Problem 100.1. (2 pts) Source: Sample FM(DM) Problem #24.
Derivatives are used as a form of insurance. True or false?
Solution: TRUE

Problem 100.2. In this problem, you should use the binomial option pricing procedure. Suppose that the exchange rate is 0.79EUR/$. Let the risk-free interest rates be $r(\$) = 0.03$ and $r(EUR) = 0.05$. Assume that $u = 1.1850$ and $d = 0.8581$ and let $T = 15$ months, $n = 3$ and $K = 0.8EUR$. Find the price of a European call with the above properties.
Solution: Note that the both the strike price and the exchange rates are denominated in euros. Hence, we must regard the $\$ interest rate $r(\$)$ as the foreign interest rate.
We are looking at three periods, each one 5 months long. The risk-neutral probabilities in this problem are
$$p^* = \frac{e^{h(r(EUR)−r(\$))} − d}{u − d} = \frac{e^{5/12(0.05−0.03)} − 0.8581}{1.1850 − 0.8581} \approx 0.46.$$
and $q^* = 1 − p^* = 0.54$.

Using the binomial pricing formula and setting $x(0) = 0.79$, we get
$$V_C(0) = e^{−r(EUR)·0.25} · \left[(p^*)^3(x(0)u^3 − K)^+ + 3(p^*)^2q^* (x(0)u^2d − K)^+ + 3p^* (q^*)^2(x(0)u^d^2 − K)^+ + (q^*)^3(x(0)d^3 − K)^+] \right]$$
$$= 0.9394 \cdot 0.46^2(0.46 \cdot 0.5146 + 3 \cdot 0.54 \cdot 0.1519) \approx 0.1.$$

Problem 100.3. Source: Problem 10.14 from the textbook.
Suppose that the current exchange rate equals $0.92$ per euro. Assume that the continuously compounded risk-free interest rate for the $\$ equals 0.04, while the continuously compounded risk-free interest rate for the euro equals 0.03. In a three-period binomial model with each period of length one quarter-year, you are given that $u = 1.2$ and $d = 0.9$. Find the price of a 9-month European call on the euro with the strike $K = 0.85$.
Solution: We can calculate the price of the call currency option in a very similar way to our calculations for options on stocks/indexes with continuously paying dividends. We simply replace the dividend yield with the foreign interest rate in our formulas. Thus, we have:

<table>
<thead>
<tr>
<th>node $uu$</th>
<th>node $ud = du$</th>
<th>node $dd$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>0.9925</td>
<td>0.9925</td>
</tr>
<tr>
<td>$B$</td>
<td>−0.8415</td>
<td>−0.8415</td>
</tr>
<tr>
<td>call premium</td>
<td>0.4734</td>
<td>0.1446</td>
</tr>
</tbody>
</table>

Using these call premia at all the earlier nodes yields:

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>node $d$</th>
<th>node $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>0.7038</td>
<td>0.5181</td>
</tr>
<tr>
<td>$B$</td>
<td>−0.5232</td>
<td>−0.3703</td>
</tr>
<tr>
<td>call premium</td>
<td>0.1243</td>
<td>0.0587</td>
</tr>
</tbody>
</table>

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The price of the European call option is $0.1243.

Note: We could have simply used the shorter “binomial” pricing formula since we were looking for the price of a European option and did not need the Δ’s and the B’s in the intermediate nodes.

Problem 100.4. (2 points)
The premium on a standard call option and a down-and-in call are the same if the barrier price exceeds the initial stock-price. True or false?

Solution: TRUE

Problem 100.5. In our usual notation, let $S(0) = 40$, $r = 0.08$, $\sigma = 0.3$, $\delta = 0$. You need to construct a 2–period forward binomial tree for the above stock.

(a) (4 pts) What are $u$ and $d$?

(b) (10 pts) Construct the binomial tree for $S$. Enter all the intermediate stock-prices.

(c) (6 pts) Consider a six-month, at-the-money European call on $S$. What is the price of that call based on the binomial tree you obtained in part (b)?

