Review: **STATIC PORTFOLIOS**

**NO INTERMEDIATE CASHFLOWS!**

**Initial Cost**  **Payoff**

**PROFIT** = **PAYOFF** - **FV}_0,T\) (INITIAL COST)

**Example:** TAKING A SIMPLE LOAN.

L... the loan amount
T... the loan term
r... continuously compounded, risk-free i.r.

=> **Initial Cost** = -L
=> **Payoff** = -Le^rT

=> **Profit** = -Le^rT + FV}_0,T\) (+L)

= -Le^rT + Le^rT = 0.\*
Example. THE OUTRIGHT PURCHASE OF A NON-DIVIDEND-PAYING STOCK

\[ S(T) \ldots \text{final asset price is a random variable} \]

\[
\text{the timeline: } \quad -S(0) \quad \bigcirc \quad S(T) \quad \text{the cashflows}
\]

\[ \Rightarrow \quad \text{Initial Cost: } S(0) \]

\[ \text{Payoff: } S(T) \quad \text{random variable} \]

\[ \text{Introduce: } \quad S \ldots \text{an independent argument} \]

\[ \ldots \text{stands for the FINAL ASSET PRICE} \]

\[ \ldots \text{“placeholder” for the random variable } S(T) \]

We can define the payoff function which describes the dependence of the investor’s payoff on \( S \leftrightarrow S(T) \).

\[ \text{Notation: } v \ldots \text{payoff function} \]

\[ \Rightarrow v(s) \ldots \text{the investor’s payoff if the final asset price equals } s \]
In the present example:

The Payoff function: \( v(s) = s \),

When we graph the payoff function, we get the Payoff DIAGRAM/CURVE:

\[
\text{Payoff} \quad \uparrow
\]

\[
\downarrow \quad s(\text{final asset price})
\]

Profit = Payoff - FV_{0T} (Initial Cost)

\[= S(T) - FV_{0T} (S(0))\]

\[= S(T) - \frac{S(0)e^{rT}}{\text{constant}}\]

Introduce the profit function:

\[v(s) = \frac{FV_{0T} (\text{Init. Cost})}{\text{Payoff}}\]

\[\Rightarrow \text{In this example, the profit function:}\]

\[s - S(0)e^{rT}\]
The profit diagram/curve:

No upper bound, i.e., unlimited growth potential.

Break Even: \[ S^* = S(0) e^{rT} \]

- \( S(0) e^{rT} \) MAXIMAL LOSS

IF PAYOFF/PROFIT is increasing (not necessarily strictly) as a function of the final asset price \( S \), we say the portfolio is long with respect to the underlying (asset).

Example: OUTRIGHT PURCHASE OF ONE SHARE OF A CONTINUOUS-DIVIDEND-PAYING STOCK W/ DIVIDEND YIELD \( s \)

- Initial Cost: \( S(0) \)

Payoff: \( e^{st} \cdot S(T) \)

the # of shares owned @ time-\( T \)

\[ v(S) = S \cdot e^{st} \]
\[ \text{Profit: } S(T) e^{rT} - S(0) e^{rT} \]

\[ a \text{ constant} \]

\[ j^* \]

The profit function is

\[ j^* = \frac{S(0) e^{rT} - S(T)}{e^{rT}} \]

\[ \text{independent argument} \]

\[ \text{break-even pt} \]

is: \[ j^* = ? \]

Solve for \( j \):

\[ j e^{rT} - S(0) e^{rT} = 0 \]

\[ \Rightarrow j e^{rT} = S(0) e^{rT} \]

\[ \Rightarrow j^* = S(0) e^{(r-S)T} \]

Try to remember this expression!