UNIVERSITY OF TEXAS AT AUSTIN

Lecture 12

Gap options.

12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

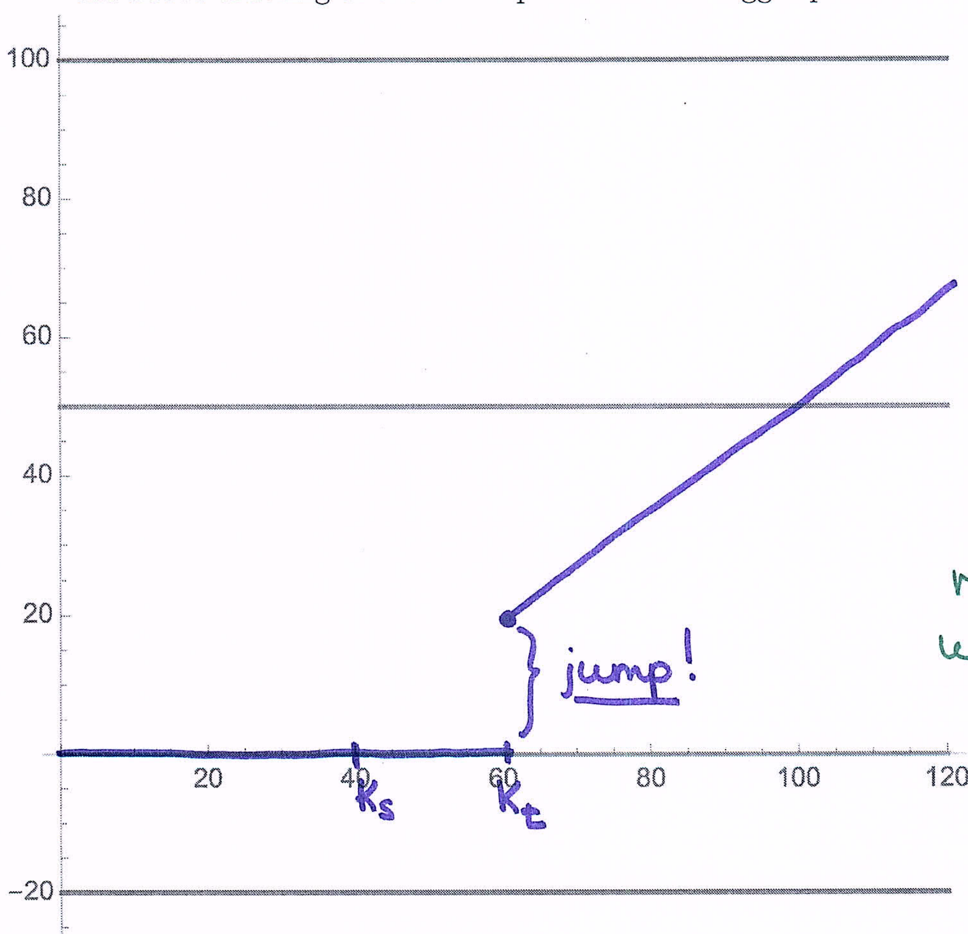
provides the payoff

$$V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]}$$

to its owner.

Problem 12.1. Consider a gap call option with $K_s \leq K_t$.

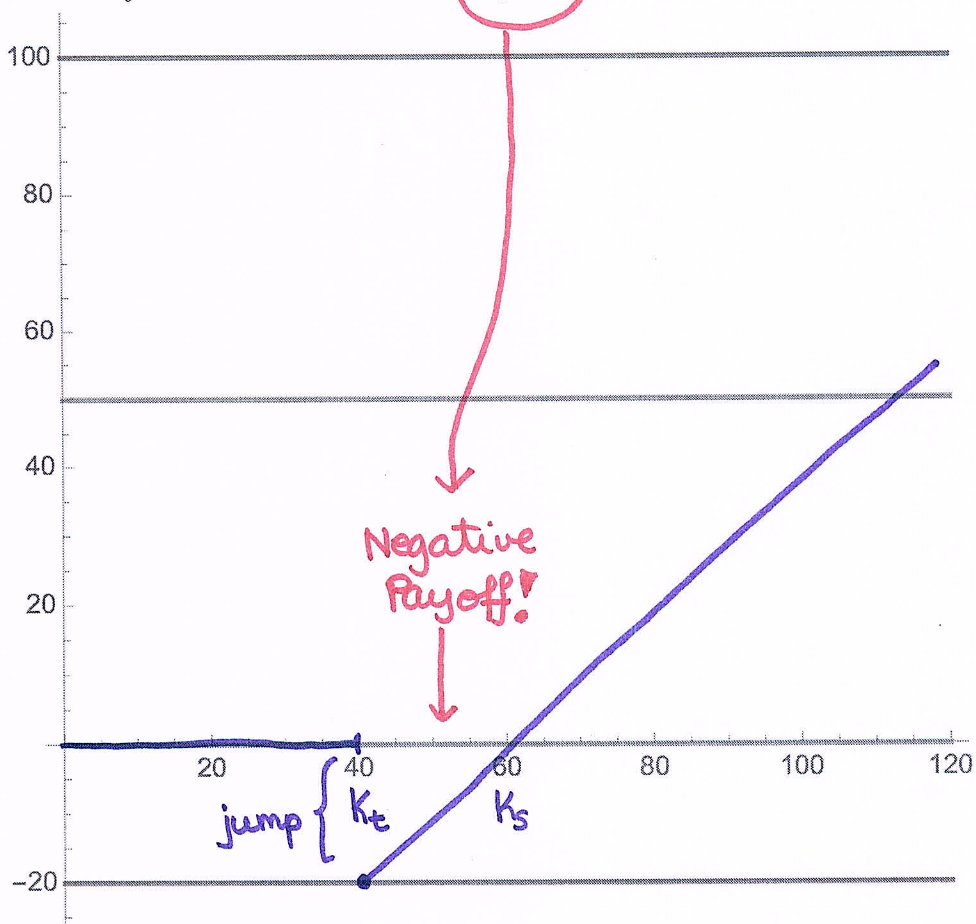
- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price?



