Stock ABC has the following characteristics:

- The current price to buy one share is 100.
- The stock does not pay dividends.
- European options on one share expiring in one year have the following prices:

<table>
<thead>
<tr>
<th>Strike Price</th>
<th>Call option price</th>
<th>Put option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>14.63</td>
<td>0.24</td>
</tr>
<tr>
<td>100</td>
<td>6.80</td>
<td>1.93</td>
</tr>
<tr>
<td>110</td>
<td>2.17</td>
<td>6.81</td>
</tr>
</tbody>
</table>

A butterfly spread on this stock has the following profit diagram.

The annual risk-free interest rate compounded continuously is 5%.

Determine which of the following will NOT produce this profit diagram.

(A) Buy a 90 put, buy a 110 put, sell two 100 puts

(B) Buy a 90 call, buy a 110 call, sell two 100 calls

(C) Buy a 90 put, sell a 100 put, sell a 100 call, buy a 110 call

(D) Buy one share of the stock, buy a 90 call, buy a 110 put, sell two 100 puts

(E) Buy one share of the stock, buy a 90 put, buy a 110 call, sell two 100 calls.
The payoff curve of the portfolio in (E). It is a Butterfly spread.

final asset price(s)

⇒ Our correct answer is (D).
Speculating on Volatility

- Options can be used to create positions that are nondirectional with respect to the underlying asset.

- Examples
  - Straddles
  - Strangles
  - Butterfly spreads

- Who would use nondirectional positions?
  - Investors who do not care whether the stock goes up or down, but only how much it moves, i.e., who speculate on volatility.
STRADDLE

- a LONG call
- a LONG put

otherwise identical (usually @ the money)

The payoff: \( V(T) = |S(T) - K| \)

This is, indeed, a bet on HIGH VOLATILITY

The initial cost: \( V_c(0) + V_p(0) \)
A: Which portfolio results in this payoff diagram?

- a LONG put w/ strike $K_p$ 
- a LONG call w/ strike $K_c$

It is a bet on high volatility!

$K_p < K < K_c$

$\{ K_p$-put $\} \cup$ strangle

$\{ K_c$-call $\} \cup$ straddle

$V_p(K_p) \leq V_p(K)$
$V_c(K_c) \leq V_c(K)$

$\frac{V_p(K_p) + V_c(K_c)}{V_p(K) + V_c(K)} \leq \frac{V_p(K)}{V_p(K) + V_c(K)}$

Initial cost of the STRANGLE $\leq$ the STRADDLE.
7.
A non-dividend paying stock currently sells for 100. One year from now the stock sells for 110. The annual risk-free rate, compounded continuously, is 6%. A trader purchases the stock in the following manner:
- The trader pays 100 today
- The trader takes possession of the stock in one year

Determine which of the following describes this arrangement.

(A) Outright purchase
(B) Fully leveraged purchase
(C) Prepaid forward contract
(D) Forward contract
(E) This arrangement is not possible due to arbitrage opportunities

8.
Joe believes that the volatility of a stock is higher than indicated by market prices for options on that stock. He wants to speculate on that belief by buying or selling at-the-money options.

Determine which of the following strategies would achieve Joe’s goal.

(A) Buy a strangle
(B) Buy a straddle
(C) Sell a straddle
(D) Buy a butterfly spread
(E) Sell a butterfly spread
16.

The current price of a non-dividend paying stock is 40 and the continuously compounded annual risk-free rate of return is 8%. The following table shows call and put option premiums for three-month European of various exercise prices:

<table>
<thead>
<tr>
<th>Exercise Price</th>
<th>Call Premium</th>
<th>Put Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>6.13</td>
<td>0.44</td>
</tr>
<tr>
<td>40</td>
<td>2.78</td>
<td>1.99</td>
</tr>
<tr>
<td>45</td>
<td>0.97</td>
<td>5.08</td>
</tr>
</tbody>
</table>

A trader interested in speculating on volatility in the stock price is considering two investment strategies. The first is a 40-strike straddle. The second is a strangle consisting of a 35-strike put and a 45-strike call.

\[ T = \frac{4}{4} \]

Determine the range of stock prices in 3 months for which the strangle outperforms the straddle.

