# More on call/put prices as functions of the strike

| | | arbitrage portfolio |
|---|---|---|
| $V_C(K) \downarrow$ | .... | Call Bull Spread |
| $V_P(K) \uparrow$ | .... | w/ $K_1 < K_2$: short the $K_1$-put, long the $K_2$-put — PUT BEAR SPREAD |

w/ $K_1 < K_2$

$V_C(K_1) - V_C(K_2) \leq PV(K_2 - K_1)$ ... short the $K_1$-call, long the $K_2$-call — CALL BEAR SPREAD

$V_P(K_2) - V_P(K_1) \leq PV(K_2 - K_1)$ ... Put Bull Spread

March 31st, 2015

**12.** You are given:

(i) $C(K, T)$ denotes the current price of a $K$-strike $T$-year European call option on a nondividend-paying stock.

(ii) $P(K, T)$ denotes the current price of a $K$-strike $T$-year European put option on the same stock.

(iii) $S$ denotes the current price of the stock.

(iv) The continuously compounded risk-free interest rate is $r$.

Which of the following is (are) correct?

T (I) $0 \le C(50, T) - C(55, T) \le 5e^{-rT}$

(II) $50e^{-rT} \le P(45, T) - C(50, T) + S \le 55e^{-rT}$

(III) $45e^{-rT} \le P(45, T) - C(50, T) + S \le 50e^{-rT}$

(A) (I) only

~~(B)~~ (II) only

~~(C)~~ (III) only

(D) (I) and (II) only

(E) (I) and (III) only

Slope Inequalities:

$V_c(50) - V_c(55) \le 5e^{-rT}$ ✓

$V_p(45) - V_c(50) + S(0) =$

$= V_p(45) - V_c(50) + V_p(50) - V_p(50) + S(0) =$

$= V_p(45) - V_p(50) + (V_p(50) - V_c(50)) + S(0) =$

$= V_p(45) - V_p(50) + (50e^{-rT} - \boxed{\cancel{F^P_{0,T}(S)}}) + \cancel{S(0)}$ $\quad S(0)$

$= V_p(45) - V_p(50) + 50e^{-rT}$ $\Rightarrow$ (III) is true.

$-5e^{-rT} \le$ $\quad \le 0$

E.

MFE Exam, Sp 2009

Problem #12.

[redacted]: Claims:

(I) $0 \overset{T}{\leq} V_c(50) - V_c(55) \overset{T}{\leq} 5e^{-rT}$

(II) $50e^{-rT} \leq V_p(45) - V_c(50) + S(0) \leq 55e^{-rT}$

(III) $45e^{-rT} \leq V_p(45) - V_c(50) + S(0) \leq 50e^{-rT}$

Which is/are true?

I. TRUE!

$$V_p(45) - V_c(50) + S(0) = \text{[Put-call parity]}$$

$$= V_p(45) - \left(V_p(50) + \underbrace{\cancel{F^P_{0,T}(S)}}_{= S(0) \text{ [NO DIVIDENDS]}} - 50e^{-rT}\right) + \cancel{S(0)} =$$

$$= \underbrace{V_p(45) - V_p(50)}_{\leq 0} + 50e^{-rT}$$

$$-5e^{-rT} \cancel{\geq} \leq$$

$$V_p(50) - V_p(45) \leq 5e^{-rT}$$

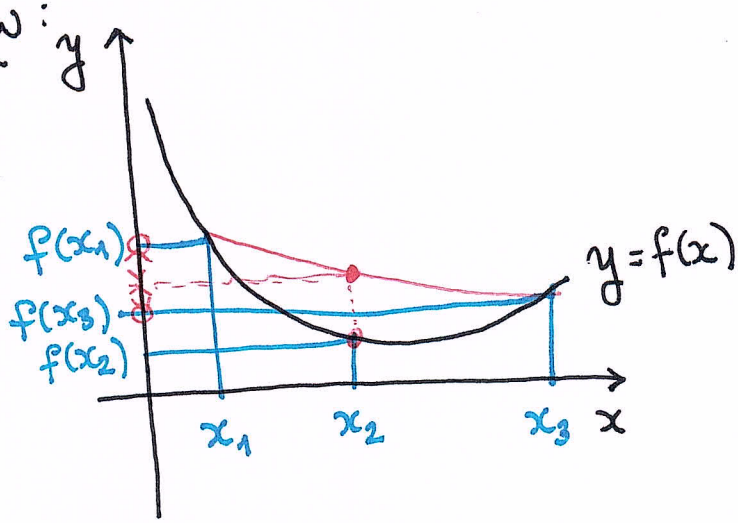
$$\Rightarrow 45e^{-rT} \leq V_p(45) - V_p(50) + 50e^{-rT} \leq 50e^{-rT}$$

$\Rightarrow$ (III) is True!

