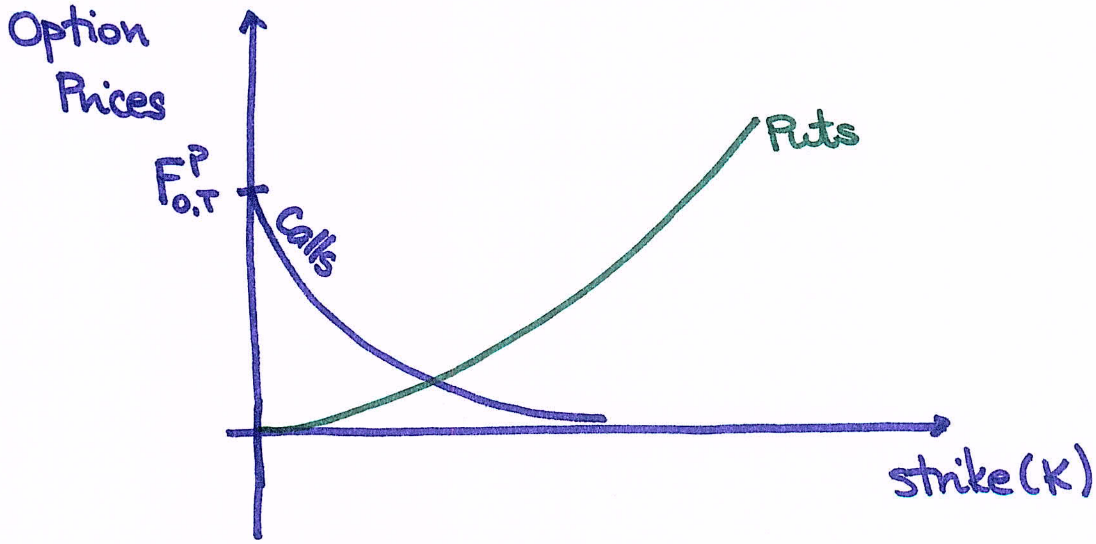
D: April 3rd, 2019.

# Convexity of Option Prices [Review].




Let $K_1 < K_2 < K_3$.

Then, for $\lambda = \frac{K_3 - K_2}{K_3 - K_1}$, we have $K_2 = \lambda \cdot K_1 + (1-\lambda) \cdot K_3$

also:

$$V_{C/P}(K_2) \leq \lambda \cdot V_{C/P}(K_1) + (1-\lambda) V_{C/P}(K_3)$$

In case that the above inequality is violated, we will exploit the arbitrage opportunity using a call/put Butterfly Spread:

- LONG $\lambda$ $K_1$-calls/puts
- SHORT 1 $K_2$-call/puts
- LONG $1-\lambda$ $K_3$-calls/puts

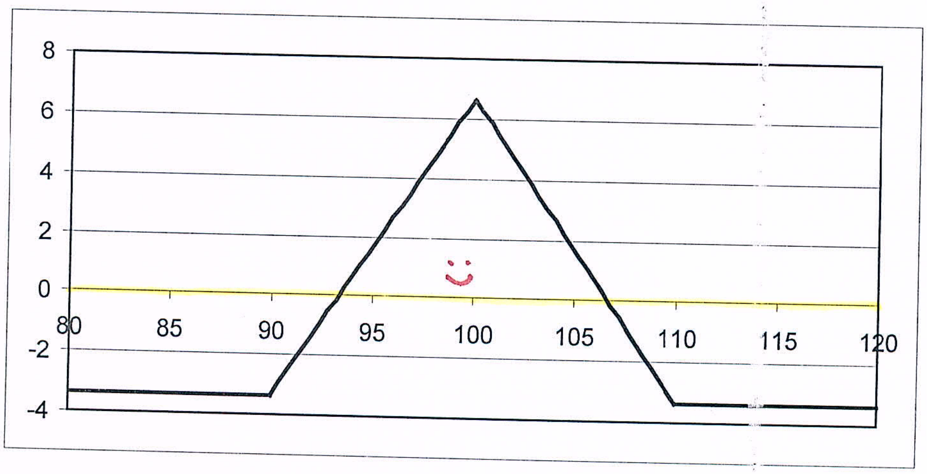
9.