(A) The strangle never outperforms the straddle.
(B) \(33.56 < S_T < 46.44\)
(C) \(35.13 < S_T < 44.87\)
(D) \(36.57 < S_T < 43.43\)
(E) The strangle always outperforms the straddle.
Focus on finding the intersection for $s \in (40, 45)$.

$s - 40 = FV(\text{Init. Cost of the straddle})$

$= 0 - FV(\text{Init. Cost of the strangle})$

$\Rightarrow s = 40 + FV(2.78 + 1.99 - 0.97 - 0.44)$

$s = 43.43. \Rightarrow \boxed{D}$
Collar: $K_p < K_c$

- a long $K_p$-put
- a short $K_c$-call

otherwise identical
2.3. **Ratio spreads.** A ratio spread is a financial position consisting of the following components:
- $m$ long calls with strike $K_1$, and
- $n$ short calls with strike $K_2$, and

with $K_1 < K_2$, $m$ and $n$ being positive constants, and the options being otherwise identical. Equivalent (in the sense of equal profit) ratio spreads can be constructed using put options only.

**Problem 2.6.** Provide an alternative name for the ratio spread in which $n = m = 1$.

**Solution:** Call bull spread.

**Problem 2.7.** Assume that $m < n$. Is the corresponding ratio spread a long or a short position with respect to the underlying?

**Solution:** It is neither.

**Problem 2.8.** Assume that $m > n$. Is the corresponding ratio spread a long or a short position with respect to the underlying?

**Solution:** It is a long position with respect to the underlying.

**Problem 2.9.** Which of the following statements is/are **incorrect**?

(a) The payoff of a call bull spread is always nonnegative.
(b) The payoff of a ratio spread is always positive.
(c) The payoff of a straddle is never negative.
(d) The payoff of a put bear spread is never negative.
(e) None of the above.

**Solution:** (b), (d)

**Problem 2.10.** Sample FM (Derivatives Markets): Problem #39.

Determine which of the following strategies creates a ratio spread, assuming all options are European.

A. Buy a one-year call, and sell a three-year call with the same strike price.
B. Buy a one-year call, and sell a three-year call with a different strike price.
C. Buy a one-year call, and buy three one-year calls with a different strike price.
Q: If \( k^* \) is the strike at which \( V_c(k^*) = V_p(k^*) \), what is it equal to (in simpler terms)?

**Put-call parity**

\[
0 = V_c(k^*) - V_p(k^*) = F_{0,T}^P(S) - PV_{0,T}(k^*)
\]

\[\Rightarrow PV_{0,T}(k^*) = F_{0,T}^P(S)\]

\[\Rightarrow k^* = FV_{0,T}(F_{0,T}^P(S)) = f_{0,T}(S)\]
Ratio Spreads. $K_1 < K_2$

- $m$ LONG calls w/ strike $K_1$ and $n$ SHORT calls w/ strike $K_2$ otherwise identical.
- $m$ and $n$ ... positive constants (usually integers).

Q: What do you get for $n = m$?
It's $m \times$ Call Bull Spread.
Q: What if $m < n$?

\[ (\text{slope} = m - n < 0) \]
\[ (\text{slope} = -m) \]

\[ \text{Payoff} \]
38.

The current price of a medical company’s stock is 75. The expected value of the stock price in three years is 90 per share. The stock pays no dividends.

You are also given

i) The risk-free interest rate is positive.
ii) There are no transaction costs.
iii) Investors require compensation for risk.

The price of a three-year forward on a share of this stock is X, and at this price an investor is willing to enter into the forward.

Determine what can be concluded about X.

(A) $X < 75$
(B) $X = 75$
(C) $75 < X < 90$
(D) $X = 90$
(E) $90 < X$

39.

Determine which of the following strategies creates a ratio spread, assuming all options are European.

(A) Buy a one-year call, and sell a three-year call with the same strike price.
(B) Buy a one-year call, and sell a three-year call with a different strike price.
(C) Buy a one-year call, and buy three one-year calls with a different strike price.
(D) Buy a one-year call, and sell three one-year puts with a different strike price.
(E) Buy a one-year call, and sell three one-year calls with a different strike price.
Q: What if \( m > m' \)?

A suitable hedge: