1.1. **Atoms and dynamics.** Before diving into the description of elementary ingredients and relationships in financial mathematics, let us turn to the structure of other (older?) disciplines.

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<td>Chemistry</td>
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</tbody>
</table>

How does financial mathematics fit into this structure?

1.2. **Role and history of financial markets.** Plainly speaking, financial markets were originally designed to bring together lenders and borrowers. Lenders might come under the guise of traditional lenders you encountered in theory of interest, but stockholders in a company might also be understood as lenders – they are offering monetary funds in return for a share in a company (and the right to its potential profits). The entities traded in a financial market are referred to as financial assets. One can read more about the particulars in Articles 19 and 20 of [1].

However, as we will see, financial markets evolved to also alleviate individual investor’s financial risks stemming from changes in prices of goods and services. As the structure of our communities became more sophisticated, financial markets followed suit.


Note that – even though its aim is to spread financial risk among a number of individuals (policy holders) – classical insurance applies to types of risk not directly associated with movement of asset prices in financial markets. It focuses on specific types of financial risks (for example, property, casualty, health, dental, personal umbrella, . . . ). We will revisit the parallels between classical insurance and financial instruments at a later time.

For now, the point of view of classical insurance will help us understand the difference between diversifiable and non-diversifiable risks in financial markets. Consider homeowner’s insurance – an insurance policy which compensates the property owner for the reduction of the value of his/her property due to a specific set of circumstances stipulated in the insurance policy (e.g., fire, theft, hurricane, earthquake, . . . ). The insurance company will address the problem of offsetting its possible liabilities in case of fire or theft in a different way that it would handle the liabilities due to a hurricane or earthquake.

One might guess that the reason is simply the amount of loss in either case. However, there is a more profound difference. While it is unlikely that a group of homeowners is going
to suffer a loss due to theft in the same insurance cycle and in the same area, a natural
disaster creates simultaneous losses across a specific area. In the former case, the premiums
accumulated from the policy holders who did not experience a loss during an insurance cycle
will contribute to the benefit payments to the policy holders who did. In the latter case, this
effect is not present, since the overwhelming majority of policy holder’s suffered a loss. One
might say that the insurance company was able to diversify the first (individual) type of risk
by insurance a whole pool of people. This was not a possibility for the second (system-wide)
type of risk.

**Problem 1.1.** Try to come up with your own example of diversifiable and non-diversifiable
risks in classical insurance.

In finance and economics, the notion of diversifiable risk is associated with the financial
risk attributable to a particular asset. One can alleviate that risk by investing in multiple
(diverse) financial instruments at the same time. A set of investments made by a particular
agent is referred to as a (**financial**) portfolio. The other type of risk is the one to which the
entire market is exposed so that the diversification of an agent’s portfolio would not reduce
the risk-exposure. This type of risk is called non-diversifiable.

**Problem 1.2.** Try to come up with your own example of diversifiable and non-diversifiable
risks in a financial market.

Frequently, though, the two arenas of protection against financial loss do overlap. One
example are the so-called *catastrophe bonds*. Read more about them at
http://en.wikipedia.org/wiki/Catastrophe_bond

**Problem 1.3. SoA Sample FM(DM) Problem #25** Determine which of the following
statements concerning risk sharing, in the context of financial risk management, is LEAST
accurate.

(a) In an insurance market, individuals that do not incur losses have shared risk with
individuals that do incur losses.
(b) Insurance companies can share risk by ceding some of the excess risk from large
claims to reinsurers.
(c) Reinsurance companies can further share risk by investing in catastrophe bonds.
(d) Risk sharing reduces diversifiable risk, more so than reducing non-diversifiable risk.
(e) Ideally, any risk-sharing mechanism should benefit all parties sharing the risk.

For more on the role of financial markets, read section 1.2 from [2].

1.3. **Points of view on financial markets.** We can loosely divide the interested parties
into three different categories:

(1) participants (individual investors, institutional investors, corporations),
(2) facilitators (market makers, traders), and
(3) researchers (us!).

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INSTRUCTOR: Mílica Čudina
Of course, we must not forget the regulators – finance is more than just Wall Street . . .

1.4. **Structure of financial mathematics.**
*What?* Financial risk within portfolios of traded entities (aka assets or securities) – such as stocks and bonds.

*How?* Nondeterministic change over time due to *market forces.*

*Where?* Financial markets.

**References**