Stock ABC has the following characteristics:

- The current price to buy one share is 100. $S(0) = 100$
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

| Strike Price | Call option price | Put option price |
|---|---|---|
| 90 | 14.63 | 0.24 |
| 100 | 6.80 | 1.93 |
| 110 | 2.17 | 6.81 |

A butterfly spread on this stock has the following profit diagram.




The continuously compounded risk-free interest rate is 5%.

Determine which of the following will NOT produce this profit diagram.

(A) Buy a 90 put, buy a 110 put, sell two 100 puts ✓

(B) Buy a 90 call, buy a 110 call, sell two 100 calls ✓ "Canonical"

(C) (Buy a 90 put, sell a 100 put) (sell a 100 call, buy a 110 call) ✓

(D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts

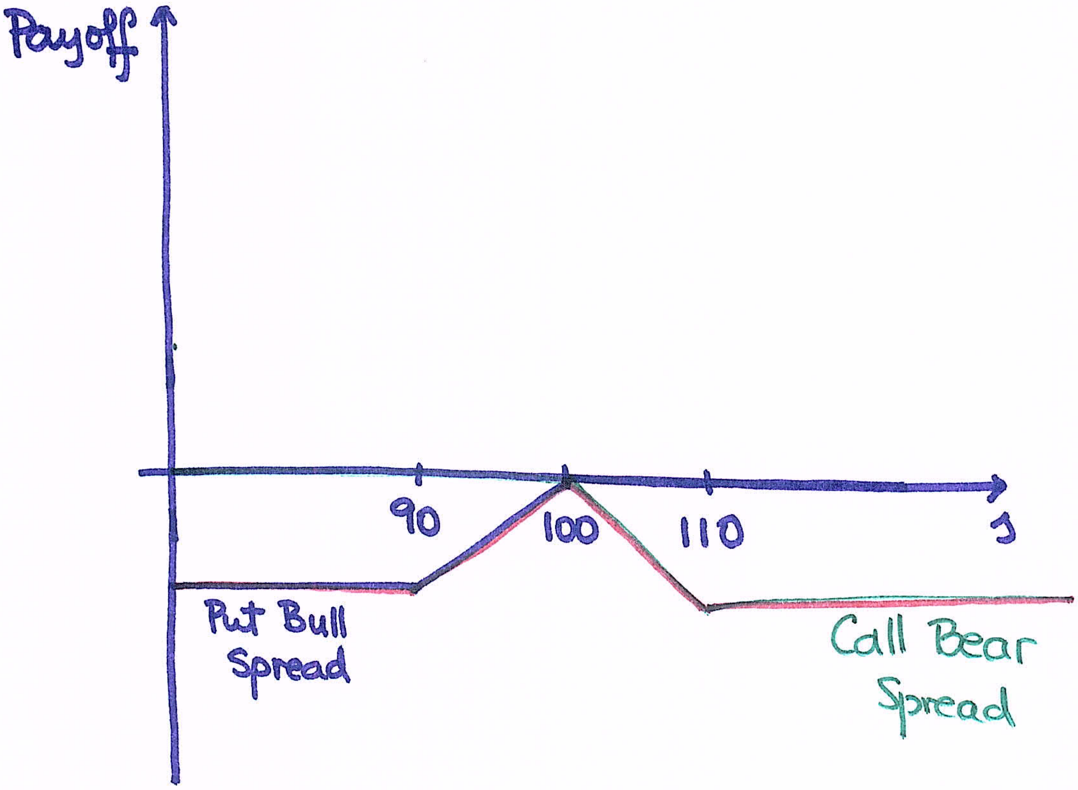
(E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls. ✓

Put·Call Parity : $\boxed{\text{long stock + long put = long call + long bond}}$



2.

(C)



(D) Slope of the payoff curve

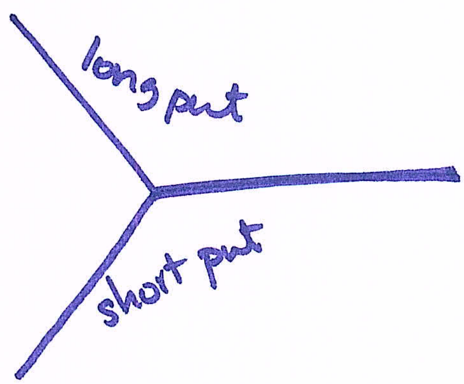
0     90     100     110

| | | | | |
|---|---|---|---|---|
| long stock | ~~+1~~ | +1 | +1 | +1 |
| long 90-call | 0 | +1 | +1 | +1 |
| short two 100-puts | +2 | +2 | 0 | 0 |
| long 110-put | ~~-1~~ | -1 | -1 | 0 |
| | +2 | | | |

$+2 \neq 0$ ⇒ Does not replicate a butterfly spread!

