3.1. **Put-call parity.** Provide your **final solution only** to the following problem(s). Each problem will be worth 2 points.

**Problem 3.1.** A company forecasts to pay dividends of $0.90, $1.20 and $1.45 in 3, 6 and 9 months from now, respectively. Given that the interest rate is \( r = 5.5\% \), how much dollar impact will dividends have on prices of 9–month options? More precisely, what is the present value of the projected dividend payments?

(a) $3.45  
(b) $3.90  
(c) $4.22  
(d) $4.50  
(e) None of the above.

**Solution:** (a)

The present value of the discrete dividends paid over the next 9 months per the above schedule is

\[
P_{0.3/4}(Div) = e^{-0.055\cdot0.25} \cdot 0.90 + e^{-0.055\cdot0.5} \cdot 1.20 + e^{-0.055\cdot0.75} \cdot 1.45 = 3.45.
\]

**Problem 3.2.** A certain common stock is priced at $42.00 per share. Assume that the continuously compounded interest rate is \( r = 10.00\% \) per annum. Consider a $50–strike European call, maturing in 3 years which currently sells for $10.80. What is the price of the corresponding 3–year, $50–strike European put option?

(a) $5.20  
(b) $5.69  
(c) $5.04  
(d) $5.84  
(e) None of the above.

**Solution:** (d)

Due to put-call parity, we must have

\[
V_P(0) = V_C(0) + e^{-rT}K - S_0 + PV_{0,T}(Div) \\
= 10.80 + e^{-0.30} \cdot 50 - 42.00 \approx 5.84.
\]

**Problem 3.3.** A certain common stock is priced at $99.00 per share and pays a continuous dividend yield of 2% per annum. Consider a $100–strike European call and put, maturing in 9 months which currently sell for $11.71 and $5.31. Let the continuously compounded risk-free interest rate be denoted by \( r \). Then,

(a) \( 0 \leq r < 0.05 \)  
(b) \( 0.05 \leq r < 0.10 \)  
(c) \( 0.10 \leq r < 0.15 \)  
(d) \( 0.15 \leq r < 0.20 \)  
(e) None of the above.

**Solution:** (c)

By the put-call parity, in our usual notation:

\[
V_C(0) = V_P(0) + e^{-\delta T}S(0) - e^{-rT}K.
\]
So,
\[ r = -\frac{1}{T} \ln \left[ \frac{1}{K} (V^R(0) + e^{-\delta T} S(0) - V_C(0)) \right] = -\frac{1}{0.75} \ln \left[ \frac{1}{100} (5.31 + e^{-0.02 \cdot 0.75} \cdot 99 - 11.71) \right] \approx 0.124. \]

**Problem 3.4.** The initial price of a non-dividend-paying stock is $55 per share. A 6-month, at-the-money call option is trading for $1.89. Let the interest rate be \( r = 0.065 \). Find the price of the European put with the same strike, expiration and the underlying asset.

(a) $0.05  
(b) $0.13  
(c) $0.56  
(d) $0.88  
(e) None of the above

**Solution:** (b)

Using put-call parity, we get
\[ V_P(0) = V_C(0) + K e^{-rT} - F_{0,T}^P(S) = 1.89 + 55(e^{-0.065 \times 0.5} - 1) = 0.1312. \]
Problem 3.5. A stock currently sells for $32.00. A 6-month European call option with strike $35.00 has a premium of $2.27. Assuming a 6% continuous dividend yield and the continuously compounded, risk-free interest rate of 4%, what is the price of the otherwise identical put option as dictated by put-call parity?

- (a) $5.05
- (b) $5.13
- (c) $5.52
- (d) $5.88
- (e) None of the above

Solution: (c)

We have

\[ V_P(0) = V_C(0) - e^{-\delta T} S(0) + K e^{-rT} \]

\[ \Leftrightarrow V_P(0) = 2.27 - 32e^{-0.06 \times 0.5} + 35e^{-0.04 \times 0.5} = 5.523. \]

Problem 3.6. A stock currently sells for $32.00. A 6-month European call option with a strike of $30.00 has a premium of $4.29, and the otherwise identical put has a premium of $2.64. Assume a 4% continuously compounded, risk-free rate. What the net present value of the dividends payable over the next 6 months?

- (a) $0.05
- (b) $0.13
- (c) $0.52
- (d) $0.94
- (e) None of the above

Solution: (d)

This problem requires the application of put-call parity. We have:

\[ S(0) - V_C(0) + V_P(0) - 30e^{-rT} = NPV(\text{dividends}) \Rightarrow NPV(\text{dividends}) = 32 - 4.29 + 2.64 - 29.406 = 0.944. \]

Problem 3.7. Source: Problem #2 from the Sample FM(DM) questions.

You are given the following information:

1. The current price to buy one share of XYZ stock is 500.
2. The stock does not pay dividends.
3. The risk-free interest rate, compounded continuously, is 6%.
4. A European call option on one share of XYZ stock with a strike price of $K$ that expires in one year costs $66.59.
5. A European put option on one share of XYZ stock with a strike price of $K$ that expires in one year costs $18.64.

Determine the strike price $K$.

- (a) $449
- (b) $452
- (c) $480
- (d) $559
- (e) None of the above

Solution: (c)
3.2. **Replicating portfolios.** Provide your complete solution to the following problem:

**Problem 3.8.** (5 points) Complete the following definition:
We say that a portfolio is a *replicating portfolio* for a certain European-style derivative security if: . . .

**Solution:** the payoff of the portfolio equals the payoff of the derivative security in all states of the world.
Provide your final answer only to the following problems. Each problem will be worth 2 points.

