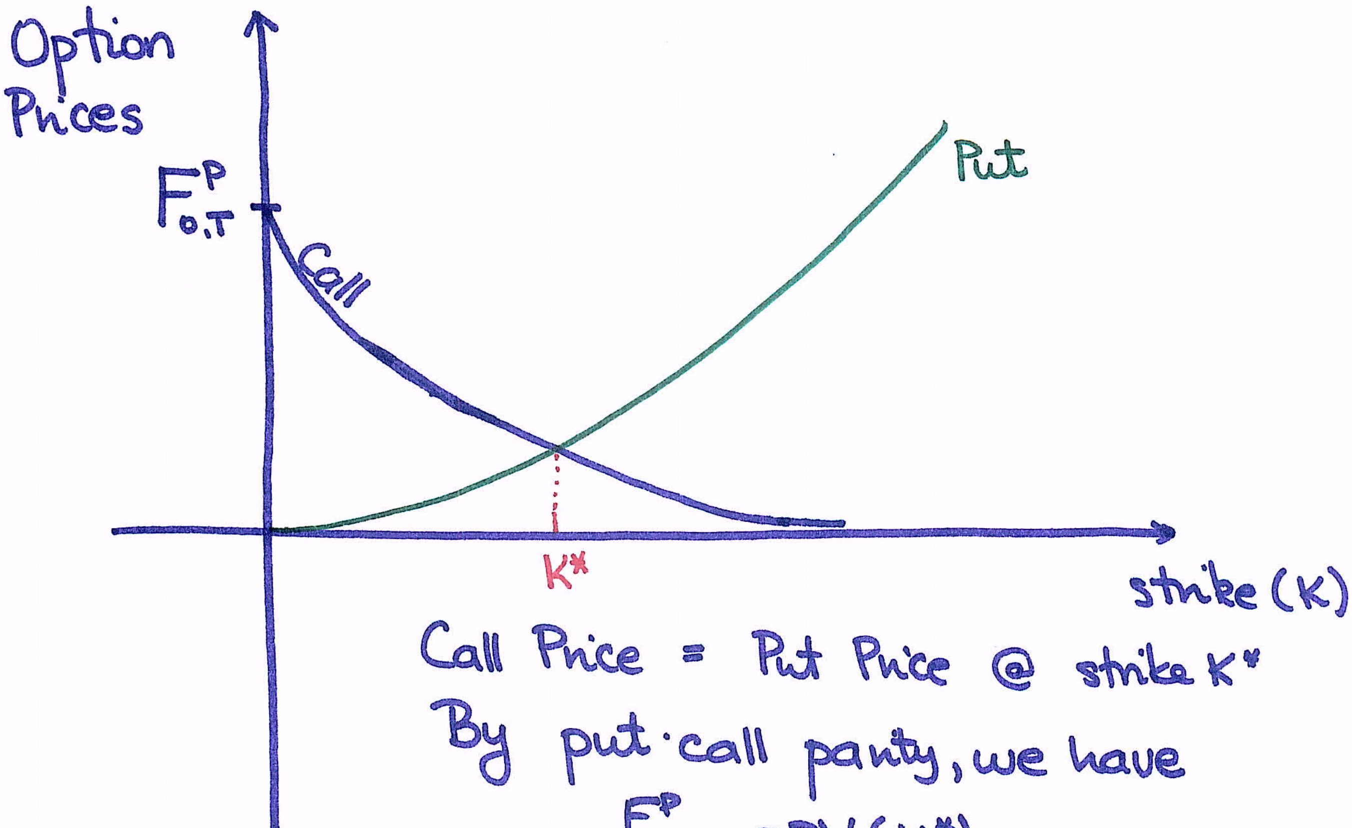
Review:

D: April 1st, 2019.

Let $K_1 < K_2$.