⇓

(D) is our answer!

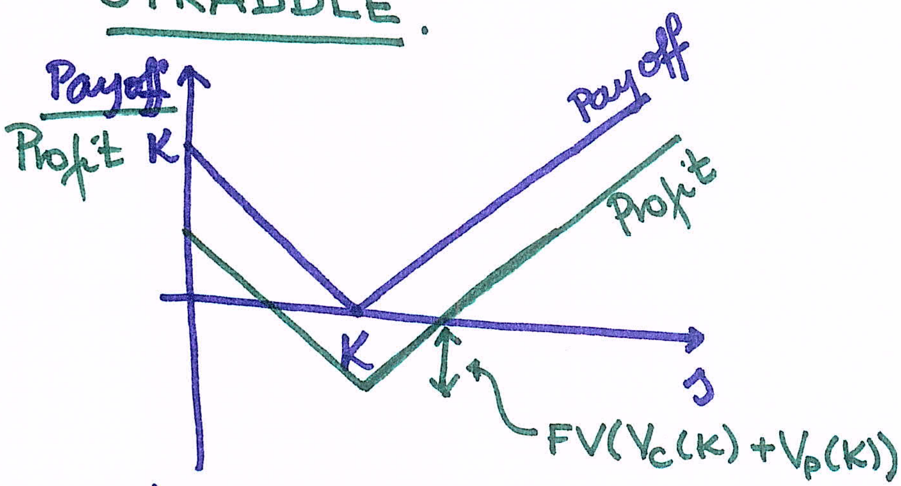


# Speculating on Volatility.

**Recall**: To speculate on LOW volatility, we can use a long butterfly spread.

To speculate on HIGH volatility:

**STRADDLE**.




The payoff f'tion: $v(s) = |s-K|$

$$= \underbrace{(s-K)_+}_{v_c(s)} + \underbrace{(K-s)_+}_{v_p(s)}$$

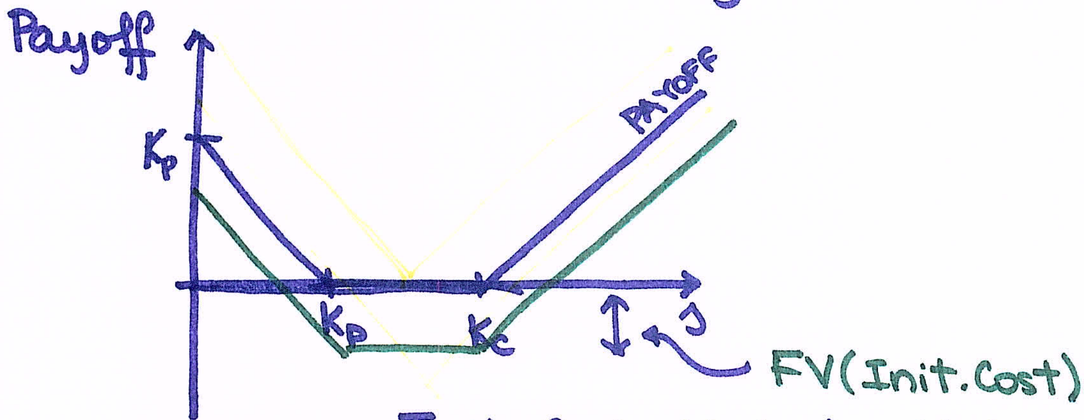
$\Rightarrow$ The straddle can be constructed as:

$\begin{cases} \cdot \text{ a LONG } K\text{-call} \\ \cdot \text{ a LONG } K\text{-put} \end{cases}$

$\Rightarrow$ Initial Cost: $\underline{V_c(K) + V_p(K)}$

## STRANGLE. (aim: To reduce the init. cost)

Take $K_p < K_c$: $\begin{cases} \cdot \text{ long the } K_p\text{-put} \\ \cdot \text{ long the } K_c\text{-call} \end{cases}$




Init. Cost: $V_c(K_c) + V_p(K_p)$

Start w/ $K_p < K < K_c$

$$\left.\begin{array}{lll} \text{Call Prices} \downarrow & \Rightarrow & V_c(K) \geq V_c(K_c) \\ \text{Put Prices} \uparrow & \Rightarrow & V_p(K) \geq V_p(K_p) \end{array}\right\} +$$

$\Rightarrow$ The init. cost of the strangle is lower than that of the straddle.

