February 24th, 2017.

Law of the Unique Price

Let A and B be two static portfolios.

Assume $V_A(T) = V_B(T)$

Then $\boxed{V_A(0) = V_B(0)}$ No arbitrage!

Def'n. Consider a European-style derivative security. A static portfolio w/ the same payoff as that of the derivative security is called its REPLICATING PORTFOLIO.

Example. "long cash call + long cash put

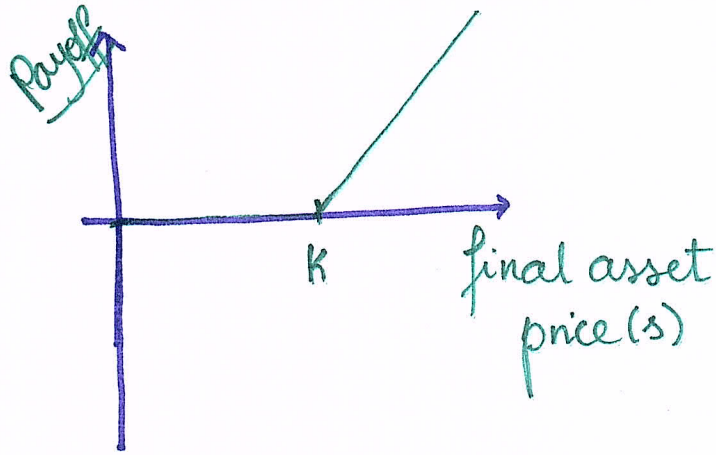
"

zero-coupon bond w/ \$1 redemption"

Example. Build a replicating portfolio for a European call w/ strike K and exercise date T consisting exclusively of digital options.

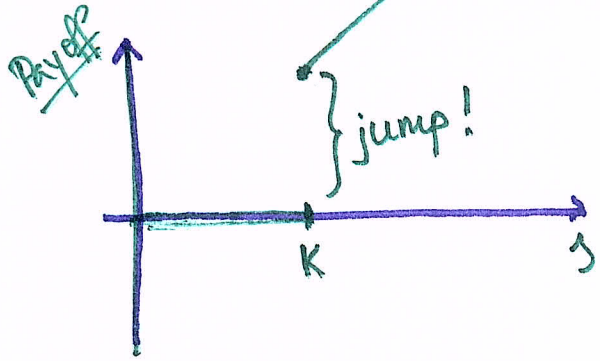
→ : The payoff of the European call:

$$V_c(T) = (S(T)-K)_+ = (S(T)-K)\mathbb{I}_{[S(T)\geq K]}$$
$$= S(T)\mathbb{I}_{[S(T)\geq K]} - K\cdot\mathbb{I}_{[S(T)\geq K]}$$



The payoff of an asset call:

$$V_{AC}(T) = S(T)\,\mathbb{I}_{[S(T)\geq K]}$$



$\Rightarrow$ · long 1 asset call
and
· short K cash calls

} replicating portfolio for a call !!!

$$\Rightarrow \boxed{V_C(0) = V_{AC}(0) - K \cdot V_{CC}(0)}$$

Prepaid forward

Initial Cost : F^P

Payoff : $S(T)$

Forward

Initial Cost : 0

Payoff : $S(T) - F$

Proposing to replicate a prepaid forward w/:

- one long forward contract
- invest $PV_{0,T}(F)$ @ the risk-free rate

This is a static portfolio!

Its payoff is : $S(T) - \cancel{F} + \cancel{FV_{0,T}(PV_{0,T}(F))}$

$= S(T)$

$\Rightarrow$ This is <u>indeed</u> a replicating portfolio for a prepaid forward!

$$\boxed{\begin{aligned} &\Rightarrow F^P = PV_{0,T}(F) \\ &\Leftrightarrow F = FV(F^P) \end{aligned}}$$

Focus on forwards on stocks:

Case #1. Non-dividend-paying

Outright Purchase

$0 \longrightarrow T$

Init. Cost: $S(0)$

Payoff: $S(T)$

Prepaid Forward

I.C.: F^P

Payoff: $S(T)$

$$F^P_{0,T}(S) = S(0)$$

$\Longrightarrow$ The forward price:

$$F_{0,T}(S) = FV_{0,T}(S(0)) = S(0)e^{rT}$$

Case #2. Continuous dividends. (δ...the dividend yield)

* continuous reinvestment of dividends *

Outright Purchase

$0 \longrightarrow T$

$\hookrightarrow$ $e^{-\delta T}$ shares $\longrightarrow$ 1 share

Prepaid forward:

$\hookrightarrow$ I.C.: $e^{-\delta T} \cdot S(0)$

I.C.: F^P 1 share

$$\Rightarrow \quad F^{P}_{0,T}(S) = S(0)e^{-\delta T}$$

$$\Rightarrow \quad F_{0,T}(S) = S(0)e^{-\delta T} \cdot e^{rT}$$

$$F_{0,T}(S) = S(0)e^{(r-\delta)T}$$

Q: Assume $r > \delta > 0$.

Nominal amts of cashflows are ordered as:

Prepaid forward < Outright Purchase < Forward

Fully Leveraged