12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $S = \{S(t), t \geq 0\}$) which given:

- an exercise date $T$;
- a *strike* price $K_s$;
- a *trigger* price $K_t$

provides the payoff

$$V_{GC}(T) = (S(T) - K_s)I_{[S(T) \geq K_t]}$$

to its owner.

**Problem 12.1.** Consider a gap call option with $K_s \leq K_t$.

- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price?
Solution:
A long position.

**Problem 12.2.** Consider a gap call option with $K_t < K_s$.

- Draw its payoff curve.
- Do you think that the word “option” is entirely appropriate in this case?
A negative payoff is possible. So, the term “option” is somewhat of a misnomer.

**Problem 12.3.** Create a replicating portfolio for the gap call option consisting of cash-or-nothing call options and asset-or-nothing call options.

**Solution:**
- long one asset-or-nothing call with trigger price $K_t$,
- short $K_s$ cash-or-nothing calls with trigger price $K_t$.

**12.2. Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by $S = \{S(t), t \geq 0\}$) which given:
- an exercise date $T$;
- a strike price $K_s$;
- a trigger price $K_t$
provides the payoff

$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]}$$
to its owner.

**Problem 12.4.** Consider a gap put option with $K_s \leq K_t$. Draw its payoff curve.
Problem 12.5. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.

Solution:
Solution:

Problem 12.6. Create a replicating portfolio for the gap put option consisting of cash-or-nothing put options and asset-or-nothing put options.

Solution:

- short one asset-or-nothing put with trigger price $K_t$, 

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• long $K_s$ cash-or-nothing puts with trigger price $K_t$.

12.3. Put-call parity for gap options.

Problem 12.7. Consider the following portfolio:

• one long gap call option with trigger price $K_t$ and the strike price $K_s$,
• one short otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?
(ii) What is the payoff of the above portfolio?
(iii) Based on your answers to the above two questions, what is put-call parity for gap options?

Solution:

(i) $V_{GC}(0) - V_{GP}(0)$

(ii) 

\[
(S(T) - K_s)I_{\{S(T) \geq K_t\}} - (K_s - S(T))I_{\{S(T) < K_t\}} = S(T) - K_s
\]

(iii) 

\[
V_{GC}(0) - V_{GP}(0) = F_{0,T}^P(S) - PV_{0,T}(K_s)
\]

Note that the right-hand side of the parity relationship does not depend on the trigger price.