MAY 5th 2015

Futures Contracts

- Standardization => Futures contract more liquid => It's easier to buy/sell them. => We are confident of the "accuracy" of the observed futures price.
  => We can use futures contracts as an UNDERLYING ASSET for a call or a put.
- Credit risk aleviated by means of a MARGIN ACCOUNT.

NOTIONAL VALUE of the investment:

$$N = n \times \text{Size of Index} \times \text{Futures Price}$$

n ↑ # of futures contracts

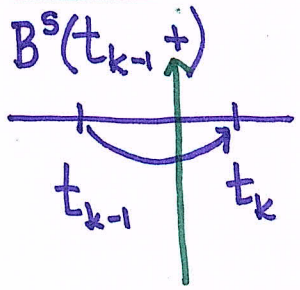
Futures Price ↑

MARGIN ACCOUNT(S) ... think of it as a savings acct, i.e.,

- EARNS INTEREST (@ the prevailing interest rate r)

INITIAL MARGIN ... the amount which needs to be initially deposited into the margin acct; usually expressed as a percentage of N

Settlement Dates [the seller's perspective]



Right-hand limit

- $B^s(t_k-) = B^s(t_{k-1}+)\, e^{r(t_k - t_{k-1})}$

Left-hand limit

The effect of interest earned.

- MARKING TO MARKET

$$B^s(t_k) = B^s(t_k-) - n \times \text{Size of Index} \times \underbrace{\left(F_{t_k,T} - F_{t_{k-1},T}\right)}_{\text{Change in the Futures price!}}$$

- The Maintenance Margin:

The comfort zone boundary of your broker; i.e., the minimal balance in the margin account (s)he is willing to accept.

$$B^s(t_k^+) = \max\left(B^s(t_k), MM\right)$$

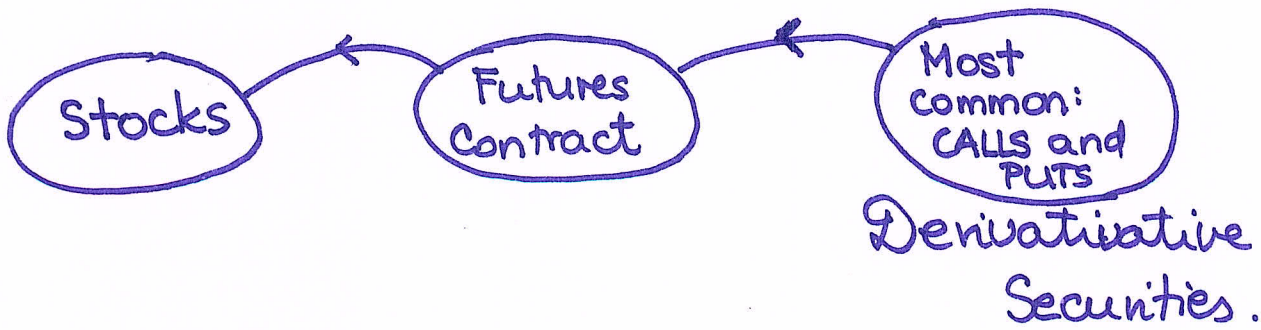
IF $B^s(t_k) < MM$, THEN

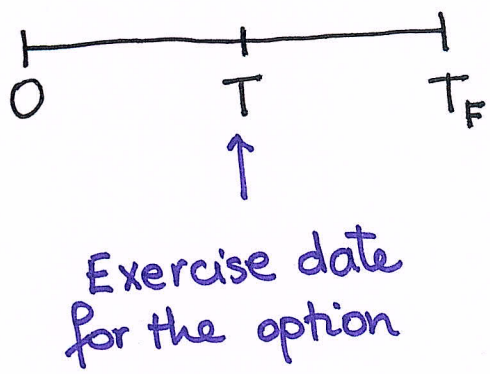
a MARGIN CALL IS MADE.

Options on Futures Contracts [John Hull's book]

Futures contracts w/ delivery date T_F.

Futures prices : F_{t,T_F} for $0 \leq t \leq T_F$ are observable quantities!





