Properties of American-Option Prices

Expiration dates for a pair of American-style contracts:

\[ V^A(t, \hat{t}) \leq V^A(t, \tilde{T}) \]

The longer the time to expiration, the more exercise dates are admissible.

In words: Longer-lived American options are worth at least as much as otherwise identical shorter-lived American options.

From now on: Omit the expiration date from our notation (unless we really need it!).

\[ V^A(t) \geq V(t) \quad \text{for all } t \leq T \]

American-option price @ time-\( t \)

otherwise identical European-option price.
American Calls: Bounds on American call prices.

\[ V_c^A(t) \geq V_c^A(t) \geq \max \left[ 0, F_{t,T}^P(S) - K e^{-r(T-t)} \right] \]

\[ V_c^A(t) \geq S(t) - K \quad \text{(possibility of early exercise)} \]

**Lower bound:**

\[ V_c^A(t) \geq \max \left[ 0, F_{t,T}^P(S) - K e^{-r(T-t)}, S(t) - K \right] \]

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**Diagram:**

\[ 0 \quad t \quad t' \quad T \]

\[ t' \in [t,T], \ldots \text{the remaining admissible early exercise dates} \]

**European early exercise dates**

\[ V_c^A(t) = \max_{t' \in [t,T]} V_c^A(t, t') \]

\[ ^{\text{exercise date}} \]

\[ ^{\text{"Fuzzy": Should be all "nice" random times.}} \]

\[ V_c^A(t) \leq \max_{t' \in [t,T]} F_{t,t'}^P(S) \leq S(t) \]

**Upper bound:**

\[ V_c^A(t) \leq S(t) \]

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American Puts:

\[ K \geq V_p^A(t) \geq V_p(t) \]

\[ K \geq V_p^A(t) \geq \max \left[ 0, K - S(t), K e^{-r(T-t)} - F_{t,T}^P(S) \right] \]