11.1. **Construction.** So far, we have looked at put-call parity for non-dividend-paying assets. Now, we will use a similar approach to obtain put-call parity for stocks that pay either discrete dividends, or a continuous dividend stream.

Let Portfolio $A$ consist of a long European call and a short European put on the same underlying asset $S$ with the same strike $K$ and the same exercise date $T$. The initial value of this portfolio is

$$V_A(0) = V_C(0) - V_P(0).$$

There are no intermediate cash-flows associated with this portfolio and its payoff at time $T$ is

$$V_C(T) - V_P(T) = S(T) - K.$$

On the other hand, let Portfolio $B$ consist of the following:

1. a **long** prepaid forward contract on $S$ for delivery at time $T$,
2. **borrowing** the present value of the strike price to be repaid at time $T$.

Then, the initial cost of this portfolio equals:

$$F_{0,T}^P(S) - PV_{0,T}(K).$$

Since there are no intermediate cash-flows associated with this portfolio, either, its payoff at time $T$ is

$$S(T) - K.$$

Since the above portfolios have the same final payoff, by the no-arbitrage principle, we conclude that their initial values must also be the same. We get the more general version of put-call parity:

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - PV_{0,T}(K).$$

11.2. **Special cases.** Our most common setting is the one with a continuously compounded interest rate $r$. In that case the put-call parity reads as

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - Ke^{-rT}.$$
• continuous dividends at the rate $\delta$:

$$V_C(0) - V_P(0) = S(0)e^{-\delta T} - Ke^{-rT}$$

**Problem 11.1. MFE Exam Spring 2007: Problem #1**

On April 30, 2007, a common stock is priced at $52.00. You are given that:

1. Dividends in equal amounts are to be paid on June 30, 2007, and on September 30, 2007.
2. A European call on the above stock with strike $K = 50$ and the exercise date in six months sells for $4.50$.
3. A European put on the above stock with strike $K = 50$ and the exercise date in six months sells for $2.45$.
4. The continuously-compounded risk-free interest rate equals 0.06.

Calculate the amount of each dividend.

**Solution:** In addition to our usual notation, we introduce $D$ to stand for the amount of each dividend payment. Then, the put-call parity reads as

$$V_C(0) - V_P(0) = S(0) - De^{-rt_1} - De^{-rt_2} - Ke^{-rT}$$

with $t_1 = 1/6$ and $t_2 = 5/12$. Solving for $D$ above, we get

$$D = \frac{S(0) - Ke^{-rT} - V_C(0) + V_P(0)}{e^{-rt_1} + e^{-rt_2}} = \frac{52 - 50e^{-0.06(1/2)} - 4.5 + 2.45}{e^{-0.06(1/6)} + e^{-0.06(5/12)}} \approx 0.73.$$