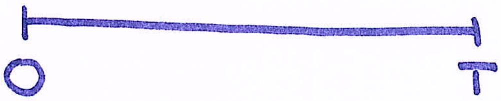
Multiple Binomial Periods.

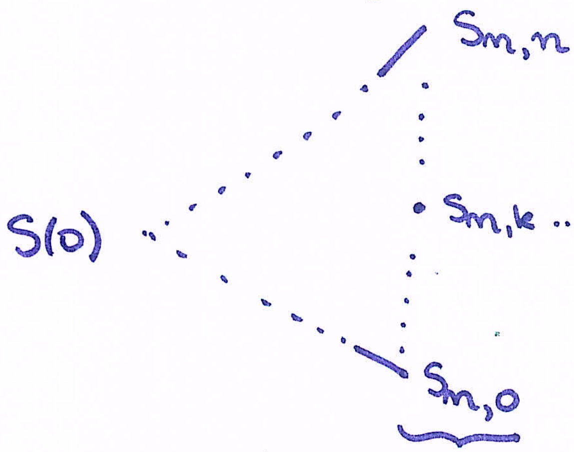
D: April 22nd, 2019.



T... exercise date of a European option

n ... # of periods

$\Rightarrow$ the length of each period is $\boxed{h = \frac{T}{n}}$



These $(n+1)$ values are the support of the rnd variable $S(T)$.

$\Rightarrow$ for every $k = 0..n$:

$$S_{n,k} = S(0)u^k \cdot d^{n-k} = S(0)\cdot\left(\frac{u}{d}\right)^{k}\cdot d^n$$

# of upsteps

Consider a European option w/ payoff f'tion $v(\cdot)$.
Then, the possible payoff values will be

$$v_{n,k} = v(s_{n,k})$$

p^*... the risk-neutral probability of an upstep.
$\Rightarrow$ the risk-neutral probab. of attaining the payoff $v_{n,k}$ is:

$$\binom{n}{k}(p^*)^k(1-p^*)^{n-k}$$

$\Rightarrow$ The risk-neutral option price is:

$$\boxed{V(0) = e^{-r \cdot T} \cdot \sum_{k=0}^{n} \left(\binom{n}{k} (p^*)^k (1-p^*)^{n-k} \cdot v_{n,k} \right)}$$

Problem. Let $r = 10\%$.

Let the initial price of a non-dividend-paying stock be \$100 per share.

You use a 5-period binomial tree to model this stock price over the next year. Let $u = 1.04$ and $d = 0.96$.

What is the price of a one-year, at-the-money European call on the above asset?

$\rightarrow$: Risk-neutral probab:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{e^{0.02} - 0.96}{0.08} = \dots = 0.7525$$

The relevant values in the stock-price tree:

$s_{5,5} = S(0) \cdot u^5 = \dots = 121.67, \quad \Rightarrow \quad v_{5,5} = 21.67,$

$s_{5,4} = S(0) \cdot u^4 \cdot d = \dots = 112.31, \quad \Rightarrow \quad v_{5,4} = 12.31,$

$s_{5,3} = S(0) \cdot u^3 \cdot d^2 = \dots = 103.67; \quad \Rightarrow \quad v_{5,3} = 3.67.$

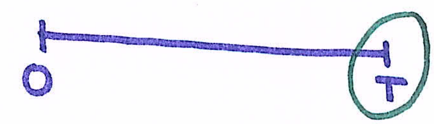
the other nodes are out-o-money.

$$\Rightarrow V_c(0) = e^{-0.10} \Big(21.67 \cdot (0.7525)^5 + 12.31 \cdot 5 \cdot (0.7525)^4 (1-0.7525) + 3.67 \cdot 10 \cdot (0.7525)^3 (1-0.7525)^2 \Big) = 10.0176$$

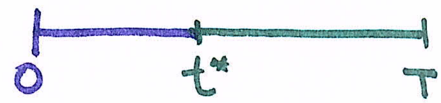
2.

Early Exercise.

- **European**: no early exercise:



- **Bermudan**: admissible early exercise on a subset of $[0,T]$



- **American**: allow early exercise for all $t \in [0,T]$

33.

Several years ago, John bought three separate 6-month options on the same stock.

- Option I was an American-style put with strike price 20.
- Option II was a Bermuda-style call with strike price 25, where exercise was allowed at any time following an initial 3-month period of call protection.
- Option III was a European-style put with strike price 30.

When the options were bought, the stock price was 20.

When the options expired, the stock price was 26.

The table below gives the maximum and minimum stock price during the 6 month period:

Time Period:	1st 3 months of Option Term	2nd 3 months of Option Term
Maximum Stock Price	24	28
Minimum Stock Price	18	22

John exercised each option at the optimal time.

Rank the three options, from highest to lowest payoff.

(A) I > II > III

(B) I > III > II

(C) II > I > III

(D) III > I > II

(E) III > II > I

I: Exercised @ the overall minimum stock price, i.e., @ 18:

Payoff: $(20-18)_+ = 2$

II: Exercised @ maximum stock price after the lockout period:

$(28-25)_+ = 3$

III. $(30-26)_+ = 4$

34.

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4. For a stock, you are given:.

(i) The current stock price is \$50.00.

(ii) $\delta = 0.08$

(iii) The continuously compounded risk-free interest rate is $r = 0.04$.

(iv) The prices for one-year European calls (C) under various strike prices (K) are shown below:

K	C	PUT		IMM. EXER
\$40	\$ 9.12	1.39	>	−10
\$50	\$ 4.91	6.79	>	5
\$60	\$ 0.71	12.20	>	10
\$70	\$ 0.00	21.10	>	20

⇒ (E)

You own four special put options each with one of the strike prices listed in (iv). Each of these put options can only be exercised immediately or one year from now.

Determine the lowest strike price for which it is optimal to exercise these special put option(s) immediately.

(A) \$40

(B) \$50

(C) \$60

(D) \$70

(E) It is not optimal to exercise any of these put options.

The owner can
- exercise the put now ⇒ value of immediate exercise
- hold onto it: it becomes a regular European put ⇒ value of the European put in the market

Use put-call parity to get put prices:

compare! :

$$V_p(40) = \underbrace{V_c(40)}_{9.12} + 40e^{-0.04} - 50e^{-0.08} = 1.39$$

5.