02/05/2018

# Forward Contracts. [cont'd]

0 — T ... delivery date

Initial Cost : 0

long forward ("buying")

$S(T)\uparrow$ $\downarrow F$ ... forward price

short forward ("selling")

$\Rightarrow$ Payoff : $S(T) - F$

$\Rightarrow$ Profit = Payoff = $S(T) - F$

* Hedging a short sale of a continuous-dividend-paying stock using a forward. *

Start w/ the unhedged position;

its profit is : $\boxed{-e^{\delta \cdot T} S(T) + S(0)e^{rT}}$

# of shares to be returned

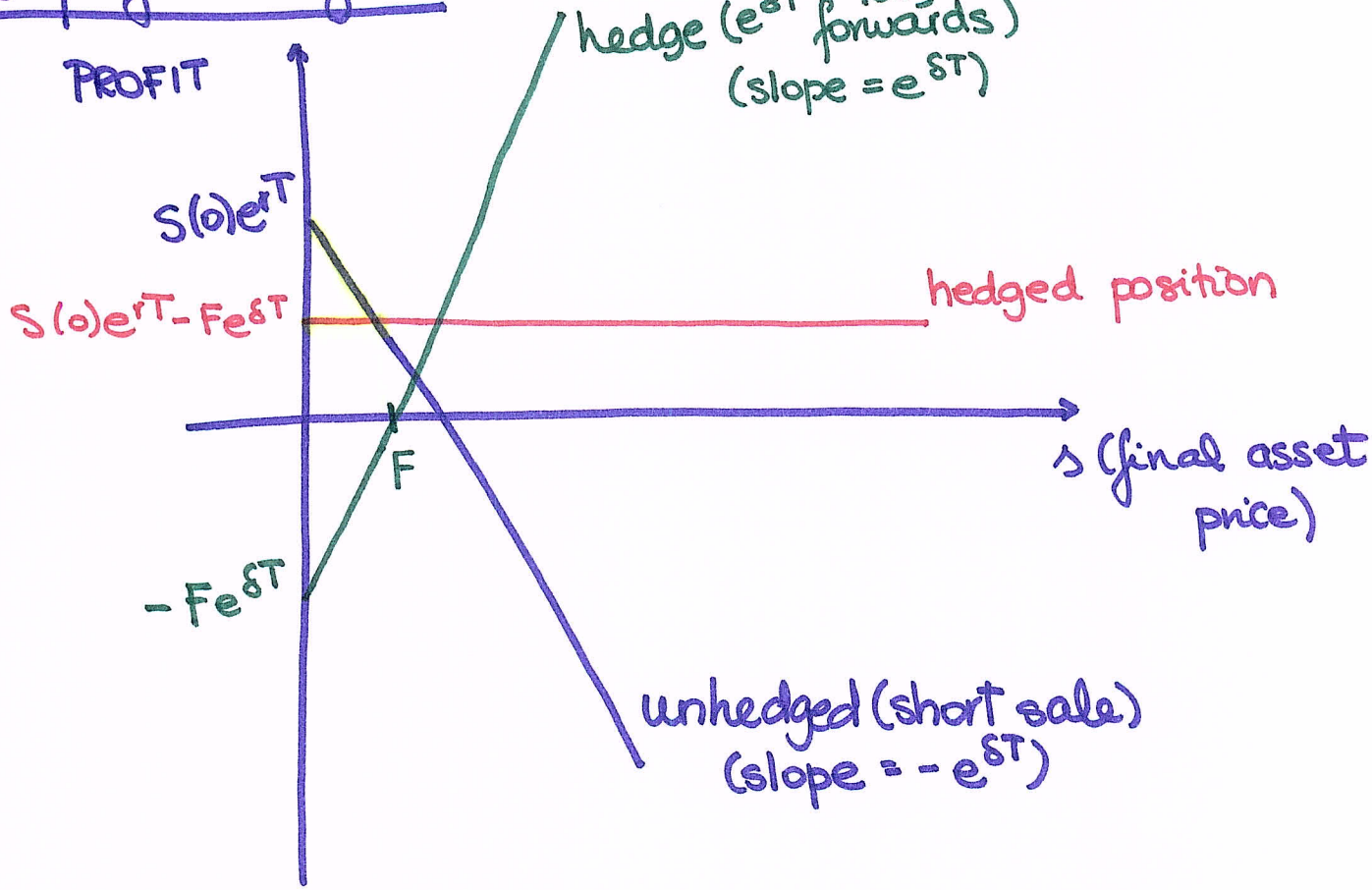
Hedge by longing $e^{\delta T}$ forward contracts, i.e., we want to BUY FORWARD exactly as many shares as need to be returned.

$$\Rightarrow \text{Profit (Hedge)} = e^{\delta T}(S(T) - F)$$

$\Rightarrow$ The profit of the total HEDGED portfolio:

$$\cancel{-e^{\delta T} S(T)} + S(0)e^{rT} + e^{\delta T}(\cancel{S(T)} - F) = \underbrace{S(0)e^{rT} - Fe^{\delta T}}_{\text{does not depend on } S(T)}$$

## The profit diagram:

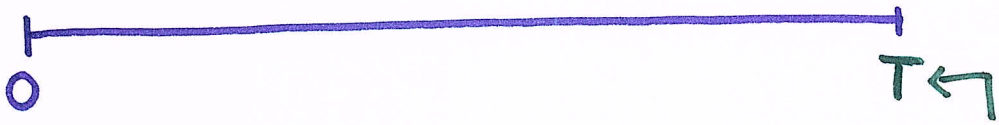


# European call options.

European → The option can be EXERCISED, i.e., the payoff can be collected only on the EXERCISE DATET.

call → Usually, gives its owner the right to buy the underlying asset.

options → Usually, the owner of the option has the right to exercise but not an obligation


The option is written, the writer of the option takes the short position.

