Option on currencies

underlying asset... the "foreign" currency (FC)

\( x(t) \) ... the worth at time-\( t \) of a single unit of FC expressed in terms of the "domestic" currency (DC)

\( r_F \) ... foreign-currency risk-free interest rate

\( r_D \) ... domestic-currency continuously compounded risk-free interest rate

**EXAMPLE**

- **Prepaid forward contract**

  \[ F_{0,T}^P (x) = e^{-r_F T} \cdot x(0) \]

  Compare to continuous-dividend-paying stocks.

  \[ S(0) \leftrightarrow x(0) \]

  \[ \delta \leftrightarrow r_F \]

- **Forward:** In general:

  \[ F_{0,T} (x) = FV_{0,T} (F_{0,T}^P (x)) \]

  w/ respect to \( r_D \)

  \[ F_{0,T} (x) = F_{0,T}^P (x) e^{r_D T} = e^{-r_F T} \cdot x(0) e^{r_D T} \]

  \[ F_{0,T} (x) = x(0) e^{(r_D - r_F) T} \]

  \( r \leftrightarrow r \)
European calls and puts on $FC$

Payoffs:
\[
V^D_c(T, x, K) = (x(T) - K)_+ \\
V^D_p(T, x, K) = (K - x(T))_+ \\
\] w/ exercise date $T$ and the strike price $K$

**Put-Call Parity for Currency Options**

\[
V^D_c(0, x, K) - V^D_p(0, x, K) = e^{-r_f T} \cdot x(0) - e^{-r_d T} \cdot K
\]

A special symmetry.

The payoff of a DC-denominated call:
\[
V^D_c(T, x, K) = (x(T) - K)_+ = x(T) \cdot K - \left( \frac{1}{K} - \frac{1}{x(T)} \right)_+
\]

Payoff of an FC-denominated put w/ strike $V_K$

\[
= \Rightarrow V^D_c(T, x, K) = x(T) \cdot K \cdot V^F_p(T, \frac{1}{x} , \frac{1}{K})
\]

No arbitrage?

\[
= \Rightarrow V^D_c(0, x, K) = x(0) \cdot K \cdot V^F_p(0, \frac{1}{x} , \frac{1}{K}) \quad \text{Put-call symmetry}
\]

\[
V^D_p(0, x, K) = x(0) \cdot K \cdot V^F_c(0, \frac{1}{x} , \frac{1}{K})
\]
9. You are given:

(i) The current exchange rate is \(0.011$$/¥. \(x(0) = 0.011\)

(ii) A four-year dollar-denominated European put option on yen with a strike price of $0.008 sells for $0.0005. \(V_P(0, x, K = 0.008) = 0.0005\)

(iii) The continuously compounded risk-free interest rate on dollars is 3%. \(r_d = 0.03\)

(iv) The continuously compounded risk-free interest rate on yen is 1.5%. \(r_y = 0.015\)

Calculate the price of a four-year yen-denominated European put option on dollars with a strike price of ¥125.

\(\frac{1}{k} = 125\)

(A) ¥35

(B) ¥37

(C) ¥39

(D) ¥41

(E) ¥43

We need to use both parity and symmetry!

\(\Rightarrow (E)\)
Exchange Options

So far: Vanilla CALLS @ time-T
the call's owner CAN pay K (the strike price) and in return receive 1 unit of the underlying asset S

\[
V_C(T) = (S(T) - K)_+
\]

Now: Exchange CALLS @ time-T
the exchange call's owner CAN give up 1 unit of the STRIKE ASSET Q and in return receive one unit of the underlying asset S.

\[
V_{EC}(T, S, Q) = (S(T) - Q(T))_+
\]

PUTS @ time-T
the exchange put's owner CAN give up 1 unit of the underlying assets and in return receive one unit of the strike asset Q

\[
V_{EP}(T, S, Q) = (Q(T) - S(T))_+
\]
Property [SYMMETRY]

\[ V_{EC}(T, S, Q) = V_{EP}(T, Q, S) \]

\[ \Rightarrow V_{EC}(0, S, Q) = V_{EP}(0, Q, S) \]

MAXIMUM OPTION

Consider two risky assets: \( S \) and \( Q \). The payoff of the maximum option is defined as:

\[ V_{\text{MAX}}(T) = \max [S(T), Q(T)] \]

Q: Financial Implementation?

\[ V_{\text{MAX}}(0) = \max [F_{0, T}^p(S), F_{0, T}^p(Q)] \]

Note:

\[ V_{\text{MAX}}(T) = \max [S(T), Q(T)] \]

\[ = S(T) + \max (0, Q(T) - S(T)) \]

\[ = S(T) + V_{EC}(T, Q, S) \]

\[ \Rightarrow \text{One replicating portfolio for the maximum option is:} \]

\[ \{ \]

\[ \bullet \text{ prepaid forward contract on } S \]

\[ \bullet \text{ exchange call w/ underlying } Q \text{ and strike asset } S \]
\[ V_{\text{MAX}}(0) = F_{0,T}^P(s) + V_{EC}(0, q, S) \]
\[ = F_{0,T}^P(s) + V_{EP}(0, S, q) \]
\[ = F_{0,T}^P(q) + V_{EC}(0, S, q) \]
\[ = F_{0,T}^P(q) + V_{EP}(0, q, S) \]

\[ F_{0,T}^P(s) + V_{EP}(0, S, q) = F_{0,T}^P(q) + V_{EC}(0, S, q) \]
\[ V_{EC}(0, S, q) - V_{EP}(0, S, q) = F_{0,T}^P(s) - F_{0,T}^P(q) \]

The **GENERALIZED Put-Call Parity**

Solve: MFE Sp 07 P#6.

Review: Payoff diagrams for calls and put