Basic risk management. An introduction to forward contracts.

5.1. Basic risk management.

Definition 5.1. We say that a portfolio is long with respect to a certain asset if its payoff function is increasing as a function of that asset’s final price. We say that a portfolio is short with respect to a certain asset if its payoff function is decreasing as a function of that asset’s final price.

Remark 5.2. Neither the increase nor the decrease above are in the strict sense.

Producers or manufacturers of goods and services (commodities) are going to be profitable if the revenue from the sale of produced goods in the market exceeds the manufacturing costs. This makes the producers have an inherent long position in their product. On the other hand, the users or buyers of commodities benefit from decrease in the price of the good; they have an inherent short position with respect to that commodity. It is oftentimes not enough to optimize the manufacturing process and/or rely on classical insurance to keep a company profitable. This is where financial hedging comes in.

5.2. Forward contracts. A forward contract is a binding agreement to buy/sell an underlying asset on an agreed-upon date in the future – the delivery date – at a fixed price set today. The forward contract specifies the following:

- the asset to be delivered,
- the quantity of the asset to be delivered,
- the delivery logistics: the time, date, place, e.g.,
- the price the buyer will pay at the time of delivery: the forward price,
- type of settlement: cash settlement or physical delivery (the former is preferable due to simplicity and lower transaction costs).

The person whose position is the long forward will be required to “buy” the underlying asset for the forward price. So, his/her payoff function is

\[ v(s) = s - F, \]

where \( F \) denotes the forward price. So, with \( S(T) \) denoting the final price of the underlying asset, the realized payoff for a long forward contract equals

\[ S(T) - F. \]

Analogously, if one has the short forward, one’s payoff is

\[ F - S(T). \]

The payoff graphs for both positions are displayed below. The blue line is the long forward payoff, and the red line is the short forward payoff. The forward price is set to be \( F = 1000 \).
5.2.1. *Forward contract versus outright purchase.* The natural question to ask at this point is:

“What is the difference between the outright purchase of the underlying asset and the long forward contract on the same asset?

First, let us go back to the way that the forward contract is agreed upon by the two parties. At no point in the discussion above did we mention the **initial cost** of the forward contract. The reason is simple – **there is no initial cost.** For a forward contract, the only action that happens at time $-0$ is the agreement reached which includes the stipulations in the forward contract. However, there are no cash-flows taking place at time $-0$. In particular, this means that the profit and the payoff curves **coincide** in the case of the forward contract.

Second, let us focus on the forward price $F$ briefly. Imagine that the party with the log forward contract wants to be certain that he/she has sufficient funds at the delivery date $T$ to pay the forward price. One way of ensuring this would be an investment at time $-0$ in a zero-coupon bond with maturity date $T$ and redemption amount $F$. We can just think about this step as if he/she simply made a deposit into a savings account today and did nothing until the delivery date – then he/she withdraws the balance and uses it to purchase the underlying asset. With the purchase of the bond coupled with the long forward contract, one creates a simple *portfolio*. Let’s try to see what the payoff curve for this portfolio is. Again, let $F = 1000$ for the sake of being able to have a concrete graph.

The blue line is the payoff of the long forward contract, and the red line is the payoff of the bond. The payoff of the entire portfolio is the **sum** of the payoffs of its components – the green line. If the green line reminds you of something, this is not accidental. In fact,
we encountered the same payoff curve in the previous lecture. It corresponded to the long stock position – the simple outright purchase of a share of stock.

We have to emphasize that looking at the profit curve is not entirely straightforward. We have to take into account that the owner of the stock is entitled to any dividend payments. On the other hand, the holder of the forward contract does not receive dividend payments. We will return to these issues soon when we discuss the ways to come up with a fair forward price.