long w/ respect to the underlying :)

Problem 12.2. Consider a gap call option with $K_t < K_s$.

- Draw its payoff curve.
- Do you think that the word "option" is entirely appropriate in this case?



Payoff: $V_{GC}(T) = (S(T) - K_s)\mathbb{I}_{[S(T) \geq K_t]}$

$$= \underbrace{S(T)\cdot\mathbb{I}_{[S(T)\geq K_t]}}_{} - K_s\cdot\underbrace{\mathbb{I}_{[S(T)\geq K_t]}}_{}$$

$$= V_{AC}(T) \quad - K_s \cdot V_{CC}(T)$$

$\Rightarrow$
- one long asset call w/ trigger K_t
- K_s short cash calls w/ trigger K_t

Problem 12.3. Create a replicating portfolio for the gap call option consisting of cash-or-nothing call options and asset-or-nothing call options.

12.2. **Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by $\mathbf{S} = \{S(t), t \geq 0\}$) which given:

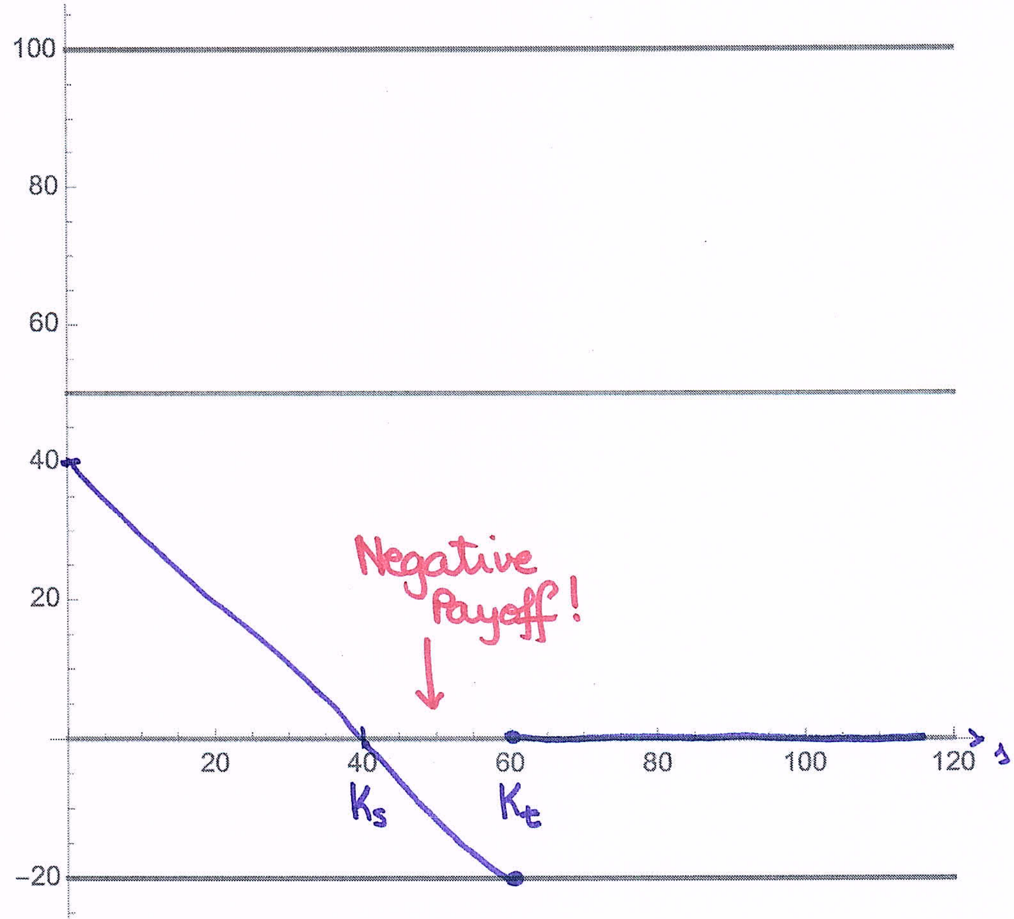
- an exercise date T;
- a **strike** price K_s;
- a **trigger** price K_t