Then:

$$0 \leq \left\{ \begin{matrix} V_C(K_1) - V_C(K_2) \\ V_P(K_2) - V_P(K_1) \end{matrix} \right\} \leq PV_{0,T}(K_2 - K_1)$$



Call Price = Put Price @ strike K^*

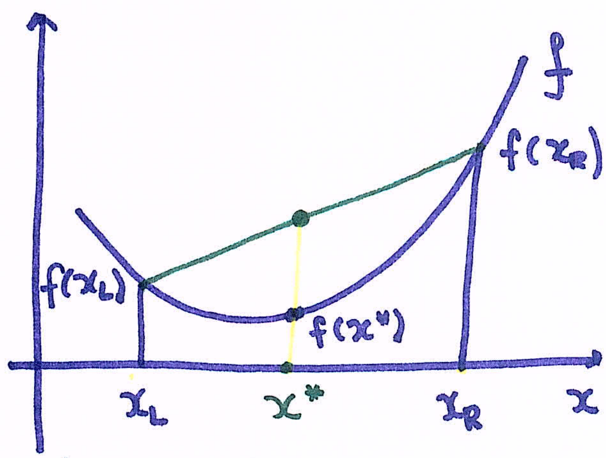
By put-call parity, we have

$$F^P_{0,T} = PV(K^*)$$

$$\Longleftrightarrow$$

$$\boxed{K^* = FV_{0,T}(F^P_{0,T}) = F_{0,T}}$$

Convex Functions.



There is a const. λ such that

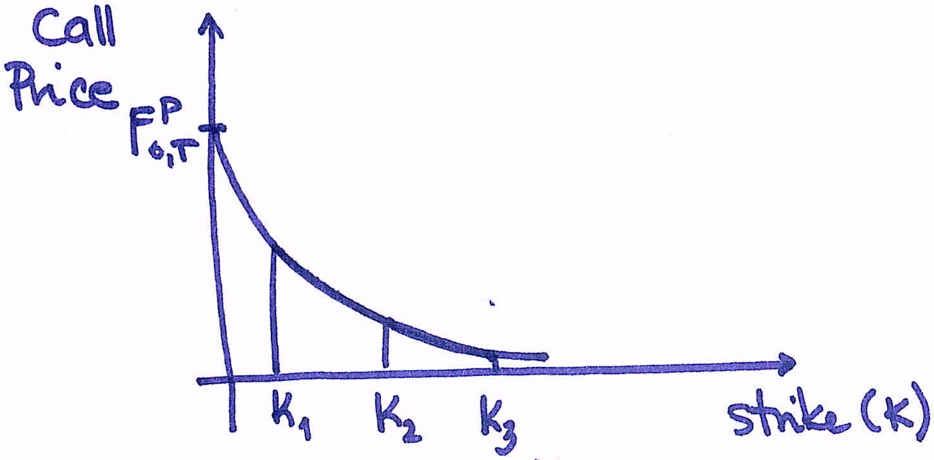
$$x^* = \lambda \cdot x_L + (1-\lambda) \cdot x_R$$

Actually: $\lambda = \dfrac{x_R - x^*}{x_R - x_L}$ and $1-\lambda = \dfrac{x^* - x_L}{x_R - x_L}$

The f'tion f is convex if:

$$\boxed{f(x^*) \leq \lambda \cdot f(x_L) + (1-\lambda) \cdot f(x_R)}$$

Call·Price Convexity...



Claim: w/ $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$

we have that

$$V_c(K_2) \leq \lambda \cdot V_c(K_1) + (1-\lambda) V_c(K_3) \qquad (CC)$$

$$\Longleftrightarrow$$

$$\frac{V_c(K_1) - V_c(K_2)}{K_2 - K_1} \geq \frac{V_c(K_2) - V_c(K_3)}{K_3 - K_2}$$

Example. Assume, to the contrary, that there exist $K_1 < K_2 < K_3$ such that

$$V_c(K_2) > \lambda V_c(K_1) + (1-\lambda) V_c(K_3) \quad \text{w/ } \lambda = \frac{K_3 - K_2}{K_3 - K_1}$$