Solution:

(a)

$$u = e^{(r-\delta)h + \sigma\sqrt{h}} = e^{0.08 \cdot 0.25 + 0.3 \cdot 0.25} = 1.1853;$$

$$d = e^{(r-\delta)h - \sigma\sqrt{h}} = e^{0.08 \cdot 0.25 - 0.3 \cdot 0.25} = 0.8781.$$

(b) In the usual notation,

$$S_d = 35.12, S_u = 47.2, S_{uu} = 46.20, S_{ud} = 41.63, S_{dd} = 30.84.$$

(c) We need to calculate the values in the replicating portfolio at the relevant nodes in order to price the European call option:

<table>
<thead>
<tr>
<th></th>
<th>$t = 0$</th>
<th>node $d$</th>
<th>node $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta</td>
<td>0.6074</td>
<td>0.1513</td>
<td>1</td>
</tr>
<tr>
<td>$B$</td>
<td>-20.187</td>
<td>-4.5736</td>
<td>-39.208</td>
</tr>
<tr>
<td>call premium</td>
<td>4.110</td>
<td>0.7402</td>
<td>8.204</td>
</tr>
</tbody>
</table>

Problem 100.6. If the interest rate is 0, then it never makes sense to early-exercise an American put option on a non-dividend paying stock.

Solution: TRUE

Problem 100.7. Consider a two-step binomial model with $S(0) = $50, $u = 2$ and $d = 0.5$. Your goal is to price an at-the-money European call option with the two periods to exercise under the assumptions that the underlying stock does not pay a dividend and that the effective interest rate per period equals $i = 25\%$.

a. (3 pts) Find the risk-neutral probability.

b. (8 pts) Find the fair price of the call.

c. (8 pts) Find the dynamic replicating portfolio which should be used at every node in the tree.

Solution:

a.

$$p^* = \frac{(1 + i) - d}{u - d} = 0.5.$$

b. We draw the binomial tree, and proceed backwards through the inner nodes. At the node where $S^u_1 = $100, we have that the value of the call is

$$v(1; 100) = \frac{1}{1.25} \left[ \frac{1}{2} [150 + 0] \right] = 60.$$
On the other hand, the remaining two final payoffs are both zero, which yields that the value of the call at the node $S^d_1 = 25$ is

$$v(1; 25) = 0.$$  

We have now reduced the pricing problem to a one-step binomial tree. The usual calculation gives us that the fair price of the above call is

$$v(0; 50) = \frac{1}{1.25} \left[ \frac{1}{2} (60 + 0) \right] = 24.$$  

c. With the usual notation, we have

$$\Delta(1; 100) = \frac{150 - 0}{1.5 \cdot 100} = 1;$$  
$$\Delta(1; 25) = 0;$$  
$$\Delta(0; 50) = \frac{60 - 0}{1.5 \cdot 50} = \frac{4}{5}.$$  

**Problem 100.8.** In our usual notation, let $S(0) = 100$, $r = 8\%$ and $\delta = 0$. Suppose that $u = 1.3$ and $d = 0.8$. Using the one-period binomial model, calculate the following:

a. (3 pts) The premium for a $105$-strike, six-month European put with the above characteristics.

b. (3 pts) The $\Delta$ in the corresponding replicating portfolio.

c. (3 pts) The amount $B$ invested in the riskless asset in the replicating portfolio.

**Solution:** See Problem 10.1.b. from McDonald!

The risk-neutral probability is

$$p^* = \frac{e^{rT} - d}{u - d} = 0.48.$$  

a. The premium for the said put is

$$V_P(0) = e^{-rT} \left[ p^* V_u + (1 - p^*) V_d \right]$$  
$$= e^{-0.08 \cdot 0.5} \left[ 0.48 \cdot 0 + 0.52 \cdot 25 \right]$$  
$$= 12.5.$$  

b.  

$$\Delta = \frac{0 - 25}{100(1.3 - 0.8)} = \frac{1}{2}.$$  

c.  

$$B = V_P(0) - \Delta(0) = 12.5 + \frac{100}{2} = 62.5.$$  

**Problem 100.9.** Consider a two-period binomial model for a non-dividend paying asset $S$ with $S(0) = 50$ and $u = 1/d = 2$. Let $i = 0.25$ denote the effective interest rate per period. You need to price a European put option on $S$ which expires at the end of the two periods and has the strike $K = 70$.