$\Rightarrow \boxed{E.}$

# Convex Functions

Review:




IF $x_1 < x_2 < x_3$, THEN THERE EXISTS $0 < \lambda < 1$

such that:

$$x_2 = \lambda \cdot x_1 + (1-\lambda) x_3$$

$$\Longleftrightarrow \quad x_2 = \lambda x_1 + x_3 - \lambda x_3$$

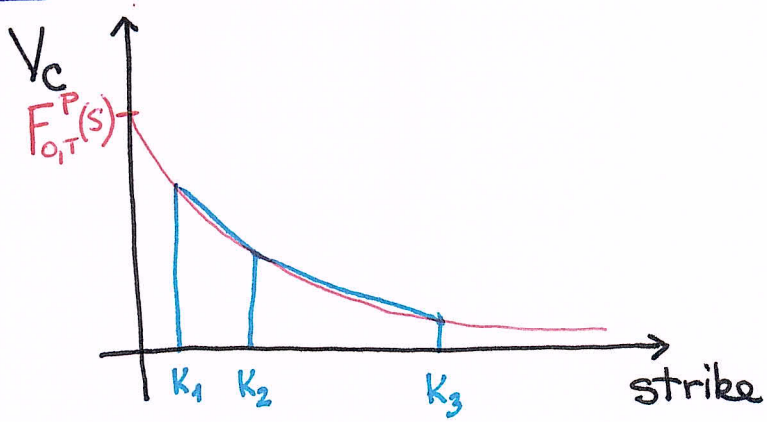
$$\Longleftrightarrow \quad x_2 - x_3 = \lambda (x_1 - x_3)$$

$$\boxed{\lambda = \frac{x_3 - x_2}{x_3 - x_1}} \qquad \boxed{1 - \lambda = \frac{x_2 - x_1}{x_3 - x_1}}$$

**Def'n.** $f$ is convex if

$$f(\lambda x_1 + (1-\lambda) x_3) \leq \lambda f(x_1) + (1-\lambda) f(x_3)$$

# Convexity of call/put prices w/ respect to the strike




**Claim:**

$$(CC) \quad \left[ \begin{array}{l} V_C(K_2) \leq \lambda V_C(K_1) + (1-\lambda) V_C(K_3) \\ \text{w/} \quad \lambda = \dfrac{K_3 - K_2}{K_3 - K_1} \end{array} \right]$$

$$\Longleftrightarrow$$

$$\frac{V_C(K_1) - V_C(K_2)}{K_2 - K_1} \geq \frac{V_C(K_2) - V_C(K_3)}{K_3 - K_2}$$

Assume TO THE CONTRARY that there exists $K_1 < K_2 < K_3$ such that

$$\underbrace{V_C(K_2)}_{\text{short it!}} > \lambda V_C(K_1) + (1-\lambda) V_C(K_3)$$

