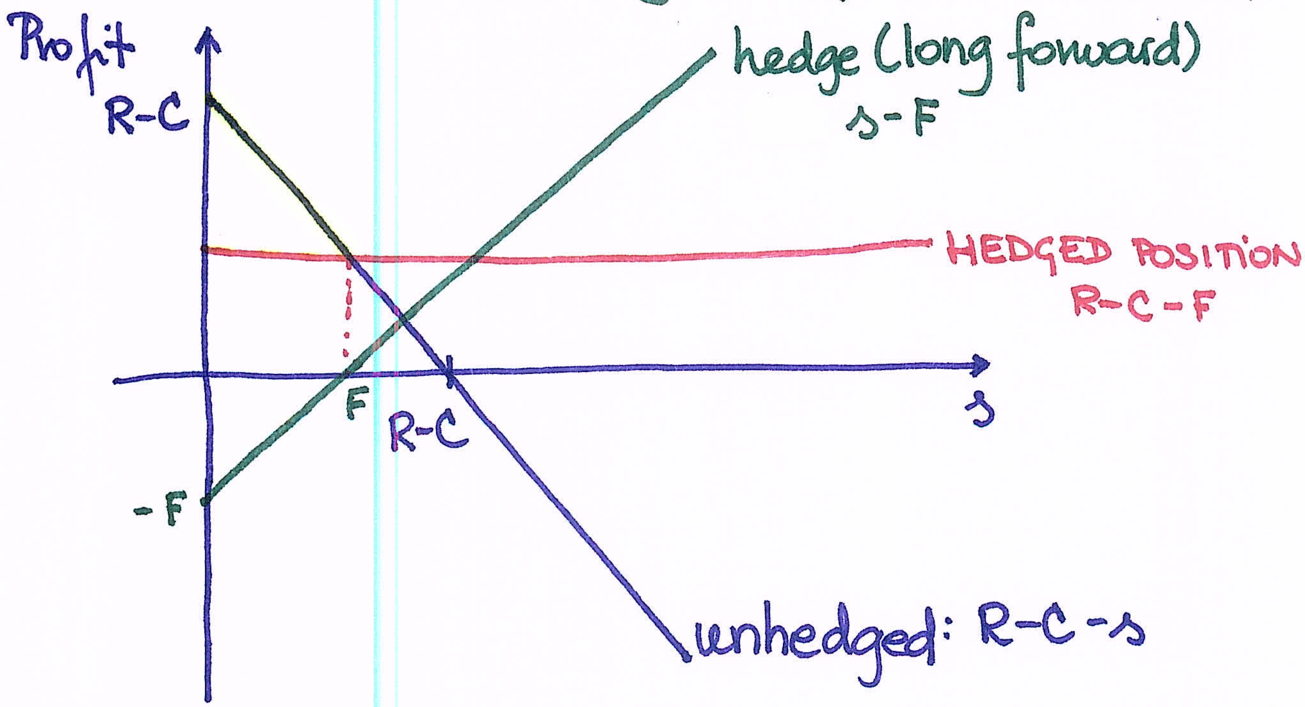
Motivation

D: Feb 15^{th}, 2019.

You start w/ an inherently short position. To hedge we can long a foward contract.



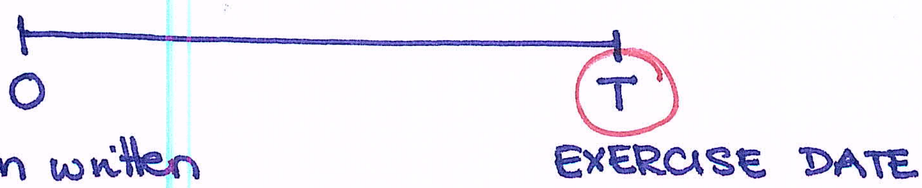
⇒ You are motivated to introduce a derivative security which includes a <u>RIGHT</u> to buy the underlying @ a pre-determined price, but it does <u>not</u> obligate you.

European Call Options.

The option can be EXERCISED, i.e., the cashflow can be collected only on the EXERCISE DATE.

Usually, this means it's a RIGHT to BUY the underlying asset.

Usually, the option's owner is not obligated to exercise the option.



At $t=0$:

The writer of the option is said to write/short the call.
The other party is said to long the call (they are the call's owner now!)

Agree on:
- the underlying asset: $S(t)$, $t \geq 0$
- T... exercise date
- K... the STRIKE/EXERCISE PRICE

Initial premium: $V_c(0)$

Goes from the long call to the writer of the call.

At $t=T$:

The call's <u>owner</u> has the RIGHT, but NOT an obligation to BUY one unit of the underlying for the strike price K.

The call's <u>writer</u> is <u>obligated</u> to do what the call's owner opts for.

