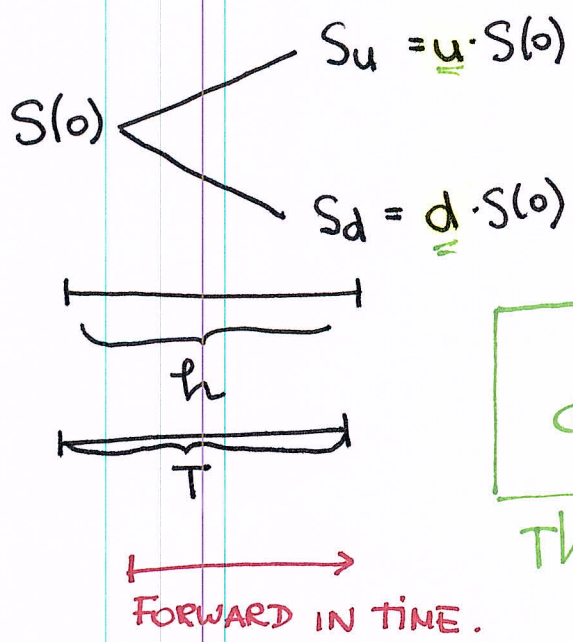
Recall : one-period binomial asset-pricing $\boxed{\text{tree}}$ MODEL.



r... continuously compounded, risk-free interest rate.

δ... dividend yield.

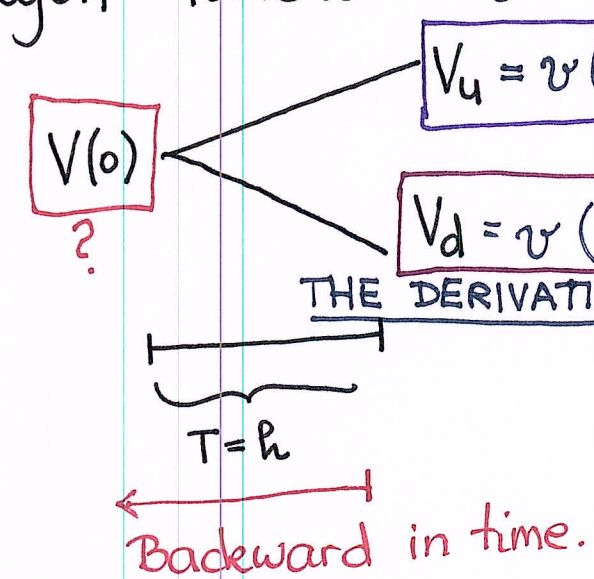
$$\boxed{d < e^{(r-\delta)h} < u}$$

The no-arbitrage condition.

THE BINOMIAL OPTION-PRICING PROCEDURE

Focus on a particular derivative security w/ a payoff function v.

FOR EXAMPLE, $v(s) = (s-K)_+$



$V_u = (S_u - K)_+$

$V_d = (S_d - K)_+$

THE DERIVATIVE-SECURITY TREE

Our market-model permits a REPLICATING PORTFOLIO consisting of
- investment in the risky asset
- a risk-free investment

for any derivative security.

More precisely:

- Δ units of the underlying
- $\boxed{B}$ invested at the risk-free rate r

THE REPLICATING-PORTFOLIO TREE

$$\Delta \cdot S(0) + B \xrightarrow[\underbrace{\quad}_{h}]{} \begin{cases} \boxed{\Delta e^{\delta h} \cdot S_u + B e^{rh}} \\ \boxed{\Delta e^{\delta h} \cdot S_d + B e^{rh}} \end{cases}$$

$$\Rightarrow \boxed{\begin{aligned} V_u &= \Delta e^{\delta h} S_u + B e^{rh} \\ V_d &= \Delta e^{\delta h} S_d + B e^{rh} \end{aligned}} \Longrightarrow \boxed{V(0) = \Delta S(0) + B}$$

NO ARBITRAGE?

Method: $\boxed{\text{PRICING BY REPLICATION}}$

$$\Rightarrow V_u - V_d = \Delta e^{\delta h}(S_u - S_d) \Rightarrow \Delta = \underbrace{e^{-\delta h}} \cdot \frac{V_u - V_d}{S_u - S_d}$$

$$\Rightarrow B e^{rh} = V_u - \Delta e^{\delta h} S_u = \dots$$

$$\dots = \frac{uV_d - dV_u}{u-d} \Rightarrow \boxed{B = e^{-rh} \cdot \frac{uV_d - dV_u}{u-d}}$$

$$V(0) = \Delta S(0) + B$$

$$= e^{-\delta h} \cdot \frac{V_u - V_d}{\underbrace{S_u - S_d}_{\cancel{S(0)}(u-d)}} \cdot \cancel{S(0)} + e^{-rh} \cdot \frac{uV_d - dV_u}{u-d}$$

$$= \frac{1}{u-d}\left[e^{-\delta h}(V_u - V_d) + e^{-rh}(uV_d - dV_u)\right]$$

$$= e^{-rh} \cdot \frac{1}{u-d}\left[e^{(r-\delta)h}(V_u - V_d) + uV_d - dV_u\right]$$

$$= e^{-rh} \cdot \frac{1}{u-d}\left[V_u\left(e^{(r-\delta)h} - d\right) + V_d\left(u - e^{(r-\delta)h}\right)\right]$$

$$= e^{-rh}\left[V_u \cdot \boxed{\frac{e^{(r-\delta)h} - d}{u-d}} + V_d \cdot \boxed{\frac{u - e^{(r-\delta)h}}{u-d}}\right]$$

Both positive !!!
ADD UP TO 1 !!!

They can be interpreted as a Probability measure on the possible states of the world.

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d}$$ ... the RISK-NEUTRAL PROBABILITY of the stock price moving up.

EXPECTED VALUE

$$V(0) = e^{-rT}\left[p^* \cdot V_u + (1-p^*)V_d\right] = e^{-rT}\,\mathbb{E}^*[V(T)]$$

come together!

$$\boxed{\text{The risk-neutral pricing formula.}}$$

14. For a one-year straddle on a nondividend-paying stock, you are given:

(i) The straddle can only be exercised at the end of one year.

(ii) The payoff of the straddle is the absolute value of the difference between the strike price and the stock price at expiration date. $V(T) = |S(T) - 50|$

(iii) The stock currently sells for \$60.00. $S(0)$

(iv) The continuously compounded risk-free interest rate is 8%.

(v) In one year, the stock will either sell for \$70.00 or \$45.00.

(vi) The option has a strike price of \$50.00.

Calculate the current price of the straddle.

(A) \$0.90

(B) \$4.80

(C) \$9.30

(D) \$14.80

(E) \$15.70

1st Payoff

$S(0) = 60$ — $S_u = 70$, $S_d = 45$

$V_u = |70 - 50| = 20$

$V_d = |45 - 50| = 5$

2nd Risk-neutral probability

$$p^* = \frac{e^{(r-\delta)h} - d}{u-d} = \frac{S(0)e^{rh} - S_d}{S_u - S_d}$$

3rd Employ the risk-neutral pricing formula