Foreign currencies.

At $t = 0$:
- Buy 1 unit of the foreign currency (FC).
- $x(t), t \geq 0$... exchange rate from FC to the DOMESTIC CURRENCY (DC); i.e., at time-$t$ we pay $x(t)$ to get 1 unit of FC

=>
- We spend $x(0)$.
- We deposit the 1 unit of FC to earn interest in a savings account.

$r_F$... the continuously compounded, risk-free interest rate for FC

At $t = T$:
- Withdraw the balance from the FC account; get $e^{r_F T} x(T)$ units of FC.
- Exchanging back to the DC, we get $-x(0) + e^{r_F T} x(T)$.
Continuous-dividend-paying stocks.

- $S(t), t \geq 0$ ... stock price at time $t$
- $\delta$ ... dividend yield

\[ \frac{t}{t+dt} \]

The shareholders receive $\delta S(t) \, dt$ in dividends for the time interval $(t, t+dt)$ per share owned!

**Convention:** With continuous dividends, **ALL** the dividends are reinvested in the SAME ASSET, IMMEDIATELY and CONTINUOUSLY!

Set:

$N(t), t \geq 0$ ... the # of shares owned at time $t$

$N(0) = n_0$ ... the # of shares purchased initially

\[ \frac{N(t)}{t+dt} \]

$dN(t) = N(t+dt) - N(t)$

\[ \frac{dN(t)}{dt} \] ... the rate of change ??
The total dividend amount paid for \((t, t+dt)\) is:

\[
\frac{N(t) \cdot S \cdot S(t) \, dt}{S(t)} \text{ per share}
\]

\[= \left( \frac{N(t) \cdot S \cdot S(t) \, dt}{S(t)} \right) \text{ per share} \]

\[= \text{The # of shares which are bought for} \]

\[
\frac{N(t) \cdot S \cdot S(t) \, dt}{S(t)} = S \cdot N(t) \, dt
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow dN(t) = S \cdot N(t) \, dt
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow \frac{dN(t)}{dt} = SN(t)
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow \frac{dN(t)}{N(t)} = S \, dt
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow d \left[ \ln (N(t)) \right] = d \left[ S \cdot t \right]
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow \ln (N(t)) = St + C
\]

\[= \text{The # of shares which are bought for} \]

\[\Rightarrow N(t) = e^{St} \cdot e^{C}
\]

\[= \text{The # of shares which are bought for} \]

\[N(t) = C_0 e^{St}
\]

Taking into account the initial condition:

\[N(t) = n_0 e^{S t}
\]

\[-n_0 \cdot S(0) + n_0 e^{S T} \cdot S(T)
\]

\[N(t) = \frac{n_0 e^{ST} - n_0 \cdot S(0)}{T}
\]
Static Portfolios

No intermediate cashflows!

"TODAY"

INITIAL COST

The time-0 cashflow from the INVESTOR'S perspective.

TIME HORIZON

THE PAYOFF

The time-T cashflow from the INVESTOR'S perspective.

THE PROFIT

\[ \text{THE PAYOFF} - FV_{0,T}(\text{INITIAL COST}) \]

IF \( \text{PROFIT} > 0 \) THEN \[ \text{GAIN}. \]

IF \( \text{PROFIT} = 0 \) THEN \[ \text{BREAK EVEN}. \]

IF \( \text{PROFIT} < 0 \) THEN \[ \text{LOSS}. \]

Example. Austin buys a zero-coupon bond redeemable @ time-T for $1 for a price denoted by \( P(0,T) \).

\[ \Rightarrow \text{Initial Cost} : P(0,T) \]

\[ \Rightarrow \text{PAYOFF} : 1 \]

\[ \Rightarrow \text{PROFIT} = 1 - \frac{FV_{0,T}(P(0,T))}{1} = 0 \]
Example. You have $300 to invest in a market index worth $100 per unit. The dividend yield is 0.02. How many units of the index will you own in six months?

\[ n_0 = 3 \]

\[ N(\frac{1}{2}) = n_0 e^{8 \cdot \frac{1}{2}} = 3e^{0.04} = ... \]

Example. Same \( s = 0.02 \) as above. How many units would you have to buy today to ensure that you own EXACTLY 1 unit in a quarter year?

\[ \Rightarrow N(\frac{1}{4}) = 1 = n_0 e^{8 \cdot \frac{1}{4}} = n_0 e^{0.005} \]

\[ \Rightarrow n_0 = e^{-0.005} \]