I. Suspect an arbitrage opportunity. :)

II. Propose an arbitrage portfolio:

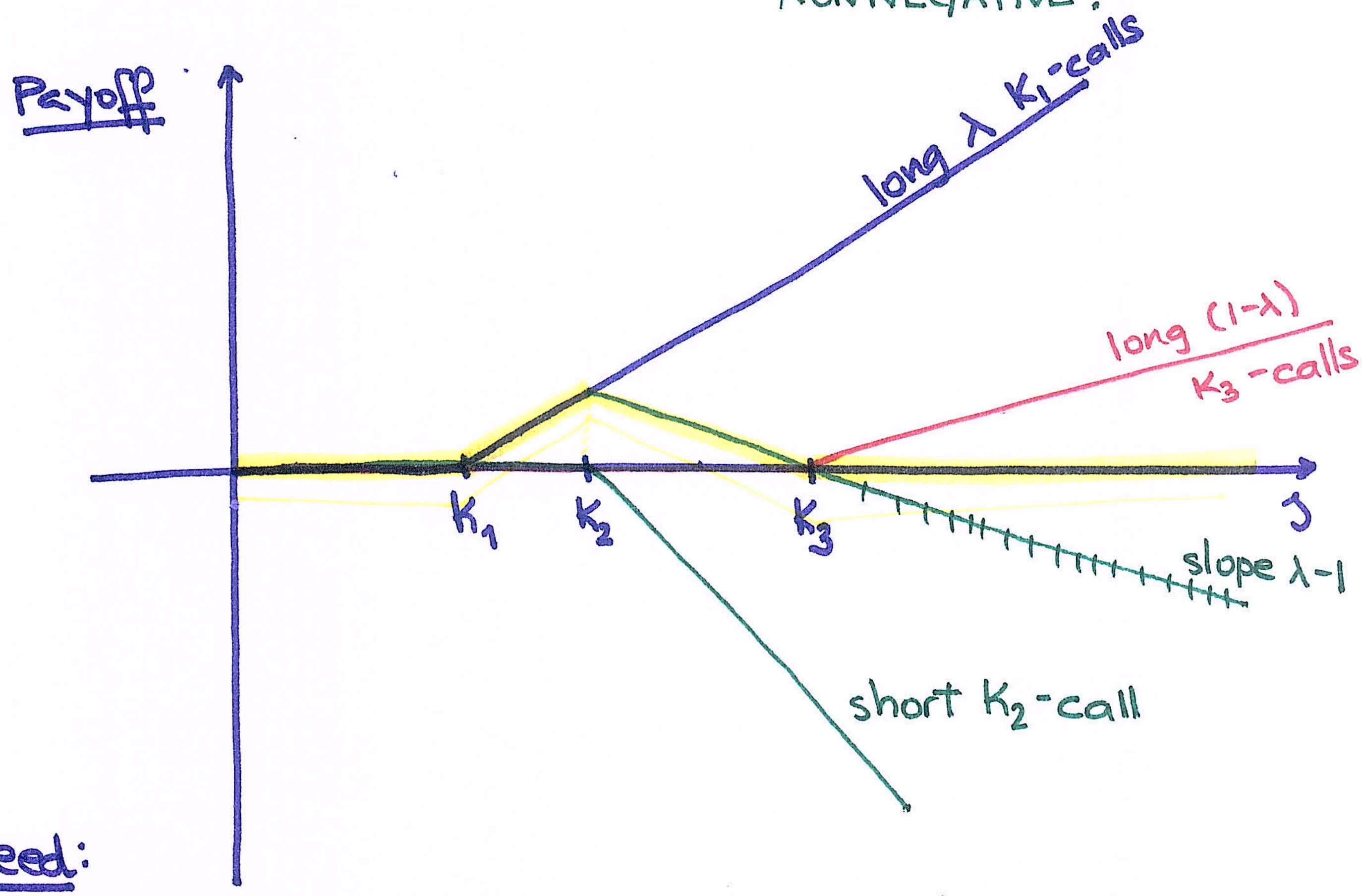
- LONG λ K_1·calls
- SHORT one K_2·call
- LONG $1-\lambda$ K_3·calls

CALL BUTTERFLY SPREAD

III. Verification.

Init. Cost $= -V_c(K_2) + \lambda \cdot V_c(K_1) + (1-\lambda) V_c(K_3) < 0$

$\Rightarrow$ It's sufficient to show that the payoff is NONNEGATIVE.