provides the payoff

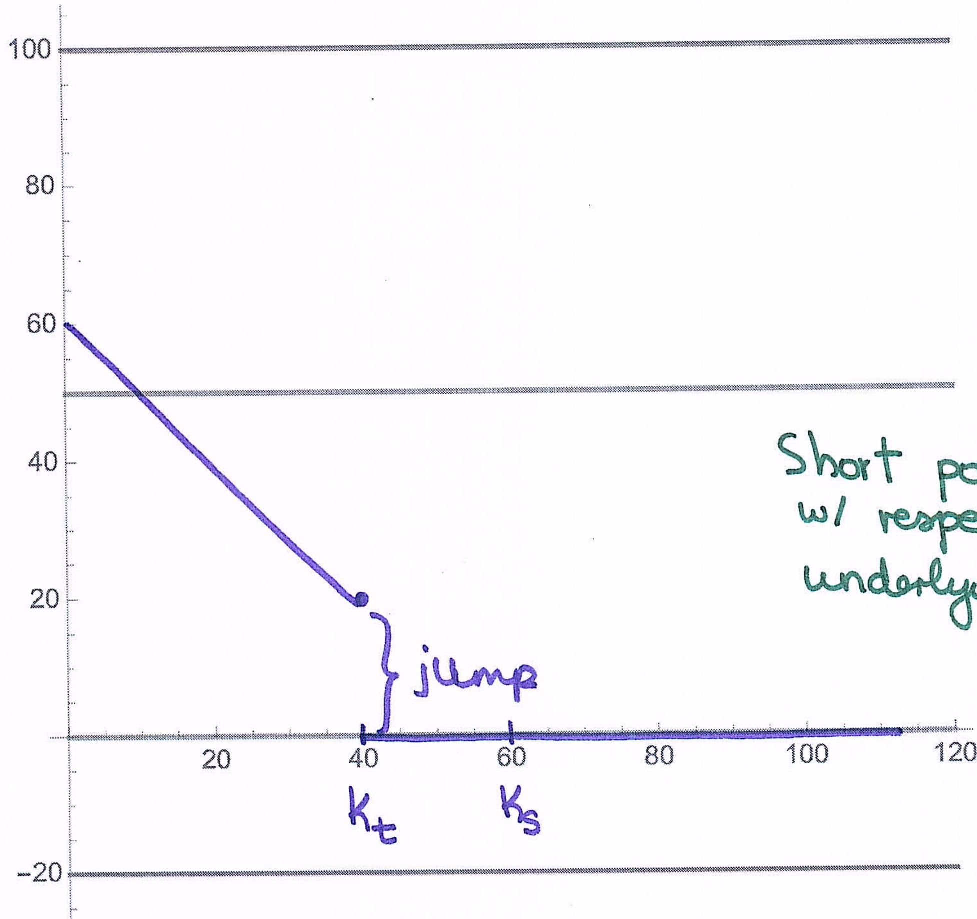
$$V_{GP}(T) = (K_s - S(T))\mathbb{I}_{[S(T)<K_t]}$$

to its owner.

Problem 12.4. Consider a gap put option with $K_s \leq K_t$. Draw its payoff curve.



Problem 12.5. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.

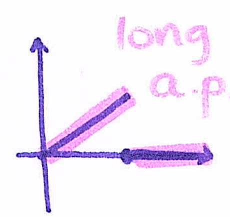


Short position w/ respect to the underlying!

<u>Payoff</u>: $V_{GP}(T) = (K_S - S(T))\mathbb{I}_{[S(T)<K_t]}$

$$= K_S \cdot \underbrace{\mathbb{I}_{[S(T)<K_t]}}_{} - \underbrace{S(T)\mathbb{I}_{[S(T)<K_t]}}_{}$$

$$= K_S \cdot V_{CP}(T) \quad - \quad V_{AP}(T)$$



$\Rightarrow$
- K_S <u>long</u> cash puts w/ ~~strike~~ trigger K_t
- one <u>written</u> asset put w/ trigger K_t

Problem 12.6. Create a replicating portfolio for the gap put option consisting of cash-or-nothing put options and asset-or-nothing put options.