I. ✓

II. Propose an ARBITRAGE PORTFOLIO

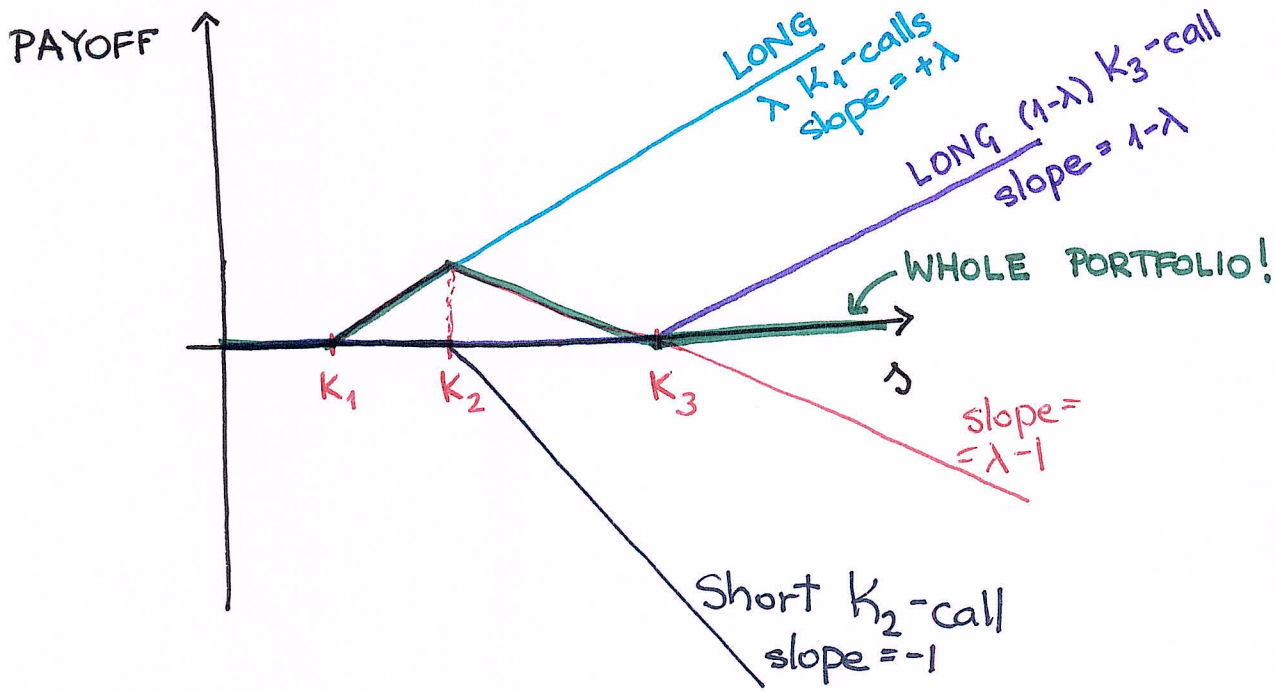
- WRITE 1 $K_2$-strike call
- LONG $\lambda$ $K_1$-strike calls
- LONG $1-\lambda$ $K_3$-strike calls

## III. Verify.

Initial Cost: $\lambda V_c(K_1) + (1-\lambda) V_c(K_3) - V_c(K_2) < 0$

$\Downarrow$

Initial inflow of money!




PAYOFF (Portfolio) $\geq 0$ $\longrightarrow$ Profit $> 0$

$\Rightarrow$ Arbitrage!

Call prices are CONVEX as functions of the strike.

Our portfolio is called a CALL BUTTERFLY SPREAD.

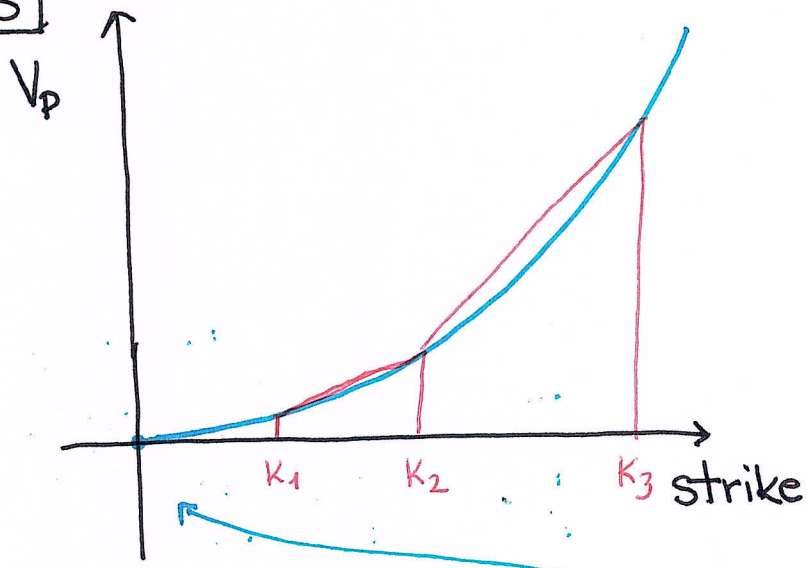
If $K_2 = \frac{1}{2}(K_1 + K_3)$ we say it's a SYMMETRIC butterfly spread.

If <u>not</u>, then ASYMMETRIC.

Q: Would one use the call butterfly spread to hedge either a long, or a short inherent position w/ respect to the underlying?

It lacks directionallity, so NO.

PUTS




Put payoff : $(K - S(T))_+$

If $K = 0$ : $(0 - S(T))_+ = 0$.

