Short Sales.

*continuous dividends *

\[ \delta \text{... dividend yield} \]

\[ \begin{align*}
\text{Init. Cost:} & \quad -S(0) \\
\text{Payoff:} & \quad -e^{ST} \cdot S(T)
\end{align*} \]

\[ \Rightarrow \text{Payoff function:} \quad \psi(s) = \frac{-e^{ST}}{\text{constant}} \]

\[ \Rightarrow \text{Profit} = \text{Payoff} - FV_{0,T}(\text{Init. Cost}) \]
\[ = -e^{ST} \cdot S(T) + FV_{0,T}(+S(0)) \]
\[ = -e^{ST} \cdot S(T) + FV_{0,T}(S(0)) \]
\[ = -e^{ST} \cdot S(T) + S(0) e^{rT} \]

\[ \text{maximal profit: } S(0) e^{rT} \]
\[ \text{break-even: } S^* = S(0) e^{(r-s)T} \]

\[ \psi(s) = -e^{ST} \cdot \delta \]

PAYOFF

Profit

maximal payoff: \( S(0) e^{rT} \)

break-even: \( S^* = S(0) e^{(r-s)T} \)

\( \delta \) (final asset price)
Fully leveraged purchase

A portfolio:

\[ \rightarrow \text{buy/long one share of stock} \]
\[ \rightarrow \text{borrow } S(0) \text{ at the risk-free interest rate to be repaid } \]
\[ \text{at time-} T \]

\[ \begin{align*}
\text{Init. cost:} & \quad + S(0) \quad \text{(purchase of asset)} \\
& \quad - S(0) \quad \text{(taking a loan)} \\
\text{Profit:} & \quad e^{sT} \cdot S(T) - FV_{0,T}(S(0)) \quad \text{const.}
\end{align*} \]

Note: For a fully-leveraged position:

\[ \text{PAYOFF = PROFIT} \]

The profit curve is identical to the profit of the outright purchase.

[Graph showing profit, loss, gain, and final asset price]
Forward Contracts.

Binding Contract.

Handshake!
AGREE ON: • the underlying asset
• the quantity
• SETTLEMENT < physical ⇒ logistics, cash
• THE DELIVERY DATE T
• THE FORWARD PRICE F

Payoff:
LONG FORWARD: BUYING FORWARD
1 unit of asset ↑ Forward Price S(T) ↓ F
SHORT FORWARD: SELLING FORWARD

⇒ Payoff (long forward) = +S(T) - F
⇒ Payoff (short forward) = F - S(T)

Initial Cost: 0

Payoff = Profit
Hedging with a forward contract.

Assume a non-dividend-paying asset.

Start:
- unhedged position, say, an outright purchase of 1 share of stock.
- the hedge is a SHORT FORWARD

Form a hedged position!

\[ F = F_{0,T}(S(0)) \]

\[ F - F_{0,T}(S(0)) \]

\[ - F_{0,T}(S(0)) \]