
5.1. **Butterfly spreads and convexity.** Please, provide your complete solution to the following problem(s):

**Problem 5.1.** (5 points) In a certain market, you are given that

- the price of a 40−strike European call option on an underlying asset $S$ and with maturity $T$ is $11$;
- the price of a 50−strike European call option on an underlying asset $S$ and with maturity $T$ is $6$;
- the price of a 55−strike European call option on an underlying asset $S$ and with maturity $T$ is $3$.

Let the risk-free interest rate be $r = 0.05$.

A trader decides to construct the following portfolio:

1. long one 40−strike call option;
2. short three 50−strike European call options;
3. long two 55−strike calls.

Suppose that at time $T = 1$ the value of the asset $S$ is $S(1) = 52$. What is the profit of the portfolio at time $T$?

Provide your final answer only for the following problems.

**Problem 5.2.** (5 points) Let $K_1 = 50, K_2 = 55$ and $K_3 = 65$ be the strikes of three European call options on the same underlying asset and with the same expiration date. Let $V_C(K_i)$ denote the price at time−0 of the option with strike $K_i$ for $i = 1, 2, 3$.

We are given that $V_C(K_1) = 16$ and $V_C(K_3) = 1$. What is the maximum possible value of $V_C(K_2)$ which still does not violate the convexity property of option prices?

(a) $V_C(K_2) = 10$
(b) $V_C(K_2) = 11$
(c) $V_C(K_2) = 13$
(d) $V_C(K_2) = 15$
(e) None of the above.

**Problem 5.3.** (5 points) You are interested in purchasing a European call option on the underlying asset $S$ which has expiration date $T = 1$ year and strike $K = 70$. Denote the price of this call by $C$.

You are given that the premiums for European call options with the same expiration date and the same underlying asset but with the strikes $K_1 = 60$ and $K_2 = 75$ are $C_1 = 10$ and $C_2 = 3$, respectively.

You know that there is no arbitrage in your market. Find the maximal price $C$ that you might be charged so that none of the convexity inequalities are violated.

(a) $C = 16/3$
(b) $C = 6$
(c) $C = 25/3$
(d) $C = 28/3$
(e) None of the above.
Problem 5.4. (5 points) Source: Problem 9.11 from “Derivatives Markets” by McDonald.
Suppose that the observed European call and European put prices are given in the following table:

<table>
<thead>
<tr>
<th>Strike</th>
<th>80</th>
<th>100</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call premium</td>
<td>22</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Put premium</td>
<td>4</td>
<td>21</td>
<td>24.80</td>
</tr>
</tbody>
</table>

Looking at the observed prices, we can suspect that there is an arbitrage opportunity in this market. Which of the following is indeed an arbitrage portfolio?
(a) Long one 80−strike call, short two 100−strike calls, long one 105−strike call
(b) Long one 80−strike call, short five 100−strike calls, long four 105−strike calls
(c) Short one 80−strike call, long five 100−strike calls, short four 105−strike calls
(d) Long one 80−strike call, short four 100−strike calls, long three 105−strike calls
(e) There is no arbitrage opportunity as a consequence of the above prices.

Problem 5.5. (5 points) Calculate the price of a long butterfly spread constructed using the following call options:
(1) a £3,925−strike call on the FTSE100 index which is being sold for £713.07;
(2) a £4,325−strike call on the FTSE100 index which is being sold for £496.46;
(3) a £4,725−strike call on the FTSE100 index which is being sold for £333.96.
Assume that the total number of the call options in your portfolio equals 4.
(a) £54.11
(b) £550.57
(c) £554.11
(d) £559.57
(e) None of the above

5.2. Collars. Please, provide your final answer only for the following problem(s):

Problem 5.6. (5 points) Please, solve the Sample MFE (Intro) Problem #59.

Problem 5.7. (5 points) Please, solve the Sample MFE (Intro) Problem #60.

Problem 5.8. (5 points) The future value in one year of the total costs of manufacturing a widget is $200. You will sell a widget in one year at its market price of \( S(1) \).
Assume that the annual effective interest rate equals 5%, and that the current price of the widget equals $230.
You now purchase a one-year, $220-strike put on one widget for a premium of $7. You sell some of the potential gain by writing a one-year, $250-strike call on one widget for a $2 premium.
What is the range of the profit of your hedged portfolio?
(a) [14.75, 44.75]
(b) [220.75, 250.75]
(c) [220, 244.75]
(d) [220, 250]
(e) None of the above.
5.3. Ratio spreads. Please, provide your final answer only for the following problem(s):

**Problem 5.9.** (5 points)
A strategy consists of longing a put on the market index with a strike of $830 and shorting a call option on the market index with a strike price of $830. The put premium is $18.00 and the call premium is $44.00. Interest rates are 0.5% effective per month. Determine the net profit if the index price at expiration is $830 (in 6 months).

(a) $0
(b) $23.67 loss
(c) $26.79 gain
(d) $28.50 gain
(e) None of the above.

**Problem 5.10.** (5 points) Solve Sample MFE (Intro) Problem #17.