(i) (3 pts) Find the values of the given option at all the nodes in the binomial tree. In particular, find the fair price at time 0 of this option.

(ii) (3 pts) Find the number of shares $\Delta$ one needs to invest in at every node in the tree in order to replicate the option.

(iii) (2 pts) If the option were American, would there be early exercise?

**Solution:** It is easy to construct the binomial tree with the given parameters and to get the risk-neutral probability $p^* = 1/2$.  

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Since the option is European, we can evaluate its price directly as

\[
V_P(0) = \frac{1}{1.25^2} \cdot \frac{1}{4} \left( (K - S_{uu})^+ + 2(K - S_{ud})^+ + (K - S_{dd})^+ \right) \\
= \frac{1}{1.25^2} \cdot \frac{1}{4} \left( (70 - 200)^+ + 2(70 - 50)^+ + (70 - 12.5)^+ \right) \\
= \frac{4}{25} \cdot 97.5 = 15.6.
\]

However, we need to find the value of the put at the other nodes of the tree as well. Evidently, the values at the leaves (notes uu, ud and dd) are precisely the payoff of the put depending on the stock price. At note \( u \), we have

\[
P_u = \frac{1}{1.25} \cdot \frac{1}{2} (0 + 20) = 8.
\]

At note \( d \), the value is

\[
P_d = \frac{1}{1.25} \cdot \frac{1}{2} (20 + 57.5) = 31.
\]

As for the \( \Delta \)s,

\[
\Delta_u = \frac{0 - 20}{\frac{3}{2} \cdot 100} = -\frac{2}{15} \\
\Delta_d = \frac{20 - 57.5}{\frac{3}{2} \cdot 25} = -1 \\
\Delta_0 = \frac{8 - 31}{\frac{3}{2} \cdot 50} = -\frac{23}{75}
\]

As for early exercise, at node \( d \), the payoff of immediate exercise would be

\[(70 - 25)^+ = 45 > P_d.
\]

So, one would create a higher profit by exercising early.

**Problem 100.10.** (5 points)

To plant and harvest 20,000 bushels of corn, Farmer Jayne incurs costs totaling $33,000. The current spot price of corn is $1.80 per bushel. What is the profit if the spot price is $1.90 per bushel when she harvests and sells her corn?

**Solution:**

\[
1.90 \cdot 20,000 - 33,000 = 5,000.
\]

**Problem 100.11.** (5 points)

KidCo Cereal Company sells Sugar Corns for $2.50 per box. The company will need to buy 20,000 bushels of corn in 6 months to produce 40,000 boxes of cereal. Non-corn costs total $60,000. What is the company's profit if they purchase call options at $0.12 per bushel with a strike price of $1.60? Assume the 6-month interest rate is 4.0% effective and the spot price in 6 months is $1.65 per bushel.

**Solution:** The future value of the initial cost of calls is

\[
1.04(0.12 \cdot 20,000) = 2,496.
\]

The payoff from the cereal boxes is $40,000 \cdot 2.50 = 100,000$. Since the spot price at expiration is greater than the strike price, the calls will be exercised, and KidCo will get 20,000 bushels of corn for

\[
1.60 \cdot 20,000 = 32,000.
\]

Altogether, their profit is

\[
100,000 - 60,000 - 32,000 - 2,496 = 5,504.
\]
Problem 100.12. (5 points) Source: SoA Sample FM (DM) Problem #30.
You are trying to decide whether to use forward contracts or futures contracts when committing to buy an underlying asset at some date in the future. Which of the following is NOT a distinguishing characteristic of futures contracts, relative to forward contracts?

(a) Contracts are settled daily, and marked-to-market.
(b) Contracts are more liquid, as one can offset an obligation by taking the opposite position.
(c) Contracts are more customized to suit the buyers needs.
(d) Contracts are structured to minimize the effects of credit risk.
(e) Contracts have price limits, beyond which trading may be temporarily halted.

