10.1. **Multiple binomial periods: American options.** Provide your **final answer** only for the following problem(s):

**Problem 10.1.** (5 points) Solve problem #44 (p.107) from the Sample MFE Problems.

**Solution:** (d)

10.2. **Properties of American options.** Please, provide your **complete solution** to the following problem(s):

**Problem 10.2.** (5 points) Solve Sample MFE Problem #26.

**Solution:** (d)

10.3. **Asian options.**

**Problem 10.3.** *Source: Problem 14.3.c. from McDonald.*

Suppose that $S(0) = 100, K = 100, r = 0.08, \sigma = 0.30, \delta = 0$ and $T = 1$.

Construct a standard two-period binomial stock price tree.

Find the price of an Asian arithmetic average price call.

**Solution:** Using the forward tree specification, $u = \exp(0.08/2 + 0.3/\sqrt{2}) = 1.2868$, $d = \exp(0.08/2 - 0.3/\sqrt{2}) = 0.84187$, and the risk neutral probability $p = (e^{0.08/2} - d)/(u - d) = 0.44716$. The two possible prices in 6 months are 128.68 and 84.19; the three possible 1 year prices are 165.58, 108.33, and 70.87.

Using the 6m and 12m prices, the possible arithmetic averages (in 1 year) are 147.13, 118.50, 96.26, and 77.53.

With $K = 100$ the up-up value is 47.1266 and the up-down value is 18.5026. There is no value in the bottom half of the tree. This gives an up value of

$$e^{-0.04} \left( p \cdot 47.1266 + (1 - p) \cdot 18.5026 \right) = 30.075$$

and an initial value of $e^{-0.04} \cdot p \cdot 30.075 = 12.921$.

**Problem 10.4.** Your portfolio consists of a long up-and-in call with the barrier at 50 and a long up-and-out call with the barrier at 50. Then, in our usual notation, the initial price of your portfolio equals:

(a) $V_C(0)$  
(b) $P_{0,T}(S)$  
(c) $S(T)$  
(d) $V_P(0)$  
(e) None of the above.

**Solution:** (a)
Problem 10.5. What is the payoff on a 75−strike Asian call option given that it is a geometric average price call? The recent prices are: 72, 76, 74, 78 and 78.

(a) $0.46  
(b) $0.56  
(c) $0.66  
(d) $0.76  
(e) None of the above.

Solution:  (b)
The payoff can be calculated as

\[
\left( (72 \cdot 76 \cdot 74 \cdot 78)^{1/5} - 75 \right)_+ \approx (75.56 - 75)_+ = 0.56.
\]
Problem 10.6. What is the payoff on a 90–strike Asian option given that it is a geometric average price put? The recent prices are: 89, 90, 91, 87, 87 and 88.

(a) $1.25  
(b) $1.35  
(c) $1.66  
(d) $1.76  
(e) None of the above.

Solution: (b)  
The payoff is calculated as  
\[(90 - (89 \cdot 90 \cdot 91 \cdot 87 \cdot 87 \cdot 88)^{1/6})_+ = (90 - 88.65)_+ = 1.35.\]

Problem 10.7. Review the notation used in class and on the slides on Asian options. Which one of the following inequalities is always correct?

(a) \((K - A(T))_+ \geq (K - G(T))_+\)  
(b) \((A(T) - K)_+ \geq (G(T) - K)_+\)  
(c) \((A(T) - K)_+ \geq (S(T) - K)_+\)  
(d) \((S(T) - K)_+ \geq (G(T) - K)_+\)  
(e) None of the above.

Solution: (b)  
We already established that \(A(T) \geq G(T)\). The equality holds iff all of the stock-prices entering the calculation are equal. This implies that (a) is not always correct while (b) is always correct.

To rule out (c) and (d), consider the simple situation in which just two stock-prices are sampled: \(S(T/2)\) and \(S(T)\). In case that \(S(T/2) < S(T)\), we get  
\[A(T) = \frac{1}{2} (S(T/2) + S(T)) < S(T) \implies (S(T) - K)_+ > (A(T) - K)_+.\]

In case that \(S(T/2) > S(T)\), we get  
\[G(T) = \sqrt{S(T/2)S(T)} > S(T) \implies (G(T) - K)_+ > (S(T) - K)_+.\]

Problem 10.8. Assume that the initial price of a share of non-dividend-paying stock equals \(S(0) = 100\) and that its volatility is given to be \(\sigma = 0.3\). The continuously compounded risk-free interest rate is \(r = 0.05\). Consider an Asian arithmetic average price call option with strike \(K = 100\) and exercise date in one year.

Using the two-period forward binomial tree, calculate the price of the above option.

(a) 12  
(b) 12.20  
(c) 13.10  
(d) 14  
(e) None of the above.

Solution: (b)  
The risk-neutral probability is  
\[p^* = 1/(1 + e^{\sigma \sqrt{T}}) = 1/(1 + e^{0.3/\sqrt{2}}) = 0.447.\]

The payoff of an Asian arithmetic average price call with strike \(K\) is, by definition, \((A(T) - K)_+.\) The forward tree up and down factors are  
\[u = e^{0.025+0.3/\sqrt{2}} = 1.27, \quad d = e^{0.025-0.3/\sqrt{2}} = 0.83.\]

Here are the payoffs resulting from the four paths through the tree:

up-up \( A_{uu} = (\frac{1}{2}(S_u + S_{uu}) - K)_+ = (\frac{u(1+u)S(0)}{2} - K)_+ = (\frac{1.27 \cdot 1.00}{2} - 100)_+ = 44.14 \)

up-down \( A_{ud} = (\frac{1}{2}(S_u + S_{ud}) - K)_+ = (\frac{u(1+d)S(0)}{2} - K)_+ = (\frac{1.27 \cdot 1.44}{2} - 100)_+ = 16.25 \)

down-up \( A_{du} = (\frac{1}{2}(S_d + S_{ud}) - K)_+ = (\frac{d(1+u)S(0)}{2} - K)_+ = (\frac{0.83 \cdot 1.00}{2} - 100)_+ = 0 \)

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down-down $A_{dd} \leq A_{du} = 0$

The risk-neutral price is

$$V(0) = e^{-rT}[(p^*)^2 \cdot A_{uu} + p^*(1-p^*)A_{ud}] = e^{-0.05} \cdot 0.447 \cdot [0.447 \cdot 44.14 + 0.553 \cdot 16.21] = 12.20.$$ 

10.4. Derivative securities.

**Problem 10.9.** (2 pts) *Source: Sample FM(DM) Problem #24.*
Derivatives are used to reduce transaction costs. *True or false?*

**Solution:** TRUE

**Problem 10.10.** (2 pts) *Source: Sample FM(DM) Problem #24.*
Derivatives are used to satisfy regulatory, tax, and accounting constraints. *True or false?*

**Solution:** FALSE