12.3. **Put-call parity for gap options.**

Problem 12.7. Consider the following portfolio:

- one **long** gap call option with trigger price K_t and the strike price K_s,
- one **short** otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?
(ii) What is the payoff of the above portfolio?
(iii) Based on your answers to the above two questions, what is **put-call parity** for gap options?

$V_{GC}(0) - V_{GP}(0)$

Portfolio's payoff:

$$V(T) = V_{GC}(T) - V_{GP}(T)$$
$$= (S(T)-K_s)\cdot \mathbb{I}_{[S(T)\geq K_t]} - (K_s - S(T))\cdot \mathbb{I}_{[S(T)<K_t]}$$
$$= (S(T)-K_s)\cdot \mathbb{I}_{[S(T)\geq K_t]} + (S(T)-K_s)\cdot \mathbb{I}_{[S(T)<K_t]}$$
$$= (S(T)-K_s)\underbrace{\left(\mathbb{I}_{[S(T)\geq K_t]} + \mathbb{I}_{[S(T)<K_t]}\right)}_{1}$$

$$V(T) = \underbrace{S(T)}_{\text{Payoff of a prepaid forward.}} - \underbrace{K_s}_{\text{Repaid loan.}}$$

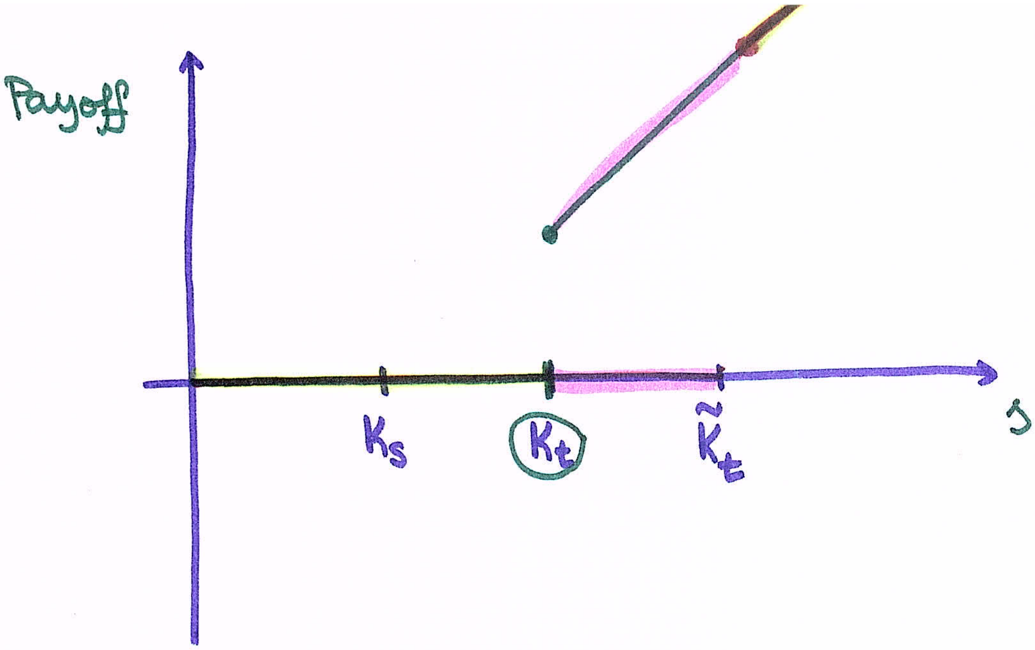
$\Rightarrow$ The time-0 price of the portfolio is:

$$V(0) = F^P_{0,T}(S) - PV_{0,T}(K_s)$$

$$\Rightarrow \quad V_{GC}(0) - V_{GP}(0) = F^P_{0,T}(S) - PV_{0,T}(K_s)$$

Gap option parity!

The trigger is irrelevant :)


Payoff
K_s
K_t
$\tilde{K}_t$
J

Options on futures.

Futures contract

$0 \qquad\qquad T_F$ (delivery date)

Futures contracts are liquid.

$\Rightarrow$ Futures prices are obseveable and we can use futures contract as an underlying asset.

We will work on calls and puts on futures.

Stocks; market indices; commodities $\leftarrow$ Futures contracts $\leftarrow$ Calls/Puts

Q: How does the futures price vary w/ time?

$0 \quad \cdots \quad t \quad \cdots \quad T_F$ delivery date

$$0 \uparrow F_{0,T_F} \qquad t \uparrow F_{t,T_F}$$

e.g., for stocks w/ cont. div:

$$F_{t,T_F}(S) = S(t)e^{(r-\delta)(T_F-t)}$$

John Hull: "Futures, options, and other derivative securities"