Foreign currencies.

01/24/2018.

At $t=0$: • Buy 1 unit of the foreign currency (FC).

$x(t)$, $t \geq 0$ ... exchange rate from FC to the DOMESTIC CURRENCY (DC); i.e., at time-t we pay $x(t)$ to get 1 unit of FC

$\Rightarrow$ • We spend $x(0)$.

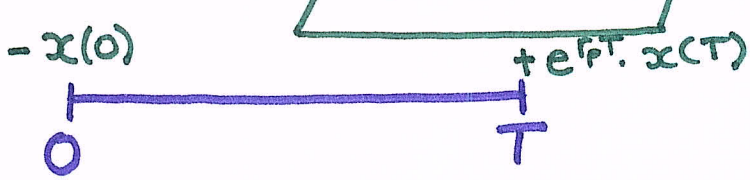
• We deposit the 1 unit of FC to earn interest in a savings account.

r_F ... the continuously compounded, risk-free interest rate for FC

At $t=T$: • Withdraw the balance from the FC account; get $e^{r_F \cdot T}$ units of FC.

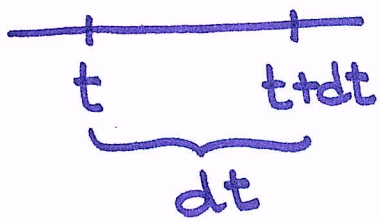
• Exchanging back to the DC, we get

$$\boxed{e^{r_F \cdot T} \cdot x(T)}$$



Continuous-dividend-paying stocks.

- $S(t)$, $t \geq 0$ ... stock price @ time-t
- δ... dividend yield



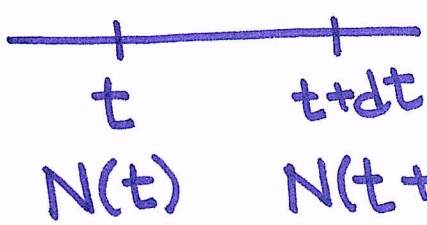
The shareholders receive $\delta S(t)\,dt$ in dividends for the time interval $(t, t+dt)$ <u>per share</u> owned!

<u>Convention</u>: With continuous dividends, ALL the dividends are reinvested in the SAME ASSET, IMMEDIATELY and CONTINUOUSLY!

Set:

<u>$N(t), t \geq 0$</u>... the # of shares owned @ time-t

$N(0) = n_0$ ... the # of shares purchased initially



$$dN(t) = N(t+dt) - N(t)$$

$\dfrac{dN(t)}{dt}$ ... the rate of change ?

The total dividend amt paid for $(t, t+dt)$ is:

$$N(t) \times \underbrace{\delta \cdot S(t)\,dt}_{\text{per share}}$$

$\Rightarrow$ The # of shares which are bought for :

$$\frac{N(t) \cdot \delta \cdot \cancel{S(t)}\,dt}{\cancel{S(t)}} = \delta \cdot N(t)\,dt$$

$$\Rightarrow \quad dN(t) = \delta \cdot N(t)\,dt$$

$$\Rightarrow \quad \frac{dN(t)}{dt} = \delta N(t)$$

$$\Rightarrow \quad \frac{dN(t)}{N(t)} = \delta\,dt \qquad \Big| \int$$

$$\Rightarrow \quad d\left[\ln(N(t))\right] = d\left[\delta \cdot t\right]$$

$$\Rightarrow \quad \ln(N(t)) = \delta t + \underbrace{C}_{\text{constant}}$$

$$\Rightarrow \quad N(t) = e^{\delta t} \cdot e^{C}$$

$$N(t) = C_0 e^{\delta t}$$

Taking into account the initial condition:

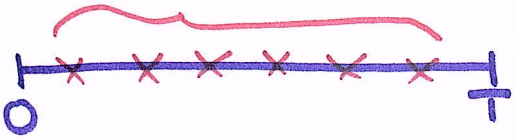
$$N(t) = n_0 e^{\delta t}$$

$$-n_0 \cdot S(0) \qquad\qquad +n_0 e^{\delta T} \cdot S(T)$$

$$0 \qquad\qquad T$$

Static Portfolios.

No intermediate cashflows!



"TODAY" — TIME HORIZON

INITIAL COST

The time-0 cashflow from the INVESTOR'S perspective.

THE PAYOFF

The time-T cashflow from the INVESTOR'S perspective

THE PROFIT

$$\text{THE PROFIT} = \underbrace{\text{THE PAYOFF} - FV_{0,T}(\text{INITIAL COST})}$$

IF PROFIT >0 THEN $\boxed{\text{GAIN}}$.

IF PROFIT $=0$ THEN BREAK EVEN.

IF PROFIT <0 THEN $\boxed{\text{LOSS}}$.

Example. Austin buys a zero-coupon bond redeemable @ time-T for \$1 for a price denoted by $P(0,T)$.

$\Rightarrow$ Initial Cost : $P(0,T)$

PAYOFF : 1

$$\Rightarrow \text{PROFIT} = 1 - \overbrace{FV_{0,T}(P(0,T))}^{=1} = 0$$

Example. You have \$300 to invest in a market index worth \$100 per unit.
The dividend yield is 0.02.
How many units of the index will you own in six months?

$\rightarrow$: $n_0 = 3$

$$N(1/2) = n_0 e^{\delta \cdot (1/2)} = 3e^{0.01} = \dots$$

$\blacksquare$

Example. Same $\delta = 0.02$ as above.
How many units would you have to buy today to ensure that you own EXACTLY 1 unit in a quarter year?

$\rightarrow$: $N(1/4) = 1 = n_0 e^{\delta(1/4)} = n_0 e^{0.005}$

$$\Rightarrow \quad n_0 = e^{-0.005}$$

$\blacksquare$