A Call on a futures contract:

- On the date t @ which the option is exercised:
  - * the option's holder acquires a LONG position in the futures contract,
  - * receives the amount $\left(F_{t,T_F} - K\right)_+$

the call option's strike

The maintenance margin : MM

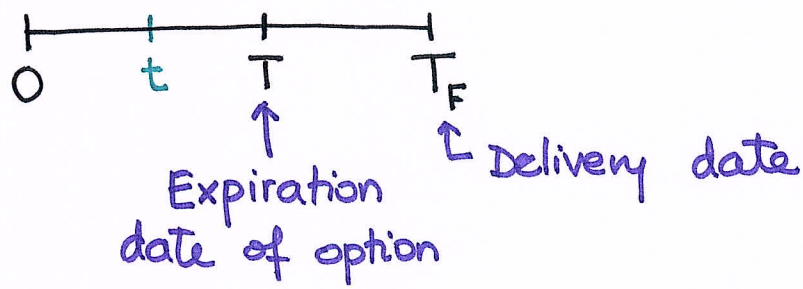
$$B^s(t_k+) = \max(B^s(t_k), MM)$$

IF $B^s(t_k) < MM$,
then a margin call is made.

$\boxed{\text{John Hull's book}}$

Options on Futures Contract

Call / Put on a Futures contract



CALL On the date t on which the option is exercised, the option's holder:

* acquires a LONG position in the underlying futures contract
* receives the amount $(F_{t,T_F} - K)_+$

the call option's strike

PUT * enters a short position in the underlying futures contract
* receive $(K - F_{t,T_F})_+$

A Put:
- a SHORT position in the futures contract,
- receive the amt: $(K - F_{t,T_F})_+$

For a pair of otherwise identical futures put/call we have the put-call parity:

$$V_C(0) - V_P(0) = F^P_{0,T}(\underbrace{\text{underlying}}_{\text{the futures contract for delivery @ time } T_F}) - Ke^{-rT}$$

Futures price = Forward price

$$\boxed{V_C(0) - V_P(0) = e^{-rT}(F_{0,T_F} - K)}$$

Mnemonic device: $\delta \longleftrightarrow r$

dividend yield $\longleftrightarrow$ continuously compounded, risk-free interest rate

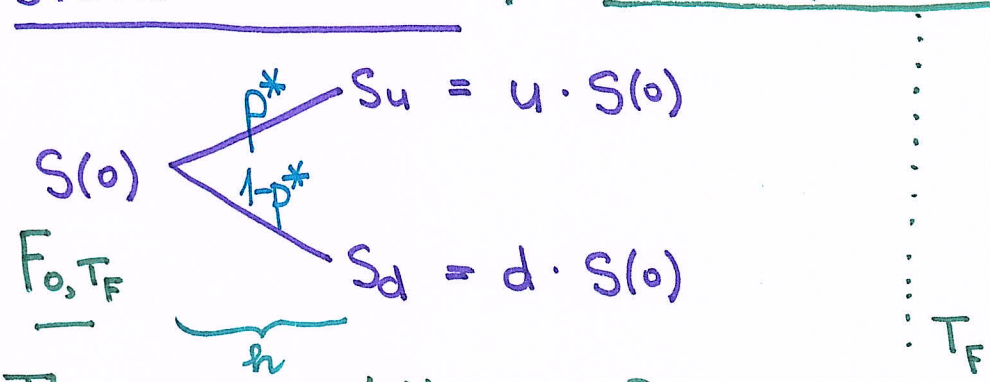
Q: What if the options are @ the money?

$\Rightarrow$ $\boxed{V_C(0) = V_P(0)}$

Binomial pricing

.... on futures on stocks

STOCK-PRICE TREE / Futures (forward)-price tree



- The no-arbitrage forward price ($t \leq T_F$):

$$F_{t,T_F}(S) = S(t)e^{(r-\delta)(T_F-t)}$$

In particular:

$$F_{0,T_F} = F_{0,T_F}(S) = S(0)e^{(r-\delta)T_F}$$

@ the up-node:

$$S_u \cdot e^{(r-\delta)(T_F-h)} = u \cdot \underbrace{S(0)e^{(r-\delta)T_F}}_{F_{0,T_F}} \cdot e^{-(r-\delta)h}$$

$$= F_{0,T_F} \cdot \underbrace{u \cdot e^{-(r-\delta)h}}_{u_F}$$

@ the down-node:

$$S_d e^{(r-\delta)(T_F-h)} = F_{0,T_F} \cdot \underbrace{d e^{-(r-\delta)h}}_{d_F}$$

$\left.\begin{array}{l} u_F \ldots \text{up} \\ d_F \ldots \text{down} \end{array}\right\}$ factors for the futures-price tree

The risk-neutral probability:

