7.1. **Put options.** A *put option* gives the owner the right – but not the obligation – to sell the underlying asset at a predetermined price during a predetermined time period. The seller of a put option is obligated to buy if asked. The mechanics of the European put option are the following:

at time $0$: (1) the contract is agreed upon between the buyer and the writer of the option, 
(2) the logistics of the contract are worked out (these include the underlying asset, the *expiration date* $T$ and the *strike/exercise price* $K$), 
(3) the buyer of the option pays a *premium* to the writer;

at time $T$: the buyer of the option can *choose* whether he/she will sell the underlying asset for the strike price $K$. The writer of the option is bound to fulfill the put-option buyer’s choice.

7.1.1. *The European put option payoff.* We already went through a similar procedure with European call options, so we will just briefly repeat the mental exercise and figure out the European-put-buyer’s profit.

If the strike price $K$ exceeds the final asset price $S(T)$, i.e., if $S(T) < K$, this means that the put-option holder is able to sell the asset for a higher price than he/she would be able to in the market. In fact, in our perfectly liquid markets, he/she would be able to purchase the asset for $S(T)$ and immediately sell it to the put writer for the strike price $K$. The payoff is, thus, $S(T) - K$.

To the contrary, if $S(T) \geq K$, the put-option owner would be better of selling the asset at the market price. So, he/she will simply walk away from the contract incurring the payoff of 0.

Combining the above two states of the world, we get the following expression for the long-put-option payoff:

$$V_P(T) = \max(K - S(T), 0) = (K - S(T))_+.$$ 

So, the payoff function for a put option is

$$v_P(s) = (K - s)_+.$$ 

For $K = 1000$, we get the payoff curve below (in blue). The buyer is supposed to pay the premium at $t = 0$. This will affect the profit curve. For instance, if the initial premium for this option with exercise date in one year equals $50 and if the continuously compounded interest rate equals $r = 0.06$, then the profit curve is the one graphed below in red.
Looking at the graph above, we see that the put-option payoff (as well as profit) is decreasing in the asset price and bounded from above by the strike price $K$.

**Example 7.1. Put option on a market index**
Consider a put option on a market index with exercise date in six months and with strike price $K = 1000$. Assume that the premium for this option equals $V_P(0) = 80$ and that the effective interest rate for the six-month period equals $i = 0.03$. The payoff function is

$$v_P(s) = (1000 - s)_+$$

and the profit function is

$$v_P(s) - V_P(0)(1 + i) = (1000 - s)_+ - 80 \cdot 1.03 = (1000 - s)_+ - 82.4.$$

Once the exercise date is reached, one gets to observe the final index value and calculate the realized payoff and profit. For instance:

1. If the final index value equals $S(T) = 1050$, the put-owner’s payoff is 0 (the option is not even exercised). The profit is, hence, $-82.40$. So, the owner of the option experiences a loss of $82.40$.
2. If the final index value equals $S(T) = 800$, the payoff is $1000 - 800 = 200$ (the option is, indeed, exercised). The profit is, hence, $200 - 82.40 = 117.60$. So, the owner of the option gains of $117.60$.

**Remark 7.2.** Two positions in the market with the the payoff of one being the exact negative payoff of the other are said to be opposites of each other. In particular,

- a purchased call is the opposite of a written call;
- a purchased put option is the opposite of a written put.

7.1.2. Problems.

**Problem 7.1.** The initial price of the market index is $900$. After 3 months the market index is priced at $915$. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of $930$, is $8.00$. What is the profit at expiration for a long put?
(a) $15.00 loss
(b) $6.90 loss
(c) $6.90 gain
(d) $15.00 gain
(e) None of the above.

