THE BINOMIAL ASSET-PRICING MODEL

Ambitious? Propose a stock-price model? ?

PRICING!

What's the PURPOSE of this model?

Investment decision (SUBJECTIVE)

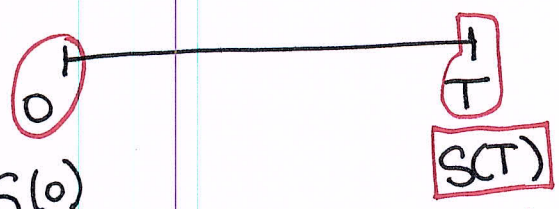
Generally, what are the desirable attributes of a model?

→ have the crucial properties of the modelled phenomenon. } e.g., stock prices should be modeled as random

→ useful / transparent / simple

↑ Depends on the stock price behaviour.

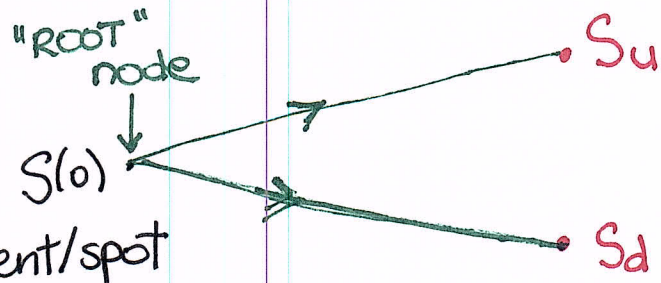
Consider: Pricing derivative securities.



T... expiration date

S(0) observeable

rnd var denoting the stock price @ time-T



current/spot asset price

"up" node } offspring

"down" node

↑ The two POSSIBLE values of S(T):

Usually: $S_u = u \cdot S(0)$ u... up factor

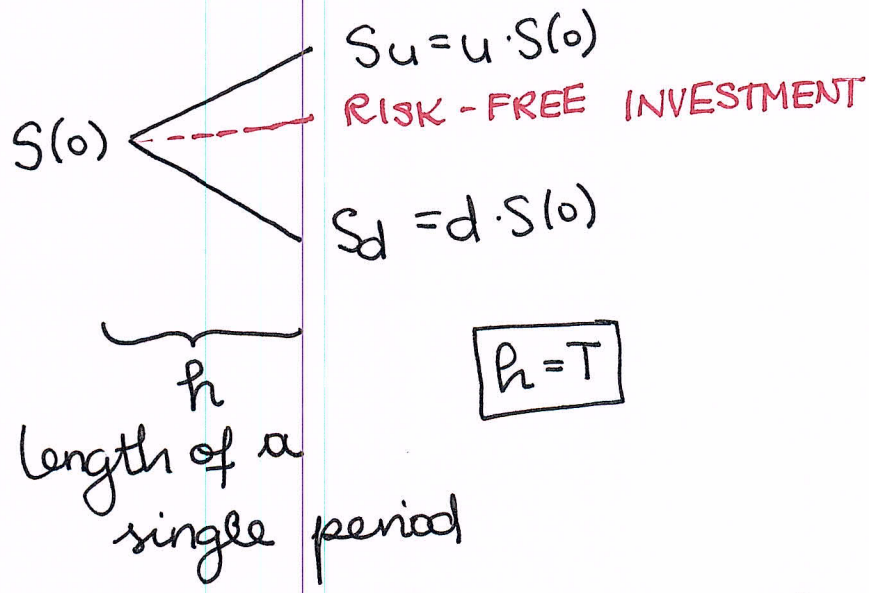
$S_d = d \cdot S(0)$ d... down factor

$$\Rightarrow \quad u = \frac{S_u}{S(0)} = \frac{S_u - S(0)}{S(0)} + 1$$

$$d = \frac{S_d}{S(0)} = \frac{S_d - S(0)}{S(0)} + 1$$

$\boxed{u > d}$

<u>Assume</u> r... continuously compounded, risk-free interest rate.



$\boxed{u > d}$

$\boxed{h = T}$

IF $S(0)$ is invested @ the risk-free rate r,
<u>THEN</u> the balance @ time-h is $\underline{S(0)\, e^{rh}}$.

IF $\underline{S(0)}$ is invested in the stock, <u>then</u> ?

We <u>only</u> work w/ continuous-dividend-paying stocks;

<u>δ... dividend yield.</u>

# of shares owned at time-h: $e^{\delta h}$

The wealth is based on the movement of the stock price:

IF "up" $\boxed{e^{\delta h} \cdot u \cdot S(0)}$

IF "down" $\boxed{e^{\delta h} \cdot d \cdot S(0)}$

We suspect the NO-ARBITRAGE CONDITION to be

$$e^{\delta h} d \cdot S(0) < e^{rh} \cdot S(0) < e^{\delta h} \cdot u \cdot S(0)$$

$$\Longleftrightarrow$$

$$\boxed{d < e^{(r-\delta)h} < u} \qquad \text{(NA)}$$

Assume TO THE CONTRARY that

$$\boxed{e^{(r-\delta)h} \leq d < u.}$$

I. Suspicion. ✓

II. Construction of an arbitrage portfolio:

BUY 1 SHARE of STOCK

III. Verification:

- Initial cost: $S(0)$
- Payoff: $e^{\delta h} \cdot S(h)$

PROFIT: IF "up": $e^{\delta h} u \cdot S(0) - e^{rh} S(0) = (e^{\delta h} u - e^{rh}) S(0)$

$$= e^{\delta h} \underbrace{(u - e^{(r-\delta)h})}_{>0} S(0)$$

IF "down": $e^{\delta h} d \cdot S(0) - e^{rh} S(0) =$

$$= (e^{\delta h} \cdot d - e^{rh}) \cdot S(0) \geq 0 \leftarrow \text{ARBITRAGE!}$$