5.2.2. Credit risk. The term credit risk signifies the risk associated with the possibility that one of the counterparties in the contract (any contract – not just a forward contract) might fail to fulfill its financial obligations. To alleviate this risk and provide a sense of security needed to conduct the trades in the first place several steps might be taken. One example is “standardization” – certain types of contracts are popular/useful enough to become exchange-traded. There are two major consequences of this step: the contracts get very detailed and standard (collateral requirements are introduced, e.g.), and the exchange guarantees the transactions in conducts. However, not all contracts are traded frequently enough to warrant the above approach. The blanket term for the contracts that are not exchange traded is over-the-counter (OTC) contracts. In their context, remedies for credit risk may include:

- credit checks,
- collateral,
- bank letter of credit, ...


Suppose stock XYZ has a current price of 100. The forward price for delivery of this stock in 1 year is 110. Also, assume there are no dividends, and that the annual effective interest rate is 10%, unless otherwise indicated (in the sequel).

Which of the following statements is FALSE?

A. The time−1 profit diagram and the time−1 payoff diagram for long positions in this forward contract are identical.
B. The time−1 profit for a long position in this forward contract is exactly opposite to the time−1 profit for the corresponding short forward position.
C. There is no comparative advantage to investing in the stock versus investing in the forward contract.
D. If the 10% interest rate were continuously compounded instead of annual effective, then it would be more beneficial to invest in the stock, rather than the forward contract.
E. If there was a dividend of 3.00 paid 6 months from now, then it would be more beneficial to invest in the stock, rather than the forward contract.

Solution:

A. TRUE
   There is no initial cost to entering a forward contract, so its payoff and profit are always equal.
B. TRUE
   This statement is always true for long and short positions in the same contract.
C. TRUE
   The profits are equal.
D. FALSE
The profit of the stock investor is always by $100e^{1.1} - 100 > 0$ below the profit of the forward-contract investor.

E. TRUE
Without any dividend payments, the two positions are equivalent as we saw in our analysis of the offered choice C. The dividend payment is given to the stock investor and not to the forward investor, so the stock investor is certainly better off.

5.2.3. Recommended problems. McDonald: #2.7, #2.8, #2.9.

5.3. Hedging with forwards.

5.3.1. Selling forward. The following graph shows the payoff curves for the producer’s position if (s)he decided to hedge by selling forward, i.e., entering a short forward contract.

![Payoff Curves](image)

The blue line corresponds to the producer’s inherent long position in the good (s)he produces; this is the unhedged position. The red line is the payoff curve for the short forward contract (with the forward price of 500 chosen arbitrarily for the purpose of being able to produce a graph). The forward contract is the hedge. The sum of the two payoffs is the payoff of the producer’s entire hedged portfolio. It is the gold line in the figure above. As we can see, the producer completely removed the randomness from his/her portfolio. The payoff of the hedged portfolio is “flat” at the forward price. The profit of the hedged portfolio is, hence, constant at the difference between the forward price and the total (fixed and variable) costs.

5.3.2. Buying forward. As we have already noted above, the buyer/user of goods/services has an inherent short position in that particular commodity.

Example 5.3. Pancakes, Inc. produces a variety of pancake products. It longed forward contract on 100 lbs of blueberries at $2.00 per pound. Revenue is $1,000 for the pancakes produced with the above blueberries. Other costs total $700. Find the profit.
Solution:

\[ 1000 - 200 - 700 = 100 \]

The following graph shows the profit curves for the buyer’s position if (s)he decideds to hedge by entering a long forward contract.

![Graph showing profit curves](image)

The blue line is the profit curve based on the fixed revenue (reduced by additional total costs) of 700; this is the unhedged position. The red line is the payoff/profit curve for the long forward contract (with the forward price of 500). The forward contract is the hedge. The sum of the two profits is the profit of the buyer’s entire hedged portfolio. It is the gold line in the figure above. Again, there is nothing random about the buyer’s hedged portfolio’s profit.

*Remark 5.4.* An asset-investor with a long/short position in the underlying also hedges using a short/long forward contract. The entire payoff/profit structure is completely analogous to the situations discussed above.