Solution: (c)

Problem 100.13. (5 points) Source: SoA Sample FM (DM) Problem #23.
In our usual notation, you are given the following zero-coupon bond prices

\[ P(0,1) = 0.96154, \quad P(0,2) = 0.91573, \quad P(0,3) = 0.85770, \quad P(0,4) = 0.78466, \quad P(0,5) = 0.69656. \]

You enter into a 5-year interest rate swap (with a notional amount of $100,000) to pay a fixed rate and to receive a floating rate based on future 1-year LIBOR rates. If the swap has annual payments, what is the fixed rate you should pay?

(a) 0.052
(b) 0.057
(c) 0.067
(d) 0.072
(e) None of the above.

Solution: (d)

Let us denote the fixed rate by \( R \). Then, in our usual notation,

\[
R = \frac{\sum_{k=1}^{5} P(0,k) r_{0}(k-1,k)}{\sum_{k=1}^{5} P(0,k)}.
\]

We have

\[
\sum_{k=1}^{5} P(0,k) r_{0}(k-1,k) = \sum_{k=1}^{5} P(0,k) \cdot \frac{P(0,k-1)}{P(0,k)} = 1 - P(0,5) = 1 - 0.69656 = 0.30344.
\]

Also,

\[
\sum_{k=1}^{5} P(0,k) = 4.21619.
\]

Finally, \( R = 0.30344/4.21619 \approx 0.072 \).

Problem 100.14. (5 points) The current price of a non-dividend-paying stock is $75 per share. You model the stock price at the end of this year using a one-period binomial model under the assumption that the stock price can either increase by \( 1/5 \) of its current value or decrease by \( 1/5 \) of its current value.

A zero-coupon bond redeemable in one year for $100 currently sells for $95.

What is the risk-neutral probability that the stock price will go up?

Solution: In our usual notation,

\[
p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{r} - d}{u - d}.
\]
The conditions on the binomial asset-pricing model yield $u = 1.2$ and $d = 0.8$. The given bond price gives us that $e^r = 100/95$. So,

$$p^* = \frac{\frac{100}{95} - 0.8}{1.2 - 0.8} = 0.6316..$$

**Problem 100.15.** (5 points) The current price of a non-dividend-paying stock is $125 per share. You model the stock price at the end of this year using a one-period binomial model under the assumption that the stock price can either increase by $20$, or decrease by $20$.

The continuously compounded risk-free interest rate is given to be 0.04.

What is the price of a one-year, $135$-strike American call option on the above stock?

**Solution:** The risk-neutral probability of the stock price going up is

$$p^* = \frac{125e^{0.04} - 105}{40} = 0.6275$$

The call option’s price is, thus,

$$V_C(0) = e^{-0.04} \times 0.6275 \times (145 - 135) = 6.0293.$$  

**Problem 100.16.** (5 points) The current price of a non-dividend-paying stock is $120 per share. You model the stock price at the end of this year using a one-period binomial model under the assumption that the stock price can either increase by $20$, or decrease by $20$.

The continuously compounded risk-free interest rate is given to be 0.04.

What is the price of a one-year, $110$-strike European option on the above stock?

**Solution:** The risk-neutral probability of the stock price going up is

$$p^* = \frac{120e^{0.04} - 100}{140 - 100} = 0.6224.$$  

The put option’s price is, thus,

$$V_P(0) = e^{-0.04} \times (1 - 0.6224) \times (110 - 100) = 3.6276.$$  

**Problem 100.17.** Suppose that the prices of zero-coupon bonds redeemable for $1 at the end of the next four quarters are $0.9891, 0.9775, 0.9515, 0.9506$.

What is the **fixed** effective quarterly interest rate of the interest-rate swap on an interest-only loan consistent with the above bond prices?

(a) 0.2246
(b) 0.0562
(c) 0.0128
(d) 0.0028
(e) None of the above.

**Solution:** (c)

$$\frac{1 - 0.9506}{0.9891 + 0.9775 + 0.9515 + 0.9506} = 0.0128.$$