$\boxed{\text{Payoff} = ?}$ $V_C(T)$ ... a random variable denoting the call's payoff

<u>Rationally</u>: the call's owner's criterion is

IF $S(T) \geq K$, <u>THEN</u> EXERCISE

$\Rightarrow S(T) - K$

IF $S(T) < K$, <u>THEN</u> DO <u>NOT</u> EXERCISE

$\Rightarrow 0$

$$\Rightarrow V_C(T) = \begin{cases} S(T) - K, & \text{IF } S(T) \geq K \\ 0, & \text{IF } S(T) < K \end{cases}$$

<u>Indicator random variables:</u>

A ... event

$$\mathbb{I}_A = \begin{cases} 1 & \text{if } A \text{ happened} \\ 0 & \text{if } A \text{ did not happened} \end{cases}$$

i.e., $\mathbb{I}_A(\omega) = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \in A \end{cases}$

elementary outcome

$\Rightarrow$ The payoff of a European call option:

$$V_c(T) = (S(T)-K)\,\mathbb{I}_{[S(T)-K \geq 0]}$$
$$= \text{MAX}[0, S(T)-K]$$

- X… severity random variable
  loss amount
- d… deductible

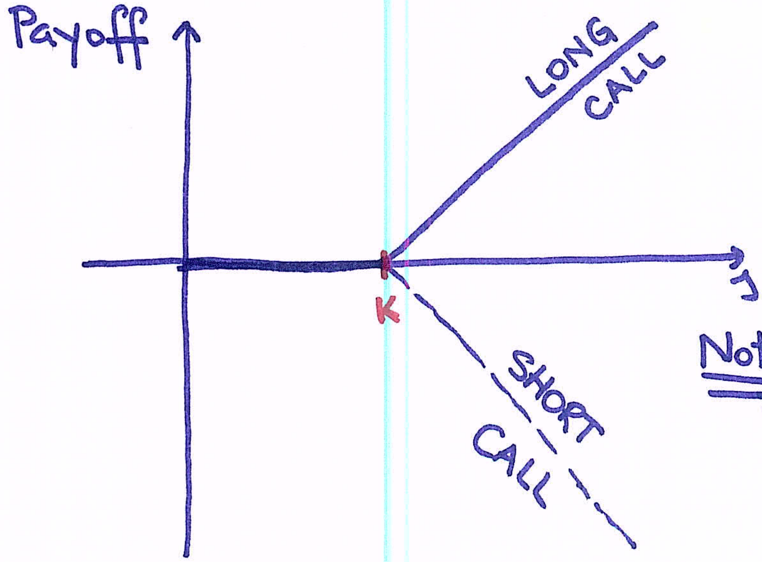
$\Rightarrow$ The <u>insurer</u> pays: $\text{MAX}[0, X-d]$

Introduce the positive-part function:

$$x \longmapsto (x)_+ := \text{MAX}[x, 0]$$

$$\Rightarrow \quad V_c(T) = (S(T)-K)_+$$

$\Rightarrow$ The payoff function: $v_c(s) = (s-K)_+$

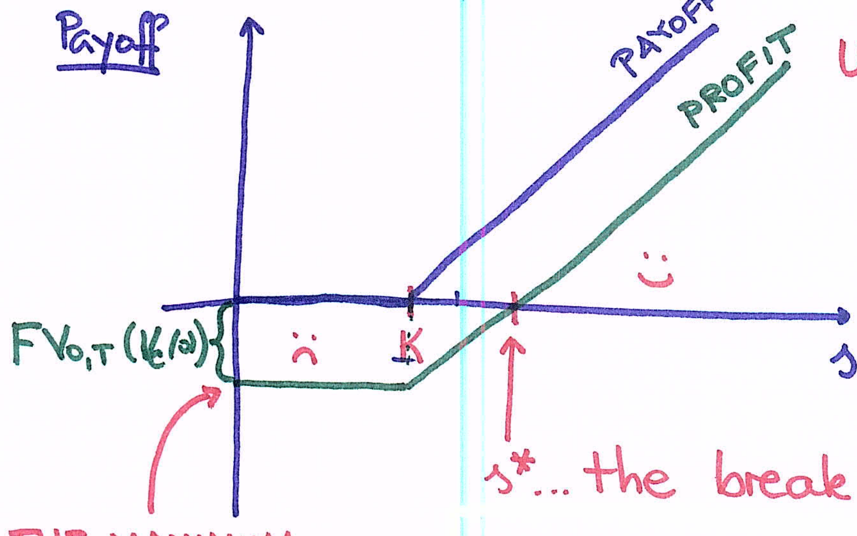


"hockey-stick functions"

<u>Note</u>: Payoff never positive, sometime it is <u>strictly</u> negative.

$\Rightarrow$ The initial positive premium $V_c(0)$.

The call's profit curve.



Unlimited growth potential

$\boxed{\text{LONG}}$ w.r.t. the underlying

THE MAXIMUM LOSS

$$s^* = K + FV_{0,T}(V_c(0))$$

Return to our motivation.

Start w/ an inherently short position.

Hedge (?) using a long call.