Currency Options. [cont'd]

03/09/2018.

Recall: exchange rate $\longleftrightarrow$ cont. div. paying stock

ccrfir for FC $\longleftrightarrow$ dividend yield

r_F $\longleftrightarrow$ δ

$r \dots r_D \dots$ ccrfir for DC

$\Rightarrow$ **Put-Call Parity**

$$V_c(0) - V_p(0) = F^P_{0,T}(x) - PV_{0,T}(K)$$

(x: exchange rate)

$$= x(0)\, e^{-r_F \cdot T} - K e^{-r_D \cdot T}$$

*** Special Symmetry ***

Notation: $V_c^D(t, x, K)$

- D: DC-denominated
- call
- valuation date
- x: exchange rate: from FC to DC
- K: strike (in DC)

for $t = 0$: put-call parity in this notation:

$$V_c^D(0, x, K) - V_p^D(0, x, K) = x(0)\, e^{-r_F \cdot T} - K e^{-r_D \cdot T}$$

$\frac{1}{x}$ ... the reverse exchange rate: from DC to FC

$$V_c^F\left(0, \frac{1}{x}, K^*\right) - V_p^F\left(0, \frac{1}{x}, K^*\right) = F^P_{0,T}\left(\frac{1}{x}\right) - PV_{0,T}(K^*)$$

K^*: FC-denominated

$$= \frac{1}{x(0)}\, e^{-r_D \cdot T} - K^* e^{-r_F \cdot T}$$

The payoff of a DC-denominated call w/ strike K:

$$V_C^D(T,x,K) = (x(T)-K)_+$$

$$= x(T)\left(1-\frac{K}{x(T)}\right)_+$$

$$= x(T)\cdot K\left(\frac{1}{K}-\frac{1}{x(T)}\right)_+$$

$\frac{1}{K}$: const.

$\frac{1}{x(T)}$: the value of the reverse exchange rate @ time-T

Payoff of a put on the DC w/ strike $1/K$

$$V_C^D(T,x,K) = \underbrace{x(T)}_{\text{exchange rate @ time-T}}\cdot \underbrace{K}_{\text{quantity}}\cdot V_P^F\left(T,\frac{1}{x},\frac{1}{K}\right)$$

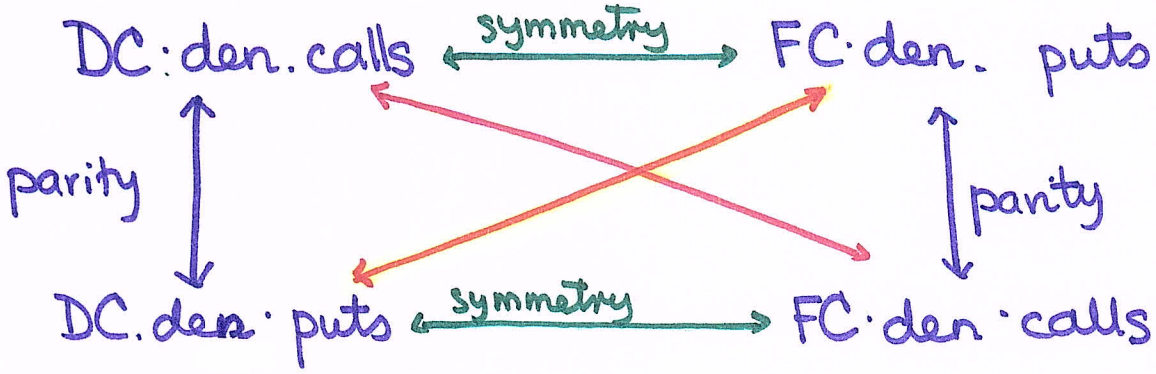
no arbitrage!

$\Rightarrow$

$$\cdot\ V_C^D(0,x,K) = x(0)\cdot K\cdot V_P^F\left(0,\frac{1}{x},\frac{1}{K}\right)$$

$$\cdot\ V_P^D(0,x,K) = x(0)\cdot K\cdot V_C^F\left(0,\frac{1}{x},\frac{1}{K}\right)$$

Put-call symmetry for currency options.


DC: den. calls
symmetry
FC· den. puts
parity
parity
DC. den · puts
symmetry
FC· den · calls

9. You are given: $...DC ¥...FC

(i) The current exchange rate is 0.011\$/¥. $x(0) = 0.011$

(ii) A four-year dollar-denominated European put option on yen with a strike price of \$0.008 sells for \$0.0005. $T=4$, $K=0.008$ $\rightarrow V_P^D(0,x,K) = 0.0005$

(iii) The continuously compounded risk-free interest rate on dollars is 3%. $r_{\$} = r_D = 0.03$

(iv) The continuously compounded risk-free interest rate on yen is 1.5%. $r_{¥} = r_F = 0.015$

Calculate the price of a four-year yen-denominated European put option on dollars with a strike price of ¥125. $T=4$

$\frac{1}{K} = \frac{1}{0.008} = 125$ w => we can use symmetry.

(A) ¥35

(B) ¥37

(C) ¥39

(D) ¥41

(E) ¥43

Put-call symmetry:

$$\underbrace{V_P^D(0,x,K)}_{\text{given}} = x(0)\cdot K \cdot \underbrace{V_C^F\left(0,\frac{1}{x},\frac{1}{K}\right)}_{?}$$

$$\Rightarrow V_C^F\left(0,\frac{1}{x},\frac{1}{K}=125\right) = \frac{1}{x(0)}\cdot\frac{1}{K}\cdot V_P^D(0,x,K)$$

$$= \frac{1}{0.011}\cdot 125\cdot 0.0005 = 5.68$$

Put-call Parity:

$$V_P^F\left(0,\frac{1}{x},\frac{1}{K}=125\right) = V_C^F\left(0,\frac{1}{x},\frac{1}{K}\right) - \frac{1}{x(0)}e^{-r_D^{0.03}\cdot T} + \frac{1}{K}e^{-r_F^{0.015}\cdot T}$$

$$= 5.68 - \frac{1}{0.011}e^{-0.03\cdot 4} + 125\,e^{-0.015\cdot 4}$$

$$= 42.77$$ ■

(4.)

Exchange Options.

- Recall: vanilla European call

  S... underlying asset ←

  K... strike price ←

  Payoff: $V_c(T) = (S(T) - K)_+$

Exchange call: S... underlying asset, Q... strike asset } BOTH RISKY ASSETS

$$V_{EC}(T, S, Q) = (S(T) - Q(T))_+$$

S: underlying asset; Q: strike asset

Exchange put:

$$V_{EP}(T, S, Q) = (Q(T) - S(T))_+$$

⇒ A special symmetry:

$$V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$$

⇒ for any $0 \le t \le T$

$$V_{EC}(t, S, Q) = V_{EP}(t, Q, S)$$

$$V_{EP}(t, S, Q) = V_{EC}(t, Q, S)$$