Option on currencies

underlying asset ... the "foreign" currency (FC)

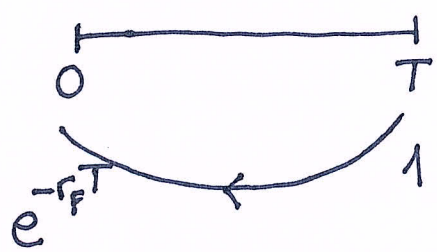
$x(t)$ ... the worth @ time-t of a single unit of FC expressed in terms of the "domestic" currency (DC)

r_F ... foreign-currency
r_D ... domestic-currency

} continuously compounded risk-free interest rate

EXAMPLE.

- Prepaid forward contract



$$F^P_{0,T}(x) = e^{-r_F T} \cdot x(0)$$

Compare to continuous-dividend-paying stocks.

$$S(0) \longleftrightarrow x(0)$$
$$\delta \longleftrightarrow r_F$$

- Forward: In general: $F_{0,T}(x) = \underbrace{FV_{0,T}}_{\text{w/ respect to } r_D}(F^P_{0,T}(x))$

$$F_{0,T}(x) = F^P_{0,T}(x) e^{r_D \cdot T} = e^{-r_F T} \cdot x(0) e^{r_D \cdot T}$$

$$\boxed{F_{0,T}(x) = x(0) e^{(r_D - r_F) \cdot T}} \qquad r \leftrightarrow r$$

- European calls and puts on FC

PAYOFFS:

$$V_C^D(T, x, K) = (x(T) - K)_+$$

valuation date

DC-denominated

$$V_P^D(T, x, K) = (K - x(T))_+$$

w/ exercise date T and the strike price K

Put-Call Parity for Currency Options

$$\boxed{V_C^D(0, x, K) - V_P^D(0, x, K) = e^{-r_F T} \cdot x(0) - e^{-r_D T} \cdot K}$$

A special symmetry.

The payoff of a DC-denominated call:

$$V_C^D(T, x, K) = (x(T) - K)_+ = x(T) \cdot K \cdot \left(\frac{1}{K} - \frac{1}{x(T)} \right)_+$$

a constant

the exchange rate in the reverse direction, i.e., DC↦FC

Payoff of an FC-denominated PUT w/ strike $1/K$

$$\Rightarrow \boxed{V_C^D(T, x, K) = x(T) \cdot K \cdot V_P^F\left(T, \frac{1}{x}, \frac{1}{K}\right)}$$

NO ARBITRAGE!

$$\Rightarrow \boxed{\begin{aligned} V_C^D(0, x, K) &= x(0) \cdot K \cdot V_P^F\left(0, \frac{1}{x}, \frac{1}{K}\right) \\ V_P^D(0, x, K) &= x(0)\, K \cdot V_C^F\left(0, \frac{1}{x}, \frac{1}{K}\right) \end{aligned}}$$

Put-call Symmetry.

9. You are given:

(i) The current exchange rate is 0.011\$/¥. $x(0) = 0.011$

$T=4$

(ii) A four-year dollar-denominated European put option on yen with a strike price of \$0.008 sells for \$0.0005. $V_P^{\$}(0, x, K=0.008) = 0.0005$

(iii) The continuously compounded risk-free interest rate on dollars is 3%. $r_{\$} = 0.03$

(iv) The continuously compounded risk-free interest rate on yen is 1.5%. $r_{¥} = 0.015$

Calculate the price of a four-year yen-denominated European put option on dollars with a strike price of ¥125.

$\frac{1}{K} = 125$

(A) ¥35

(B) ¥37

(C) ¥39

(D) ¥41

(E) ¥43

$V_P^{¥}(0, \frac{1}{x}, \frac{1}{K}=125) = ?$

DC-den. calls $\xleftrightarrow{\text{SYMMETRY}}$ FC-den. puts

$\updownarrow$ PARITY $\qquad$ $\updownarrow$ PARITY

DC-den. puts $\xleftrightarrow{\text{SYMMETRY}}$ FC-den. calls

We <u>need</u> to use BOTH parity and symmetry!

$\Rightarrow$ E.

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Exchange Options

So far: Vanilla

CALLS @ time-T

the call's owner

CAN

pay K (the strike price)

and in return

receive 1 unit of the underlying asset S

PAYOFF:

$$V_C(T) = (S(T)-K)_+$$

Now: Exchange

CALLS @ time-T

the exchange call's owner

CAN

give up 1 unit of the STRIKE ASSET Q

and in return

receive one unit of the underlying asset S.

PAYOFF:

$$V_{EC}(T, S, Q) = (S(T)-Q(T))_+$$

underlying asset

strike asset

PUTS @ time-T

the exchange put's owner

CAN

give up 1 unit of the underlying asset S

and in return

receive one unit of the strike asset Q

$$V_{EP}(T,S,Q) = (Q(T)-S(T))_+$$

Property [SYMMETRY]

$$V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$$

$$\Rightarrow \boxed{V_{EC}(0, S, Q) = V_{EP}(0, Q, S)}$$

MAXIMUM OPTION

Consider two risky assets: S and Q.
The payoff of the maximum option is defined as:

$$\boxed{V_{MAX}(T) = \max[S(T), Q(T)]}$$

Q: Financial Implementation?

$$V_{MAX}(0) \geq MAX\left[F^P_{0,T}(S), F^P_{0,T}(Q)\right]$$

Note:

$$\begin{aligned} V_{MAX}(T) &= \max[S(T), Q(T)] \\ &= S(T) + \max(0, Q(T) - S(T)) \\ &= S(T) + V_{EC}(T, Q, S) \end{aligned}$$

$\Rightarrow$ One REPLICATING PORTFOLIO for the maximum option is:

- prepaid forward contract on S
- exchange call w/ underlying Q and strike asset S

$$\Rightarrow \quad V_{MAX}(0) = F^P_{0,T}(S) + V_{EC}(0, Q, S)$$

$$= F^P_{0,T}(S) + V_{EP}(0, S, Q)$$

$$= F^P_{0,T}(Q) + V_{EC}(0, S, Q)$$

$$= F^P_{0,T}(Q) + V_{EP}(0, Q, S)$$

$$F^P_{0,T}(S) + V_{EP}(0, S, Q) = F^P_{0,T}(Q) + V_{EC}(0, S, Q)$$

$$\boxed{V_{EC}(0, S, Q) - V_{EP}(0, S, Q) = F^P_{0,T}(S) - F^P_{0,T}(Q)}$$

The <u>GENERALIZED Put-Call Parity</u>

<u>Solve</u>: MFE Sp07 P#6.

<u>Review</u>: Payoff diagrams for calls and put