Forward Contracts. [cont'd]

Initial Cost: 0

T...delivery date

long forward (buying)

\[ S(T) \uparrow \downarrow F \quad \text{forward price} \]

short forward ("selling")

\[ \Rightarrow \text{Payoff: } S(T) - F \]

\[ \Rightarrow \text{Profit} = \text{Payoff} = S(T) - F \]

* Hedging a short sale of a continuous-dividend-paying stock using a forward. *

Start w/ the unhedged position; its profit is:

\[ -e^{ST} S(T) + S(0)e^{rT} \]

\# of shares to be returned

Hedge by longing \( e^{ST} \) forward contracts, i.e., we want to buy forward exactly as many shares as need to be returned.

\[ \Rightarrow \text{Profit (Hedge)} = e^{ST}(S(T) - F) \]

\[ \Rightarrow \text{The profit of the total HEDGED portfolio:} \]

\[ -e^{ST} S(T) + S(0)e^{rT} + e^{ST}(S(T) - F) = \frac{S(0)e^{rT} - Fe^{ST}}{c} \quad \text{(1)} \]
The profit diagram:

- **Profit**
  - $S(0)e^{rT}$
  - $S(0)e^{rT} - F_{e^{rT}}$
  - $-F_{e^{rT}}$

- **Hedge (e^{rT} long forwards)**
  - Slope = $e^{rT}$

- **Unhedged (short sale)**
  - Slope = $-e^{rT}$

- **Hedged position**

- $S$ (final asset price)
European call options. Usually, the owner of the option has the right to exercise but not an obligation.

The option can be EXERCISED, i.e., the payoff can be collected only on the EXERCISE DATE.

The option is written, the writer of the option takes the short position.

Agree: • T... the EXERCISE DATE
      • K... the STRIKE/EXERCISE PRICE

The call's owner, i.e., the agent w/ a long call has a right but not an obligation to buy 1 unit of the underlying for the strike price K.

Payoff =

\[
\begin{align*}
\text{IF } S(T) \geq K & \text{, THEN EXERCISE} \\
& \Rightarrow S(T) - K \\
\text{IF } S(T) < K & \text{, THEN DO NOT EXERCISE} \\
& \Rightarrow 0
\end{align*}
\]
\[ \text{Payoff} = \begin{cases} S(T) - K, & \text{if } S(T) \geq K \\ 0, & \text{if } S(T) < K \end{cases} \]

\[ V_c(T) = (S(T) - K) \cdot \mathbb{I}_{[S(T) \geq K]} \]

**Indicator random variable:**

\[ \mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happen} \end{cases} \]

**Note:**

\[ S(T) \geq K \iff S(T) - K \geq 0 \]

\[ V_c(T) = (S(T) - K) \mathbb{I}_{[S(T) - K \geq 0]} = \text{MAX} [S(T) - K, 0] \]

**Introduce the positive part function:**

\[ (x)_+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} = x \vee 0 \]

\[ X \ldots \text{loss amount variable} \]
\[ d \ldots \text{deductible} \]

**Premt. Amt.:**

\[ (X - d)_+ \]

\[ V_c(T) = (S(T) - K)_+ \]
The payoff function: $\nu_c(s) = (s - K)^+$

**Diagram:**
- Payoff axis labeled as Payoff.
- Graph showing a long call and a short call.
- Final asset price labeled as $s$.
- Note: Payoff never positive, can be negative.
- They need to be compensated up front with a premium on the call.