**Problem 3.9.** Denote the continuously compounded interest rate by \( r \). Let \( V_{CC}(0) \) denote the price of a cash call on the asset \( S \) with strike \( K \) and exercise date \( T \). Let \( V_{CP}(0) \) denote the price of a cash put on the asset \( S \) with strike \( K \) and exercise date \( T \). Then,

\[
V_{CC}(0) + V_{CP}(0) =
\]

(a) \( e^{-rT} \)
(b) 1
(c) \( e^{rT} \)
(d) \( F_{0,T}^P(S) \)
(e) None of the above

**Solution:** (a)

**Problem 3.10.** Denote the continuously compounded interest rate by \( r \). Let \( V_{AC}(0) \) denote the price of an asset call on the asset \( S \) with strike \( K \) and exercise date \( T \). Let \( V_{AP}(0) \) denote the price of an asset put on the asset \( S \) with strike \( K \) and exercise date \( T \). Then, regardless of whether \( S \) pays dividends or not,

\[
V_{AC}(0) + V_{AP}(0) =
\]

(a) \( Ke^{-rT} \)
(b) \( S(0) \)
(c) \( F_{0,T}(S) \)
(d) \( F_{0,T}^P(S) \)
(e) None of the above

**Solution:** (d)

**Problem 3.11.** Which of the following statements does **NOT** accurately reflect the relationship between various derivative securities and “synthetic” forward contracts?

(a) Forward = stock – zero-coupon bond
(b) Zero-coupon bond = stock – forward
(c) Prepaid forward = forward – zero-coupon bond
(d) Stock = forward + zero-coupon bond
(e) All of the above are accurate.

**Solution:** (c)

3.3. **Currency options.** Provide your complete solution to the following problem(s):

**Problem 3.12.** (5 points) Suppose that the exchange rate is 0.95 USD per euro, and that the euro-denominated continuously compounded interest rate is 4%, while the dollar-denominated continuously compounded interest rate is 6%. The price of a 1-year 0.93-strike European call on the euro is $0.0571. What is the price of the corresponding European put?

**Solution:** Note: See Problem 9.4. in McDonald!
We can make use of the put-call-parity for currency options:

\[
V_{P}(0) = -e^{-r_T}x(0) + V_{C}(0) + e^{-r_T}K
\]

\[
\iff
V_{P}(0) = -e^{-0.04} \times 0.95 + 0.0571 + e^{-0.06} \times 0.93 = -0.91275 + 0.0571 + 0.87584 = 0.0202.
\]

A $0.93 strike European put option has a value of $0.0202.

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**Instructor:** Milica Ćudina
Problem 3.13. (8 points) The price of a 6–month dollar denominated call option on the euro with a $0.90 strike is $0.0404. The price of an otherwise equivalent put option is $0.0141. Assume that for the dollar we have $r = 5\%$.

a. (5 pts) What is the 6–month dollar-euro forward price?

b. (3 pts) If the euro-denominated annual continuously compounded interest rate is 3.5\%, what is the spot exchange rate?

Solution:

a. We can use put-call-parity to determine the forward price:

$$V_C(0) - V_P(0) = e^{-rT}F_{0,T}(x) - Ke^{-rT}$$

$$\Leftrightarrow F_{0,T}(x) = e^{rT}[V_C(0) - V_P(0) + Ke^{-rT}] = e^{0.05\times0.5}[0.0404 - 0.0141 + 0.9e^{-0.05\times0.5}]$$

$$\Leftrightarrow F_{0,T}(x) = 0.92697.$$  

b. Given the forward price from above and the pricing formula for the forward price, we can find the current spot rate:

$$F_{0,T}(x) = x(0)e^{(r-r_f)T} \Leftrightarrow x_0 = F_{0,T}(x)e^{-(r-r_f)T} = 0.92697e^{-(0.05-0.035)0.5} = 0.92.$$  

Problem 3.14. (5 points) Assume that the current exchange rate is $1.3$ per euro. The continuously compounded interest rate for the euro is 0.03, while continuously compounded interest rate for the USD is 0.04.

Let the price of an at-the-money USD-denominated European call on on the euro with exercise date in 6 months be equal to 0.053

What is the price of an at-the-money Euro-denominated put on the USD with the exercise date in 6 months?

Solution: Let $x$ denote the exchange rate from euros to dollars. We are given that $x(0) = 1.3$. Using the put-call symmetry/duality for options on currencies, we get

$$V_{P}^{Euro}(0,1/x(0)) = (1/x(0)^2V_{C}^{USD}(0,x(0)) \approx 0.031.$$  

3.4. Chooser options. Provide your final answer only for the following problems.

Problem 3.15. (2 points) The initial price of a chooser option is greater than or equal to the price of a regular European call on the same asset with the same strike and exercise date. True or false?

Solution: TRUE

Problem 3.16. (5 points) Consider a chooser option on a stock $S$ whose current price is $100$ per share. Assume that we are using our usual notation, i.e., let

$$V_{CH}(0,t^*,T,K)$$

denote the time–0 price of a chooser option with choice date $t^*$, exercise date $T$ and strike price $K$. Then, the following inequality holds:

(a) $V_{CH}(0,t^*,T,K) \leq V_P(0,T,K)$

(b) $V_{CH}(0,t^*,T,K) \leq V_C(0,T,K)$

(c) $\max(V_P(0,T,K),V_C(0,T,K)) \leq V_{CH}(0,t^*,T,K)$

(d) $V_{CH}(0,t^*,T,K) < \max(V_P(0,T,K),V_C(0,T,K))$

(e) None of the above

Solution: (c)