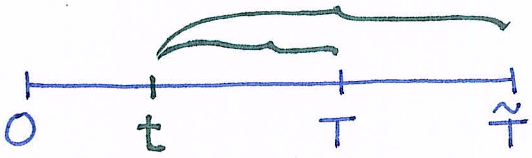
Properties of American-Option Prices



$\underbrace{T < \tilde{T}}$ Expiration dates for a pair of American-style contracts.

$V^A(t, \cdot)$ ← PRICE OF THE CONTRACT @ time-t.

t ↑ the valuation date

$\cdot$ ↑ the expiration date

$$V^A(t,T) \leq V^A(t,\tilde{T})$$

↑ The longer the time to expiration, the more exercise dates are admissible.

In words: Longer-lived American options are worth @ least as much as otherwise identical shorter-lived American options.

From now on: Omit the expiration date from our notation (unless we really need it!).

$$V^A(t) \geq V(t) \quad \text{for all } t \leq T$$

$V^A(t)$ ↑ American-option price @ time-t

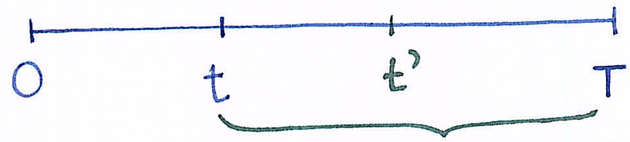
$V(t)$ ↑ otherwise identical European-option price.

American Calls: Bounds on American-call prices.

$$\left.\begin{aligned} &V_C^A(t) \geq V_C(t) \geq \mathrm{MAX}\left[0,\ F_{t,T}^P(S) - Ke^{-r(T-t)}\right] \\ &V_C^A(t) \geq S(t) - K \quad \text{(due to possibility of early exercise)} \end{aligned}\right\}$$

Lower bound:

$$V_C^A(t) \geq \mathrm{MAX}\left[0,\ F_{t,T}^P(S) - Ke^{-r(T-t)},\ S(t) - K\right]$$



$t' \in [t,T]$ ... the remaining admissible early exercise dates

$$V_C^A(t) \text{ "="} \max_{t' \in [t,T]} V_C(t, t')$$

European (V_C); t': exercise date

$\max_{t'\in[t,T]}$: "Fuzzy" : Should be all "nice" random times.

$$V_C^A(t) \leq \max_{t' \in [t,T]} F_{t,t'}^P(S) \leq S(t)$$

Upper bound: $V_C^A(t) \leq S(t)$

American Puts:

$$K \geq V_P^A(t) \geq V_P(t)$$

$$\boxed{K \geq V_P^A(t) \geq \mathrm{MAX}\left[0,\ K - S(t),\ Ke^{-r(T-t)} - F_{t,T}^P(S)\right]}$$