7.1. **Two binomial periods: American options.** Please, provide your complete solutions to the following problems:

**Problem 7.1.** (10 points) Find the current price of a one-year, $110-strike American put option on a non-dividend-paying stock whose current price is \( S(0) = 100 \). Assume that the continuously compounded interest rate equals \( r = 0.06 \).

Use a two-period binomial tree with \( u = 1.23 \), and \( d = 0.86 \) to calculate the price \( V_P(0) \) of the put option.

**Problem 7.2.** (10 points) Solve problem #49 (p.116) from the Sample MFE Problems.

**Problem 7.3.** (20 points) Consider a two-period binomial model for a non-dividend paying asset \( S \) with \( S(0) = 50 \) and \( u = 1/d = 2 \). Let \( i = 0.25 \) denote the effective interest rate per period. You need to price a European put option on \( S \) which expires at the end of the two periods and has the strike \( K = 70 \).

(i) (10 pts) Find the values of the given option at all the nodes in the binomial tree. In particular, find the no-arbitrage price at time 0 of this option.

(ii) (8 pts) Find the number of shares \( \Delta \) one needs to invest in at every node in the tree in order to replicate the option.

(iii) (2 pts) If the option were American, would there be early exercise?

**Problem 7.4.** (10 points)

Consider a one-period forward binomial tree with \( h = 1 \), \( S(0) = 100 \), \( r = 0.08 \), \( \sigma = 0.3 \), \( \delta = 0.08 \).

(a) (5 pts) Find the expression \( V^A_C(0, K) \) for the time=0 price of the American call option on \( S \) with strike \( K \) and maturity at the end of the period.

(b) (3 pts) Determine the condition for the strike \( K \) to be such that early exercise occurs?

(c) (2 pts) In particular, is there early exercise for \( K = 70 \)?