Commodity Swaps

Recall: Forwards

\[ 0 \quad T \]

Delivery date

In general w/ SWAPS:

\[ x_1, x_2, \ldots, x_k, \ldots, x_n \] ← SWAP PRICES

\[ 0 \quad T_1, T_2, \ldots, T_k, \ldots, T_n \] ← SWAP DATES

On any swap date \( T_k \):

If Physical Settlement:

Buyer

\[ x_k \downarrow \uparrow 1 \text{ unit of the asset} \]

Seller

The NET-EFFECT for the buyer:

\[ S(T_k) - x_k \]

CASH SETTLEMENT is feasible

the market price of the asset
Focus on two swap dates (generalization is easy).

Q: What are the acceptable pairs of $x_1$ and $x_2$?

The ones which do NOT introduce arbitrage opportunities!

Existing environment:

- **Forward prices**: $F_{0,T_1}$ and $F_{0,T_2}$
- **Term structure of interest rate**, i.e.,
  \[
  \begin{align*}
  r_0(0, T_1) \\
  r_0(0, T_2)
  \end{align*}
  \]
  **YEARLY EFFECTIVE SPOT RATES**

reference time

the end of the investment period

the start of the investment period
Prepaid swaps (auxiliary contracts)

\[
\begin{align*}
x^p & \quad \text{prepaid-swap price} \\
0 & \quad T_1 \quad T_2 \\
\text{PAYMENT DATE} & \quad \text{delivery dates}
\end{align*}
\]

A replicating portfolio for a prepaid swap is:

\[
\begin{align*}
\{ & \quad \text{a prepaid forward w/ delivery date } T_1 \\
& - \quad \| \quad - \quad \| \quad - \quad \| \quad - \quad T_2 \\
\Rightarrow & \quad x^p = F^p_{0,T_1} + F^p_{0,T_2} = PV_{0,T_1} (F_{0,T_1}) + PV_{0,T_2} (F_{0,T_2})
\end{align*}
\]

\[
\Rightarrow \quad x^p = \frac{F_{0,T_1}}{(1 + r_0(T_1))^T_1} + \frac{F_{0,T_2}}{(1 + r_0(T_2))^T_2}
\]

NO ARBITRAGE!

Example. Oil delivery @ \( T_1 = 1 \) and \( T_2 = 2 \).
The forward prices for oil delivery in one year and in two years are $20 and $21, resp.
The effective annual yield rates for zero-coupon bonds redeemable in one year and in years are 6% and 6.5%, resp.
What is the prepaid-swap price for oil delivery in one year and in two years?

\[
x^p = \frac{F_{0,1}}{(1 + 0.06)^1} + \frac{F_{0,2}}{(1 + 0.065)^2} = \frac{20}{1.06} + \frac{21}{1.065^2} = 37.383
\]
Credit risk. Cash availability \( \implies \) "Regular" swaps are more "appealing" than their prepaid counterparts.

In general: \( x_1 \) and \( x_2 \) must satisfy

\[
x^p = \frac{x_1}{(1 + r_0(0, T_1))^{T_1}} + \frac{x_2}{(1 + r_0(0, T_2))^{T_2}}
\]

Level Swaps \( \implies \) the level swap price

\[
x^p = x \cdot \left[ \frac{1}{(1 + r_0(0, T_1))^{T_1}} + \frac{1}{(1 + r_0(0, T_2))^{T_2}} \right]
\]

Example [cont’d]

For the oil swap: \( x^p = 37.383 \).

\( \implies \) The level swap price is:

\[
x = \frac{37.383}{\frac{1}{1.06} + \frac{1}{(1.065)^2}} = \ldots = 20.483
\]

NOTE: \( F_{0,1} = 20 < x = 20.483 < F_{0,2} = 21 \)

Generally: \( F_{0,T_1} < x < F_{0,T_2} \)

The effect: Overpayment at time-\( T_1 \) and underpayment at time-\( T_2 \)

The interpretation: A \( \text{LOAN} \) from the buyer to the seller \( @ F_0(1.2) \) implied forward rate for \( F_{0,2} \).
Why Swap Interest Rates?

- Interest rate swaps permit firms to separate credit risk and interest rate risk
  - By swapping its interest rate exposure, a firm can pay the short-term interest rate it desires, while the long-term bondholders will continue to bear the credit risk
Interest-rate swap

Recall: **Time-varying** interest rates

Random, **time-varying** interest rates, aka FLOATING INTEREST RATES $\Rightarrow$ swaps on interest rates.

$L$ .... loan amount

Charged a floating interest rate: $R_{t_{k-1}, t_k}(t_{k-1}, t_k)$

What is the payment amount to be paid at $t_k$?

The payment amount: $A_{t_k} = P_{t_k} + I_{t_k}$

The **NOTIONAL PRINCIPAL**

$w/ I_{t_k} = OLB_{t_{k-1}} \cdot R_{t_{k-1}, t_k}(t_{k-1}, t_k)$
An Example of an Interest Rate Swap (cont’d)

- On net, XYZ pays 6.9548%

\[
XYZ \text{ net payment} = - \text{LIBOR} + \text{LIBOR} - 6.9548\% = -6.9548\%
\]

Floating Payment \quad Swap Payment
The simplest (non-trivial) case

INTEREST-ONLY PMTS

=> The notional principal:

\[ \text{OLB}_{t_k} = L \]

NPV (all pmts using \( R \)) = NPV (all pmts using implied forward rates)

\[
\sum_{k=1}^{n} \left[ (L \cdot R \cdot P(0, t_k)) \right] = \sum_{k=1}^{n} \left[ L \cdot r_0(t_{k-1}, t_k) \cdot P(0, t_k) \right]
\]

Zero-coupon bond prices:

\[
\sum_{k=1}^{n} r_0(t_{k-1}, t_k) \cdot P(0, t_k)
\]

=> \( R = \frac{\sum_{k=1}^{3} r_0(t_{k-1}, t_k) \cdot P(0, t_k)}{\sum_{k=1}^{3} P(0, t_k)} \)

For yearly installments:

\[
R = \sum_{k=1}^{n} \left( \frac{P(0, t_k)}{\sum_{k=1}^{m} P(0, t_k)} \cdot r_0(t_{k-1}, t_k) \right)
\]

Solve Sample
FM (DM) Problem #23.