(PC) $V_P(K_2) \leq \lambda V_P(K_1) + (1-\lambda) V_P(K_3)$ w/ $\lambda = \dfrac{K_3 - K_2}{K_3 - K_1}$

$$\Longleftrightarrow$$

$$\frac{V_P(K_2) - V_P(K_1)}{K_2 - K_1} \leq \frac{V_P(K_3) - V_P(K_2)}{K_3 - K_2}$$

The ARBITRAGE PORTFOLIO in the case that (PC) does not hold:

- <u>Short</u> 1 $K_2$-put
- <u>long</u> $\lambda$ $K_1$-puts
- <u>long</u> $(1-\lambda)$ $K_3$-puts

} PUT BUTTERFLY SPREAD

<u>Tasks for @ home</u>: 1° Draw the payoff curve.

2° Compare it to the payoff curve of the call butterfly spread w/ the same $K_1, K_2, K_3$.

3° Compare the initial costs between the call and the put butterfly spreads.

# Speculating on Volatility

Changes in the price without taking into account the direction.

- Options can be used to create positions that a nondirectional with respect to the underlying asset
- Examples
  - Straddles
  - Strangles
  - Butterfly spreads

high vol

low vol

symmetric / asymmetric

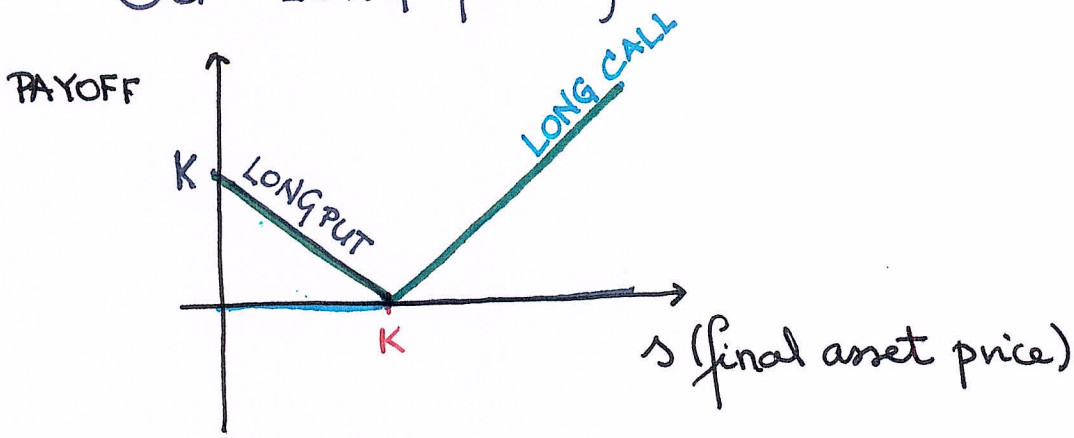
payoff are laterally symmetric

- Who would use nondirectional positions?
  - Investors who do not care whether the stock goes up or down, but only how much it moves, i.e., who speculate on volatility

# STRADDLE

- a LONG call
- a LONG put

} w/ identical <u>inputs</u>



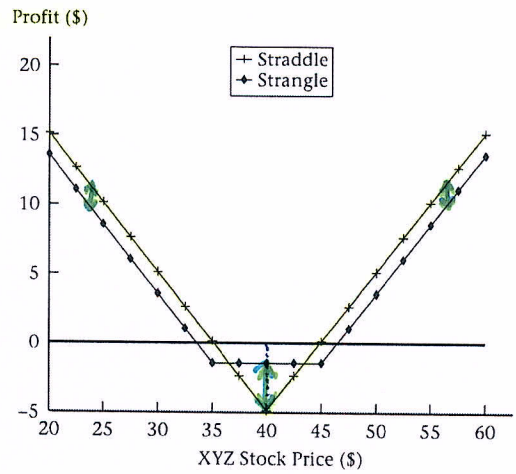

The payoff of the straddle: $V(T) = |S(T) - K|$

... A <u>bet</u> on HIGH VOLATILITY ...

<u>Initial cost</u> : $V_c(0) + V_p(0)$ (time)

# Strangles

- Buying an out-of-the-money call and put with the same time to expiration




- A strangle can be used to reduce the high premium cost, associated with a straddle

$K_p < K < K_c$

$$\left.\begin{array}{c} K_p\text{-strike put} \\ + \\ K_c\text{-strike call} \end{array}\right\} \text{STRANGLE}$$

$$\left.\begin{array}{c} K\text{-strike put} \\ + \\ K\text{-strike call} \end{array}\right\} \text{straddle}$$

$$\left.\begin{array}{l} V_p(K_p) \leq V_p(K) \\ V_c(K_c) \leq V_c(K) \end{array}\right\} +$$

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Cost of strangle $\leq$ Cost of straddle.