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{\overset{1}{\cancel{e^{(r-\delta)h}}} - d_F \cancel{e^{(r-\delta)h}}}{u_F \cdot \cancel{e^{(r-\delta)h}} - d_F \cancel{e^{(r-\delta)h}}} = \frac{1-d_F}{u_F-d_F}$$

$$\delta \longleftrightarrow r$$

SAMPLE MFE

46. You are to price options on a futures contract. The movements of the futures price are modeled by a binomial tree. You are given:

(i) Each period is 6 months. $h = \frac{1}{2}$

(ii) $u/d = 4/3$, where u is one plus the rate of gain on the futures price if it goes up, and d is one plus the rate of loss if it goes down. $u_F/d_F = 4/3$

(iii) The risk-neutral probability of an up move is 1/3. $p^* = \frac{1}{3}$

(iv) The initial futures price is 80. $F_{0,T_F} = 80$

(v) The continuously compounded risk-free interest rate is 5%. $r = 0.05$

$T = 1 \quad n = 2$

Let C_I be the price of a 1-year 85-strike European call option on the futures contract, and C_{II} be the price of an otherwise identical American call option.

Determine $C_{II} - C_I$.

$$\frac{1}{3} = p^* = \frac{1 - d_F}{u_F - d_F}$$

$u_F/d_F = 4/3$

(A) 0

(B) 0.022

(C) 0.044

(D) 0.066

(E) 0.088

$$\frac{1}{3} = \frac{\frac{1}{d_F} - 1}{\frac{u_F}{d_F} - 1}$$

$$\frac{1}{3} = \frac{\frac{1}{d_F} - 1}{\frac{1}{3}}$$

$$\frac{1}{9} = \frac{1}{d_F} - 1$$

$$\Rightarrow d_F = \frac{9}{10}$$

$$\Rightarrow u_F = \frac{12}{10}$$

1st The tree

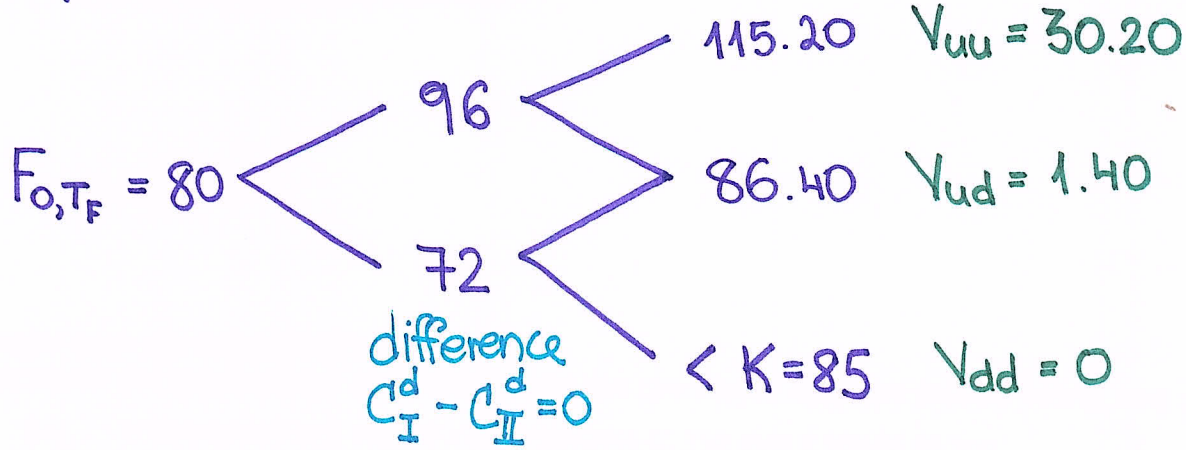
2nd $C_I = ?$, $C_{II} = ?$ as usual

3rd $C_{II} - C_I$ = answer.

$$\frac{1}{3} = p^* = \frac{1-d_F}{u_F-d_F} \cdot \frac{d_F}{d_F} = \underbrace{\frac{\frac{1}{d_F}-1}{\frac{u_F}{d_F}-1}}_{\frac{1}{3}}$$

$$\frac{1}{d_F} - 1 = \frac{1}{9} \Rightarrow d_F = \frac{9}{10} \Rightarrow u_F = \frac{12}{10}$$

<u>The futures-price tree</u>:



Price C_I and C_{II} $\Rightarrow$ answer $= C_{II} - C_I$

Try to work efficienty (focus on the <u>difference</u>)!