7.

A non-dividend paying stock currently sells for 100. One year from now the stock sells for 110. The continuously compounded risk-free interest rate is 6%. A trader purchases the stock in the following manner:

- The trader pays 100 today
- The trader takes possession of the stock in one year

Determine which of the following describes this arrangement.

(A) Outright purchase

(B) Fully leveraged purchase

(C) Prepaid forward contract

(D) Forward contract

(E) This arrangement is not possible due to arbitrage opportunities

8.

Joe believes that the volatility of a stock is higher than indicated by market prices for options on that stock. He wants to speculate on that belief by buying or selling at-the-money options.

$= K = S(0)$

Determine which of the following strategies would achieve Joe's goal.

X (A) Buy a strangle : long $K_p$·put & long $K_c$·call : $K_p \neq K_c$ X

(B) Buy a straddle

X (C) Sell a straddle low vol.

X (D) Buy a butterfly spread low vol.

X (E) Sell a butterfly spread a multitude of strikes again?

16.

The current price of a non-dividend paying stock is 40 and the continuously compounded risk-free interest rate is 8%. The following table shows call and put option premiums for three-month European of various exercise prices:

$r = 0.08$

$T = 1/4$

| Exercise Price | Call Premium | Put Premium |
|---|---|---|
| 35 | 6.13 | 0.44 |
| 40 | 2.78 | 1.99 |
| 45 | 0.97 | 5.08 |

A trader interested in speculating on volatility in the stock price is considering two investment strategies. The first is a 40-strike straddle. The second is a strangle consisting of a 35-strike put and a 45-call.

Determine the range of stock prices in 3 months for which the strangle outperforms the straddle.

Profit(Strangle) $\geq$ Profit(Straddle)

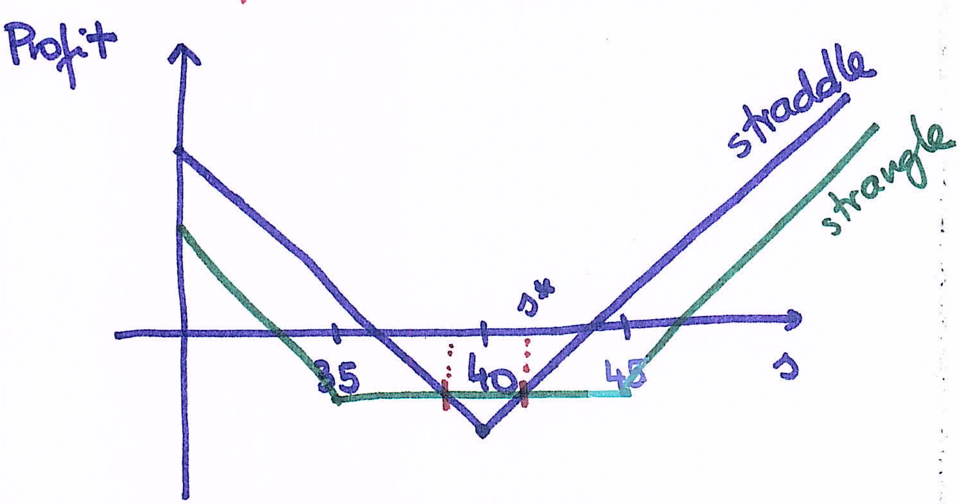
(A) The strangle never outperforms the straddle.

(B) $33.56 < S_T < 46.44$

(C) $35.13 < S_T < 44.87$

(D) $36.57 < S_T < 43.43$

(E) The strangle always outperforms the straddle.




$$S^* - 40 - FV(4.77) = -FV(1.41)$$

$$S^* = 40 + FV(3.36) = 40 + 3.36e^{0.02} = 43.43$$

$\Rightarrow$ (D)