Agree:
- T... the EXERCISE DATE
- K... the STRIKE/EXERCISE PRICE

The call's owner, i.e., the agent w/ a long call has a right but not an obligation to buy 1 unit of the underlying for the strike price K.

The premium is paid

long call

↓

short call

$$\boxed{\text{Payoff} = ?}$$

IF $S(T) \geq K$, THEN EXERCISE

$\Rightarrow S(T) - K$

IF $S(T) < K$, THEN DO NOT EXERCISE

$\Rightarrow 0$

$$\Rightarrow \text{Payoff} = \begin{cases} S(T)-K, & \text{if } S(T) \geq K \\ 0, & \text{if } S(T) < K \end{cases}$$

$$\boxed{V_c(T) = (S(T)-K) \cdot \mathbb{I}_{[S(T) \geq K]}}$$

<u>Indicator random variable</u>:

A... an event

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases}$$

<u>Note</u>: $S(T) \geq K \iff S(T) - K \geq 0$

$$V_c(T) = (S(T)-K)\mathbb{I}_{[S(T)-K \geq 0]}$$

$$= \text{MAX}[S(T)-K, 0]$$

Introduce the POSITIVE PART FUNCTION:

$$(x)_+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} = x \vee 0$$

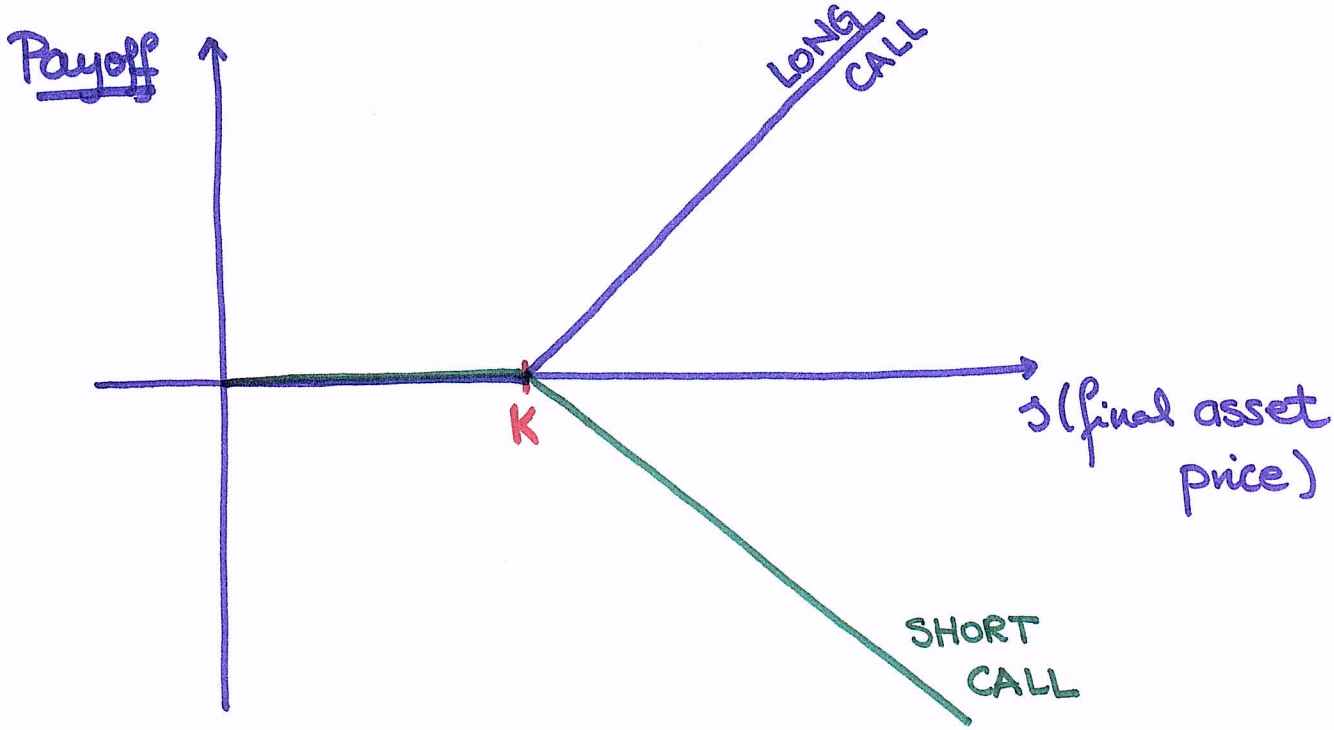
X... loss amt rnd variable

d... deductible

Pmt Amt : $(X-d)_+$

$$V_c(T) = (S(T)-K)_+$$

$\Rightarrow$ The payoff function: $v_c(s) = (s-K)_+$




$\downarrow$

Note: Payoff never positive, can be negative.

$\Downarrow$

They need to be compensated up front w/ a premium on the call.