European Call Options.

- Review:
  1. Buyer (long) at 0
  2. Call premium $V_c(0)$
  3. Writer (short)
  4. $K$... strike/exercise price

Exercise date $T$:

Condition for exercise:

$$S(T) \geq K$$

Payoff $V_c(T) = (S(T) - K)_+$

$\Rightarrow$ The payoff function:

$$V_c(s) = (s - K)_+$$

The call option is LONG with respect to the underlying.
Hedging a short-sale with a call option.

- short sale of 1 share of non-div. stock \( \{ \text{CAP.} \) 
- long call

Payoff

\[ \text{combined profit} \]

\[ \text{short sale} \]

\[ -K \]

Algebraically:

\[ \frac{-\sigma + (s-K)+}{\text{short sale}} + \frac{(s-K)+}{\text{long call}} = \begin{cases} -\sigma + (s-K) = -K & \text{IF } s \geq K \\ -\sigma & \text{IF } s < K \end{cases} \]

\[ = -\min (s, K) \]
Covered/naked option writing.

Covered... the option's writer takes an appropriate position in the underlying asset.
Naked... do not cover the option, but just write it

- written call option (no dividends)
- long/buy the underlying

\[ \text{A covered call.} \]

Payoff

\[ \text{Long underlying} \]

\[ \min(s, K) \]

\[ \text{covered call} \]

\[ \text{written (short) call} \]
European put options.

Usually, a right to sell.

\[ K \] ... strike price ...

The condition for exercise is:

\[ K > S(T) \]

The put's owner has the RIGHT, but not an obligation to

\{ SELL 1 unit of the underlying for the strike price \( K \) \}

\{ The writer is obligated to do whatever the owner of the put decides. \}

\[ \text{Payoff} = \begin{cases} K - S(T), & \text{if } S(T) < K \\ 0, & \text{if } S(T) \geq K \end{cases} \]

\[ V_p(T) = (K - S(T))_+ \]
The put's payoff function is:

\[ v_p(s) = (K-s)_+ \]

\[ \Rightarrow \text{Profit curve:} \]

Bounded both from below & above.
SHORT w/ respect to the underlying.