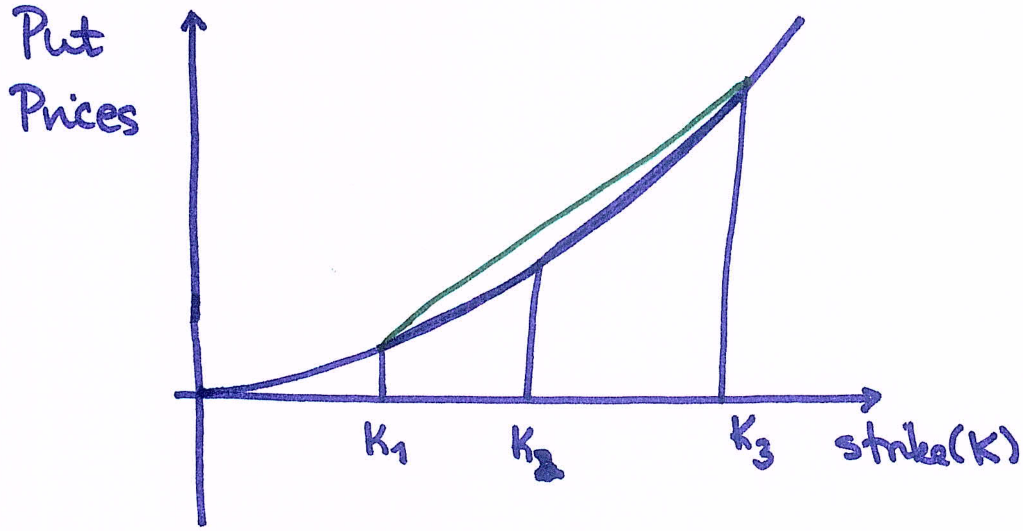
Indeed:

Payoff ≥ 0 $\Rightarrow$ Profit > 0 $\Rightarrow$ It's an arbitrage portfolio!

- If $K_2 = \frac{1}{2}(K_1 + K_3)$, it's a symmetric butterfly spread.
- Otherwise, it's asymmetric.
- Lacks directionality $\Rightarrow$ not a very good hedge.

  It can be used to speculate on low volatility.

Put · Price Convexity..



Claim: w/ $\lambda = \dfrac{K_3 - K_2}{K_3 - K_1}$

we have

$$V_p(K_2) \leq \lambda \cdot V_p(K_1) + (1-\lambda) V_p(K_3) \qquad \text{(PC)}$$

$$\Longleftrightarrow$$

$$\frac{V_p(K_2) - V_p(K_1)}{K_2 - K_1} \leq \frac{V_p(K_3) - V_p(K_2)}{K_3 - K_2}$$

Q: If you observe $K_1 < K_2 < K_3$ such that (PC) does not hold, how do you construct an arbitrage portfolio?

$\left.\begin{array}{l} \bullet \text{ long } \lambda \ K_1\text{-puts} \\ \bullet \text{ short } 1 \ K_2\text{-put} \\ \bullet \text{ long } 1-\lambda \ K_3\text{-puts} \end{array}\right\}$ PUT Butterfly Spread

ToDo: • Convince yourselves:
Put butterfly spreads have the same PAYOFF as the call butterfly spread.

$\Rightarrow$ The payoff of the put butterfly spread is nonnegative.

$\Rightarrow$ It is, indeed, an arbitrage portfolio.

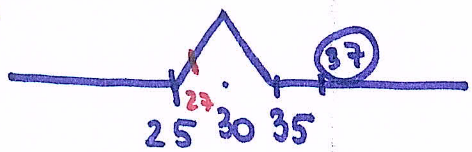
67.

Consider the following investment strategy involving put options on a stock with the same expiration date.

i) Buy one 25-strike put
ii) Sell two 30-strike puts
iii) Buy one 35-strike put

Symmetric Put Butterfly spread.

Calculate the payoffs of this strategy assuming stock prices (i.e., at the time the put options expire) of 27 and 37, respectively.



(A) –2 and 2
(B) 0 and 0
(C) 2 and 0
(D) 2 and 2
(E) 14 and 0

68.

For a non-dividend-paying stock index, the current price is 1100 and the 6-month forward price is 1150. Assume the price of the stock index in 6 months will be 1210.

Which of the following is true regarding forward positions in the stock index?

(A) Long position gains 50
(B) Long position gains 60
(C) Long position gains 110
(D) Short position gains 60
(E) Short position gains 110

7.