Solution: (c)
The profit from a position is defined as the position’s payoff minus the future value of the initial cost.
If \( S(T) = 915 \) denotes the price of the market index at time \( T = 0.25 \) (i.e., in three months), then the payoff of the long put is \( (K - S(T))_+ \), where \( K = 930 \) denotes the strike of the put. So, since \( K > S(T) \), the payoff is
\[
(930 - 915)_+ = 15.
\]
The future value of the initial put premium is
\[
8(1 + 0.004)^3 = 8.0964.
\]
So, the profit is
\[
\]

Problem 7.2. Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4\% convertible semiannually, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

Solution: (b)
Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the “break-even” point. We solve for \( s \) in
\[
(K - s)_+ - V_P(0) \left( 1 + \frac{i^{(2)}}{2} \right) = (1000 - s)_+ - 74.20(1.02) = 0.
\]
The solution is \( s = 924.32 \).

Method II. This is the more pedestrian method. The long-put profit is
\[
(K - s)_+ - V_P(0) \left( 1 + \frac{i^{(2)}}{2} \right) = (1000 - s)_+ - 74.20(1.02).
\]
The short-put profit is the exact negative of the expression above, i.e.,

\[-(K - s)_+ + V_P(0) \left(1 + \frac{i^{(2)}}{2}\right) = -(1000 - s)_+ + 74.20(1.02).\]

So, algebraically, we need to solve for \(s\) in the equation

\[(1000 - s)_+ - 74.20(1.02) = -(1000 - s)_+ + 74.20(1.02) \iff 2(1000 - s)_+ = 2 \cdot 74.20(1.02) \iff (1000 - s)_+ = 74.20(1.02).\]

We get the same answer as above, of course.

**Problem 7.3.** Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000—cartons’ worth in six months. His total costs are $12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of $13 per carton and another with the strike price of $15 per carton. Their premiums are $0.15 and $0.18, respectively.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals $14. What would Farmer Shaun’s profit be if he had decided to hedge using the $13-strike put versus his profit if he had decided to use the $15-strike put to hedge?

**Solution:** Farmer Shaun’s unhedged position has the following profit:

\[10,000(S(T) - 12)\]

where \(S(T)\) stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would long the put. So, the profit of the $13-strike-put hedge would be:

\[10,000(13 - S(T))_+ - 10,000 \times 0.15 \times 1.04.\]

The profit of the $15-strike-put hedge would be:

\[10,000(15 - S(T))_+ - 10,000 \times 0.18 \times 1.04.\]

The profit of the hedged position with the given \(S(T) = 14\) in the first case equals

\[10,000(14 - 12 - 0.15 \times 1.04) = 18,440.\]

For the second insurance strategy, the profit is

\[10,000(14 - 12 + (15 - 14) - 0.18 \times 1.04) = 28,128.\]

7.1.3. **Suggested problems.** McDonald: #2.3, #2.5, #2.14.
7.2. **Moneyness.** The moneyness of an option reflects whether an option would cause a positive, negative or zero cashflow were to be exercised immediately. More precisely, at any time \( t \in [0, T] \), an option is said to be:

1. **in-the-money** – the owner of the option would receive a strictly positive cashflow were the option exercised immediately;
2. **at-the-money** – the owner of the option would receive a zero cashflow were the option exercised immediately;
3. **out-of-the money** – the owner of the option would receive a strictly negative cashflow were the option exercised immediately.

**Example 7.3. Moneyness of a put option**
Consider a put option with strike \( K = 100 \). If the initial price \( S(0) \) of the underlying asset equals:

1. 95 – then the option is in-the-money;
2. 100 – then the option is at-the-money;
3. 105 – then the option is out-of-the-money.

Imagine that we are half-way through the life of the option, i.e., we have reached time \( T/2 \). We can observe the price of the underlying asset at that time too. We denote this value by \( S(T/2) \), and we can also state that at time \( -T/2 \)

1. if \( S(T/2) > 100 \) the put option is out-of-the-money;
2. if \( S(T/2) = 100 \) the put option is at-the-money;
3. if \( S(T/2) < 100 \) the put option is in-the-money.